CRYOGENIC ENGINEERING

Prof. Milind D. Atrey

Department of Mechanical Engineering,
IIT Bombay

Lecture No - 31

Earlier Lecture

- In the earlier lecture, we have seen a Pulse Tube (PT) cryocooler in which the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling.

- This gas tube is called as **Pulse Tube** and this phenomenon is called as **Pulse Tube action**.

- PT systems can be classified based on the
  - Stirling type or GM type
  - Geometry and Operating frequency
  - Phase shift mechanism

Earlier Lecture

- In the year 1990, Ray Radebaugh of NIST, proposed Phasor analysis of a Pulse Tube Cryocooler.

- There exists a phase angle between the mass flow rate at the cold end and the pressure vector.

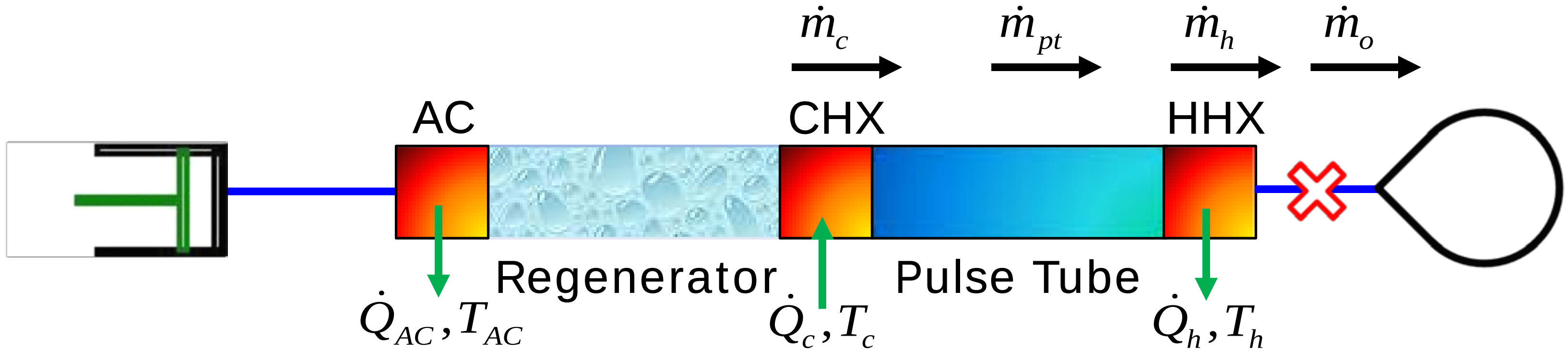
- This phase angle depends on the dimensions, frequency, $\mathbf{p_1}$ and the other operating parameters.

Outline of the Lecture

Topic : Cryocoolers

- Phasor Analysis (contd)
- Phasor Diagrams
- PT Classification based on Phase Shift Mechanism

Introduction



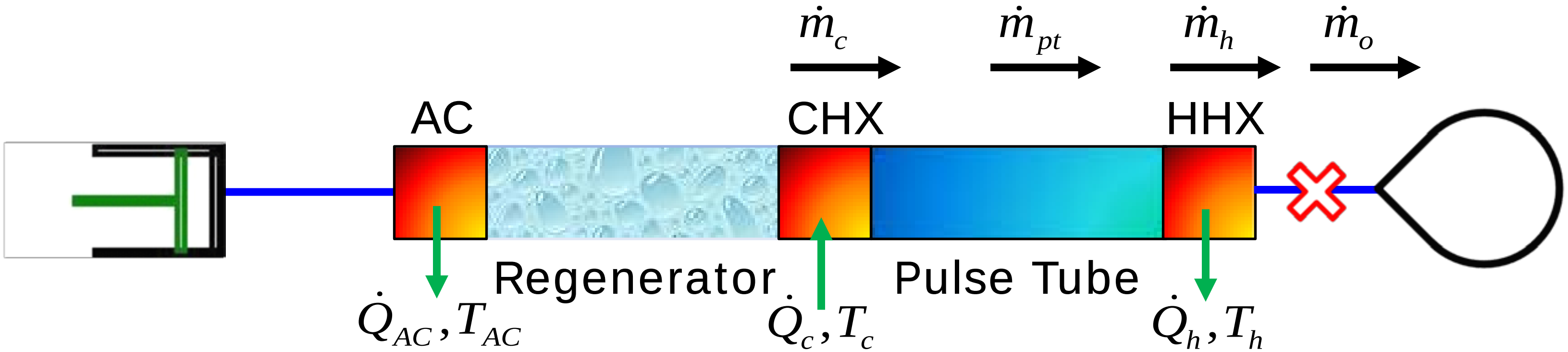
- In the earlier lecture, Phasor analysis of an Orifice Pulse Tube Cryocooler (OPTC) working on a monatomic gas was explained.

- The pressure (**p**) and the temperature (**T**) variations in the PT are assumed to be sinusoidal.

$$p = p_0 + p_1 \cos(\omega t)$$

$$T = T_0 + T_1 \cos(\omega t)$$

Introduction



- At any cross section of the Pulse Tube, using the adiabatic law, we have

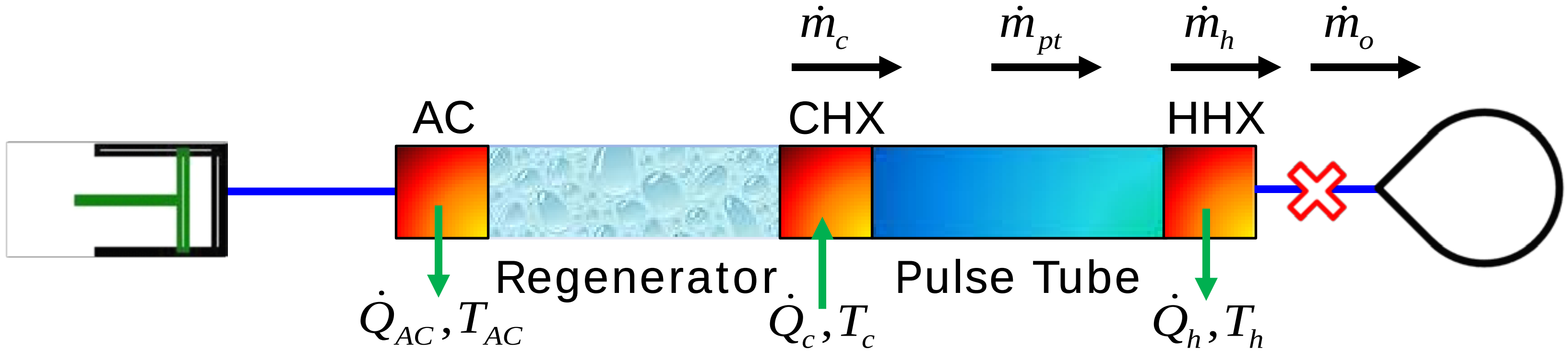
$$\frac{T_1}{T_0} = \frac{2}{5}\left(\frac{p_1}{p_0}\right)$$

- Mass flow rates at the Hot end and the Orifice are equal.

$$\dot{m}_h = \dot{m}_o$$

- Mass flow rate through the orifice is $\dot{m}_h = \boxed{C_1} p_1 \cos(\omega t)$

Introduction

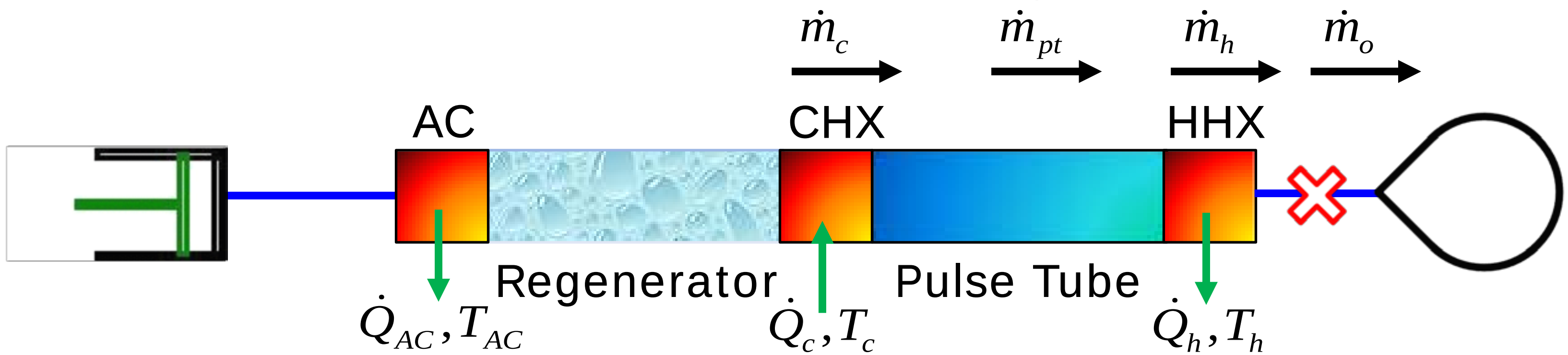


- The mass flow rate at the cold end (**m_c**) is as given below.

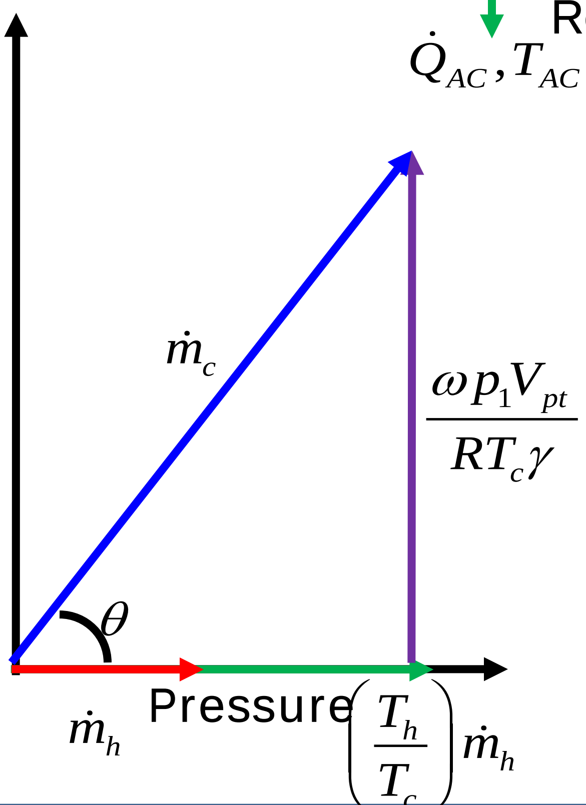
$$\dot{m}_c = \boxed{\frac{\omega p_1 V_{pt}}{\gamma R T_c}} \cos\left(\omega t + \boxed{\frac{\pi}{2}}\right) + \boxed{\frac{T_h}{T_c} C_1 p_1} \cos\left(\omega t\right)$$

- It is clear that vector **m_c** is a sum of two vectors which are at **90°** to each other.

Phasor Analysis



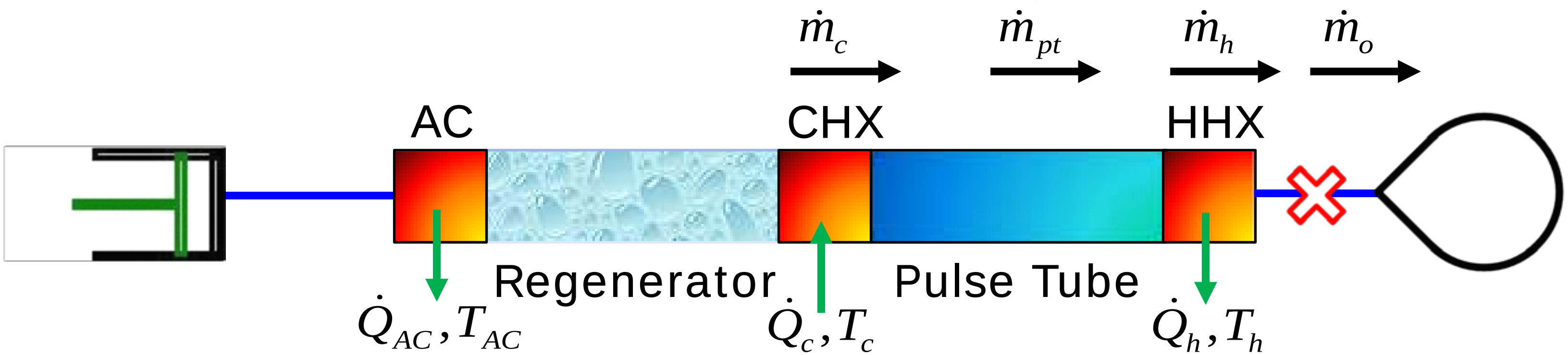
$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} \dot{m}_h$$



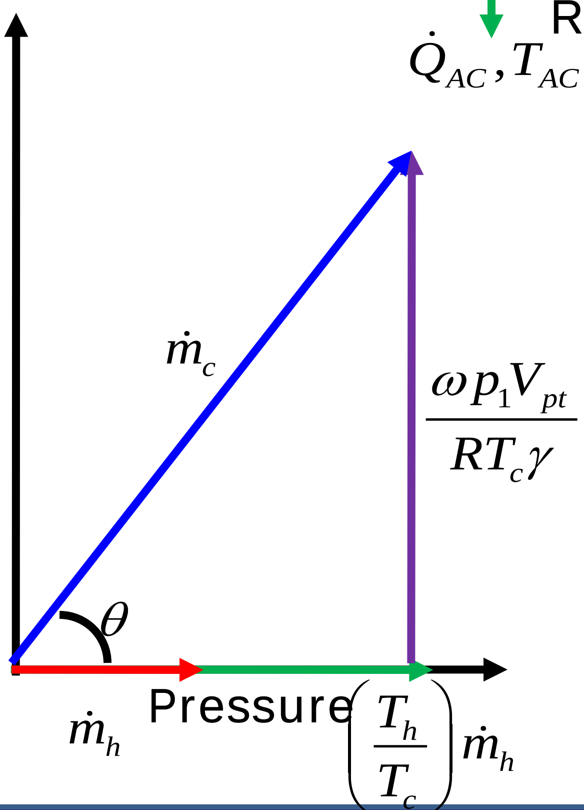
- Plotting these two vectors, we have the figure as shown.

- This diagram is called as the **Phasor diagram** of Pulse Tube.

Phasor Analysis



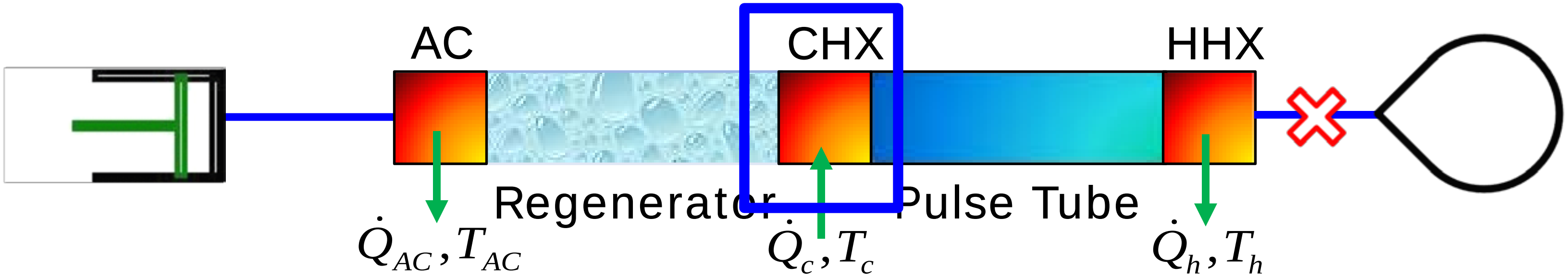
$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} \dot{m}_h$$



- It is clear that there exists a phase angle between **m$_c$** and pressure vector.

- The importance of phase angle is explained in this lecture.

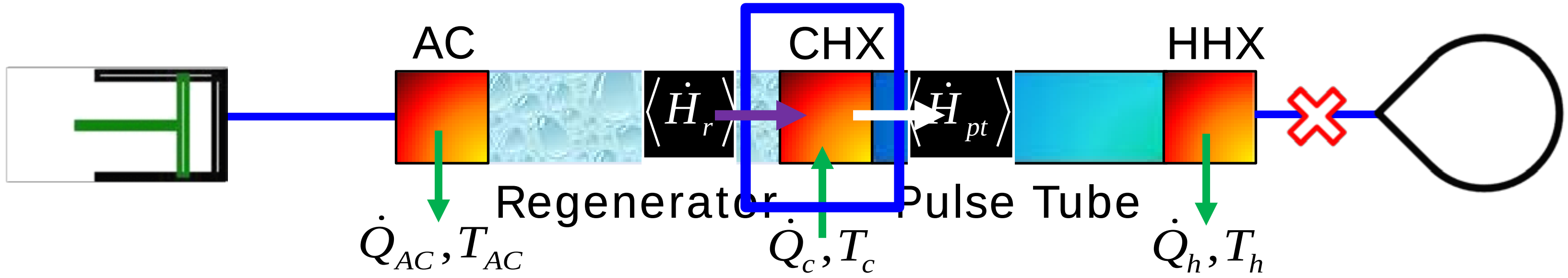
Phasor Analysis



- Going ahead with the analysis, consider a control volume enclosing the Cold End Heat Exchanger (CHX) as shown in the figure.

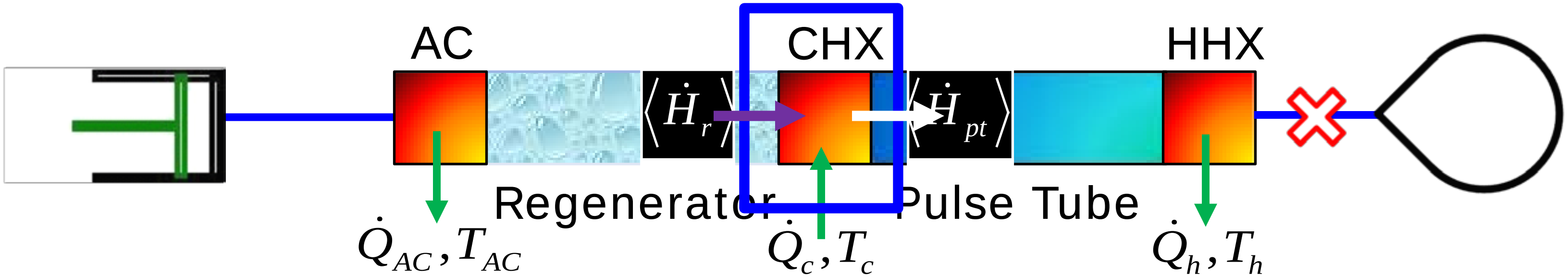
- Let **<H>** and **<Q>** denote the time averaged enthalpy flow and heat flow respectively, across the control volume.

Phasor Analysis



- Let us assume the following.
  - **<H$_r$>** be the average enthalpy flow in the Regenerator.
  - **<H$_{pt}$>** be the average enthalpy flow in the Pulse Tube.
  - **<Q$_c$>** be the average heat flow or the heat lifted at the Cold End (refrigeration effect).

Phasor Analysis



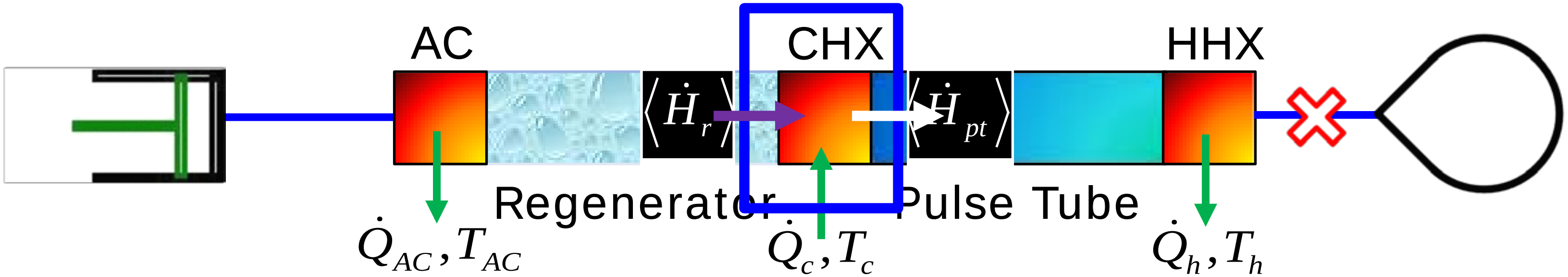
- Applying **1st** law of thermodynamics to this control volume, we have

$$\dot{Q}_c = \langle \dot{H}_{pt} \rangle - \langle \dot{H}_r \rangle$$

- If the net heat energy stored by the regenerator in a cycle is zero, it is called as perfect regeneration.

- That is, the heat energy lost by the gas during pressurization is same as the heat energy gained by the gas during depressurization.

Phasor Analysis



- Applying **1st** law of thermodynamics to this control volume, we have

$$\dot{Q}_c = \langle \dot{H}_{pt} \rangle - \cancel{\langle \dot{H}_r \rangle}$$

- Assuming a perfect regeneration in the regenerator, we have

$$\langle \dot{H}_r \rangle = 0$$

- Therefore, the heat lifted (**Q_c**) is the enthalpy flow into the Pulse Tube.

$$\dot{Q}_c = \langle \dot{H}_{pt} \rangle$$

Phasor Analysis

- At any cross section of the OPTC, the definition of enthalpy is as given below. $\dot{H} = \dot{m} C_p T$

- Let **τ** be the total time period of one cycle. Let **C_p** be a constant and the time average enthalpy is

$$\langle \dot{H} \rangle = \frac{C_p}{\tau} \int_0^{\tau} \dot{m} \boxed{T} dt \qquad \boxed{T} = T_0 + T_1 \cos(\omega t)$$

- Substituting, temperature **T** into the above equation, we get

$$\langle \dot{H} \rangle = \frac{C_p}{\tau} \int_0^{\tau} \dot{m} \left(T_0 + T_1 \cos\left(\omega t\right)\right) dt$$

Phasor Analysis

$$\langle \dot{H} \rangle = \frac{C_p}{\tau}\int_0^{\tau} \dot{m}\left(T_0 + \boxed{T_1}\cos(\omega t)\right)dt$$

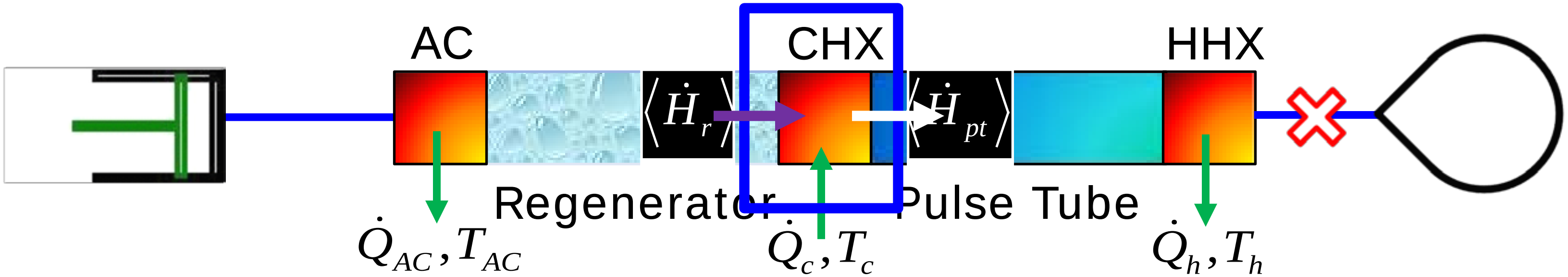
- From the adiabatic law, we have $\frac{\boxed{T_1}}{T_0} = \frac{2}{5}\left(\frac{p_1}{p_0}\right)$

- Eliminating $\mathbf{T_1}$ from both the above equations, we get

$$\langle \dot{H} \rangle = \frac{C_p}{\tau}\int_0^{\tau} \dot{m}\left(T_0 + \frac{2}{5}\left(\frac{p_1}{p_0}\right)T_0\cos(\omega t)\right)dt$$

$$\langle \dot{H} \rangle = \cancel{\frac{C_p}{\tau}\int_0^{\tau} \dot{m}T_0\,dt} + \frac{C_p}{\tau}\int_0^{\tau} \frac{2\dot{m}}{5}\left(\frac{p_1}{p_0}\right)T_0\cos(\omega t)\,dt$$

Phasor Analysis



- From the **1st** law, the heat lifted (**Q_c**) at the cold end of the PT is the enthalpy flow of the PT.

$$\dot{Q}_c = \left\langle \dot{H}_{pt} \right\rangle$$

- This is obtained by substituting **m_c** and **T_c** in the following equation.

$$\left\langle \dot{H} \right\rangle = \frac{C_p}{\tau} \int_0^{\tau} \frac{2\boxed{\dot{m}}}{5} \left(\frac{p_1}{p_0} \right) \boxed{T_0} \cos(\omega t)\, dt$$

Phasor Analysis

$$\left\langle \dot{H} \right\rangle = \frac{2T_c C_p p_1}{5\tau p_0} \int_0^{\tau} \boxed{\dot{m}_c} \cos\left(\omega t\right) dt$$

$$\boxed{\dot{m}_c} = -\frac{\omega p_1 V_{pt}}{\gamma R T_c} \sin\left(\omega t\right) + \frac{T_h}{T_c} C_1 p_1 \cos\left(\omega t\right)$$

$$\left\langle \dot{H} \right\rangle = \frac{2T_c C_p p_1}{5\tau p_0} \left[\cancel{-\frac{\omega p_1 V_{pt}}{2\gamma R T_c} \int_0^{\tau} \sin\left(2\omega t\right) dt} + \frac{T_h}{T_c} C_1 p_1 \int_0^{\tau} \cos^2\left(\omega t\right) dt \right]$$

- In the above equation, the first term is zero. This is because, the sine function when integrated over one full cycle vanishes.

Phasor Analysis

$$\left\langle \dot{H} \right\rangle = \frac{\cancel{2T_c} C_p p_1}{5\tau p_0}\left[\frac{C_1 p_1 T_h}{\cancel{2T_c}}\int_0^\tau \left(1+\cos\left(2\omega t\right)\right)dt\right]$$

- Rearranging, we have

$$\left\langle \dot{H} \right\rangle = \frac{C_1 C_p T_h p_1^2}{5\tau p_0}\left[\int_0^\tau dt + \cancel{\int_0^\tau \cos\left(2\omega t\right)dt}\right]$$

- Here again, the second term is zero. This is because, the cosine function when integrated over one full cycle vanishes.

$$\left\langle \dot{H} \right\rangle = \frac{C_1 C_p T_h p_1^2}{5\cancel{\tau} p_0}\left[\cancel{\tau}\right] = \frac{C_1 C_p T_h p_1^2}{5 p_0}$$

Phasor Analysis

- Therefore, the time averaged enthalpy is given as

$$\left\langle \dot{H} \right\rangle = \frac{C_1 C_p T_h p_1^2}{5 p_0}$$

- We know that the mass flow rate at the hot end (**m$_h$**) is a vector and it is as given below.

$$\dot{m}_h = C_1 p_1 \cos\left(\omega t\right)$$

- The magnitude of this vector is given by

$$\left|\dot{m}_h\right| = C_1 p_1$$

Phasor Analysis

- Using **<H>** and **$|m_h|$** equations, we have.

$$|\dot{m}_h| = C_1 p_1$$

$$\langle \dot{H} \rangle = \frac{\boxed{C_1 p_1} C_p T_h p_1}{5 p_0}$$

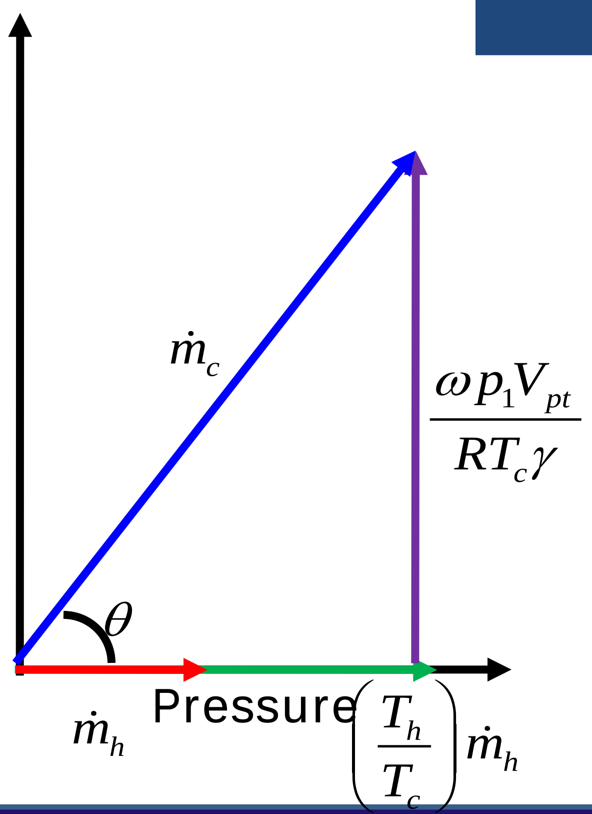
$$\langle \dot{H} \rangle = \frac{\boxed{|\dot{m}_h|} C_p T_h p_1}{5 p_0}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{T_h |\dot{m}_h|}{T_c |\dot{m}_c|}$$

$$\boxed{|\dot{m}_h|} = \frac{T_c |\dot{m}_c| \cos\theta}{T_h}$$

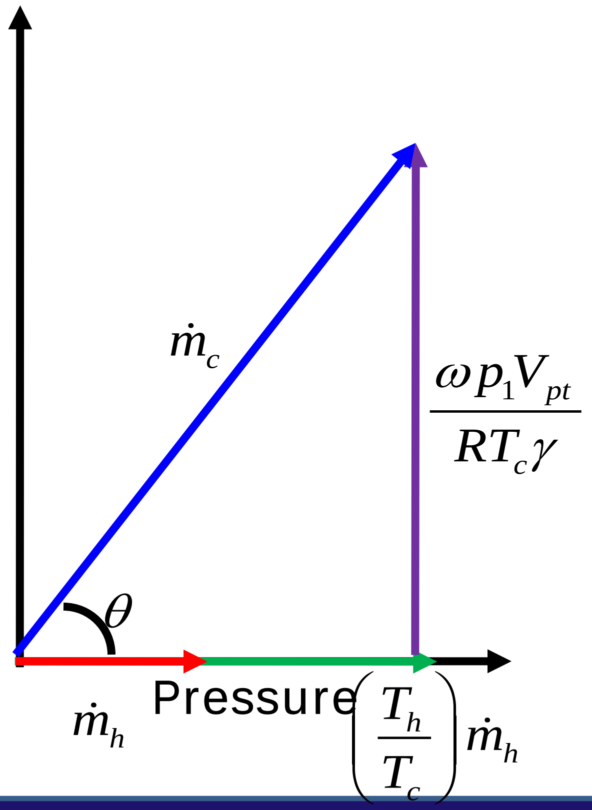
$$\langle \dot{H} \rangle = \left(\frac{T_c |\dot{m}_c| \cos\theta}{\cancel{T_h}} \right) \left(\frac{C_p \cancel{T_h} p_1}{5 p_0} \right)$$

$$\langle \dot{H} \rangle = \left(\frac{C_p}{5} \right) \left(\frac{p_1}{p_0} \right) T_c |\dot{m}_c| \cos\theta$$



Phasor Analysis

- For a monatomic gas, **γ = 5 / 3**. Therefore, the specific heat at constant pressure (**C_p**) is given by the following equation.



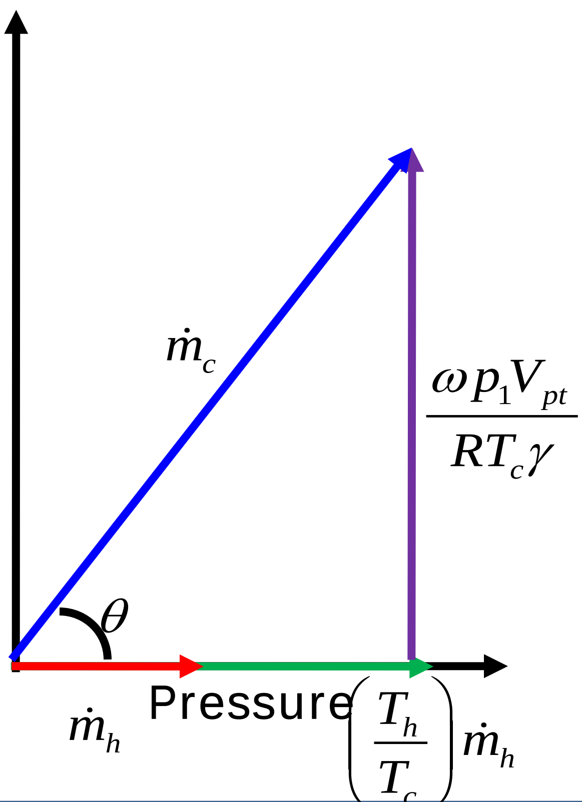
$$\boxed{C_p} = \frac{\Re\gamma}{\gamma - 1} = \frac{5\Re}{2}$$

$$\langle \dot{H} \rangle = \left(\frac{\boxed{C_p}}{5}\right)\left(\frac{p_1}{p_0}\right) T_c \left|\dot{m}_c\right| \cos\theta$$

$$\dot{Q}_c = \langle \dot{H} \rangle = \frac{1}{2}\Re T_c \left(\frac{p_1}{p_0}\right)\left|\dot{m}_c\right| \cos\theta$$

Phasor Analysis

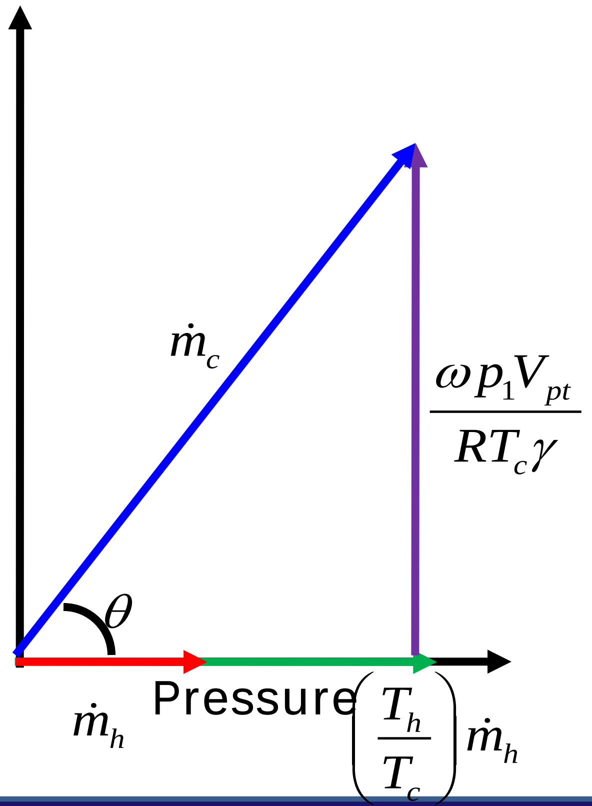
$$\dot{Q}_c = \left(\frac{\Re T_c p_1 |\dot{m}_c|}{2 p_0} \right) \cos\theta$$



- From the adjacent equation, it is clear that the heat lifted at the cold end (**Q_c**) is dependent on
  - **$|m_c|$**
  - **p_1 / p_0**
  - **T_c**
  - phase angle.
- For a given design and operating parameters, the **Q_c** is maximum when phase angle is minimum.

Phasor Analysis

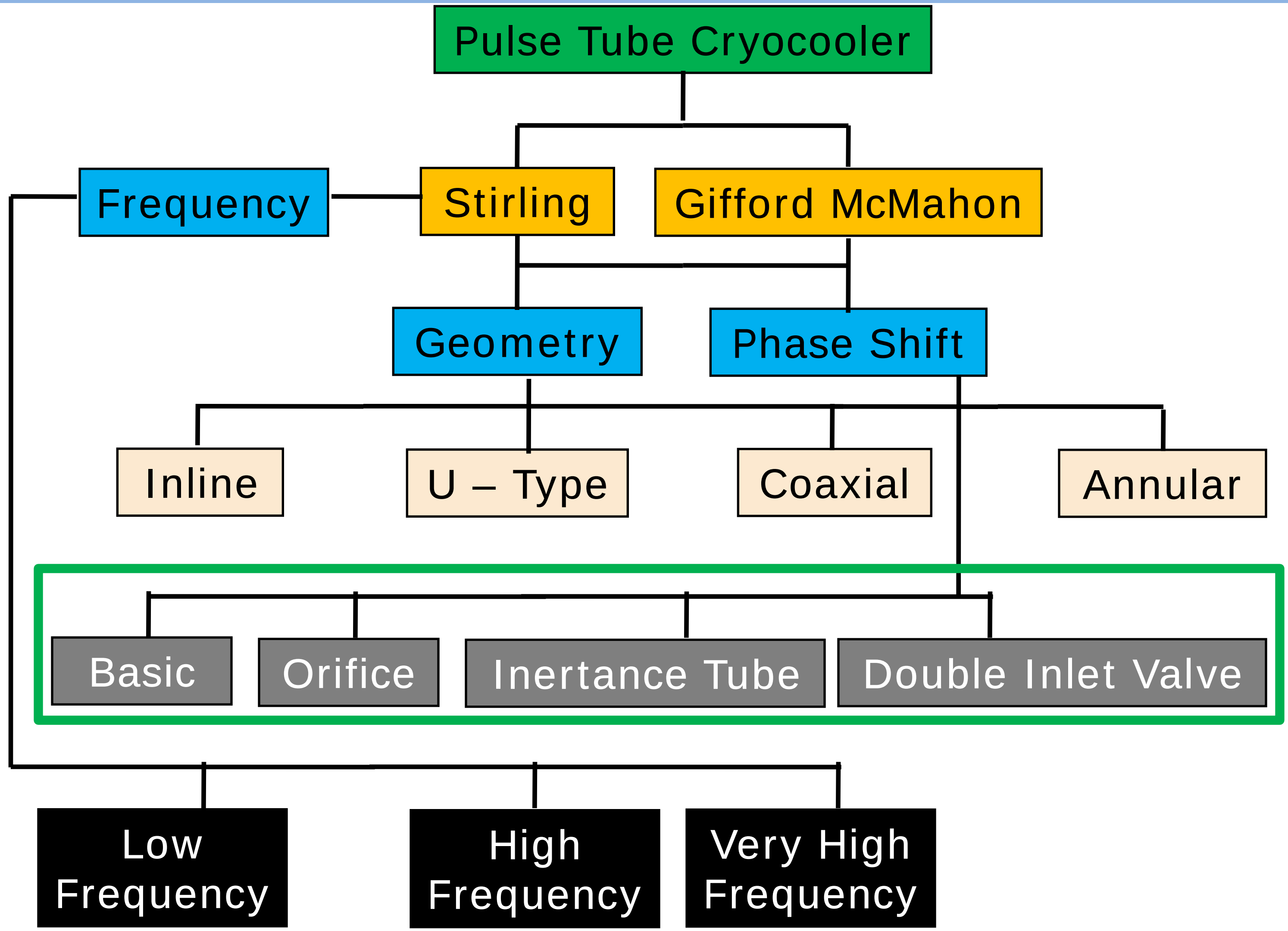
$$\dot{Q}_c = \left(\frac{\Re T_c p_1 \left| \dot{m}_c \right|}{2 p_0} \right) \cos\theta$$



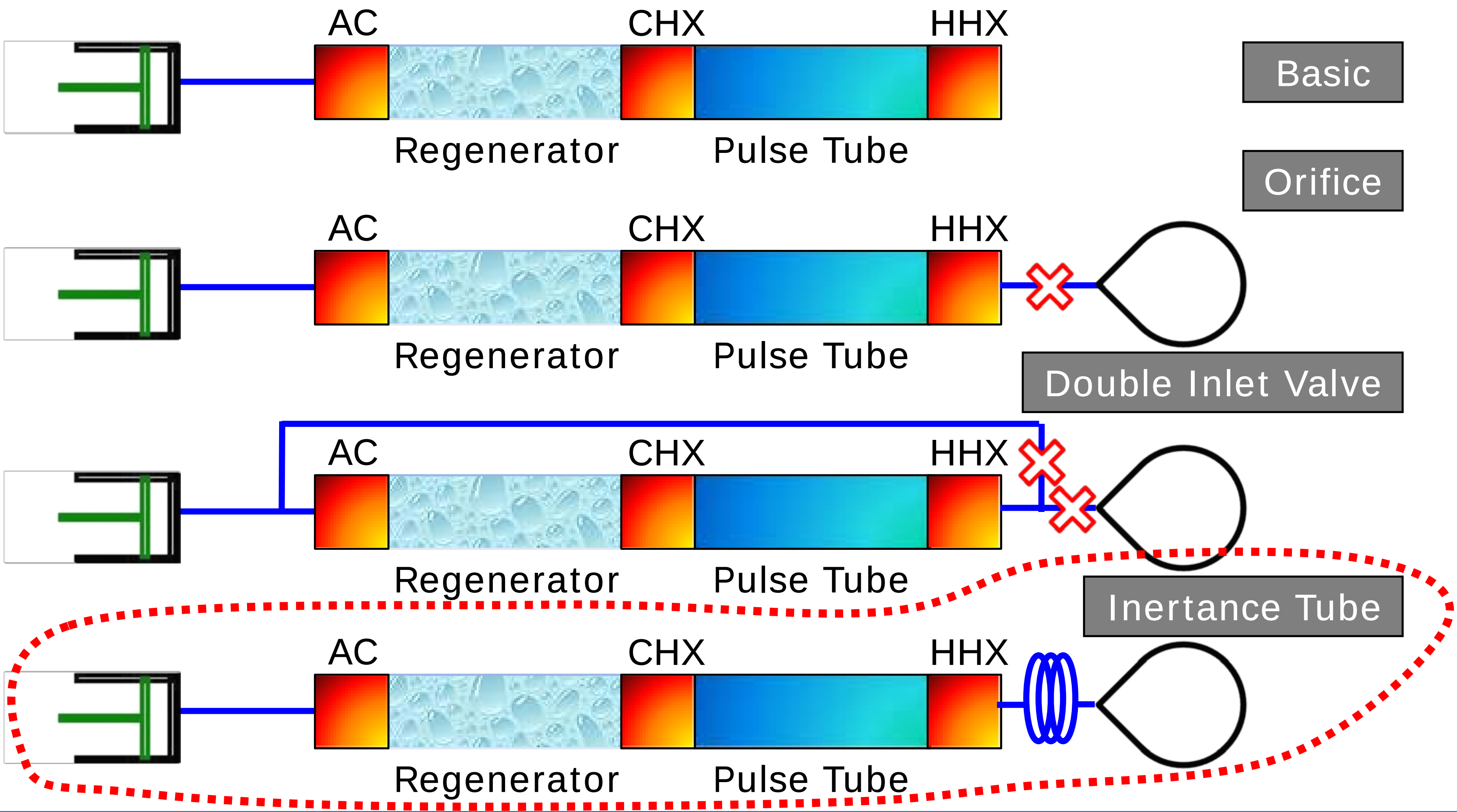
- Various phase shifting mechanisms have been developed in order to minimize the phase angle.

- It is important to note that the phase angle can be minimized, and in certain cases it can be made zero.

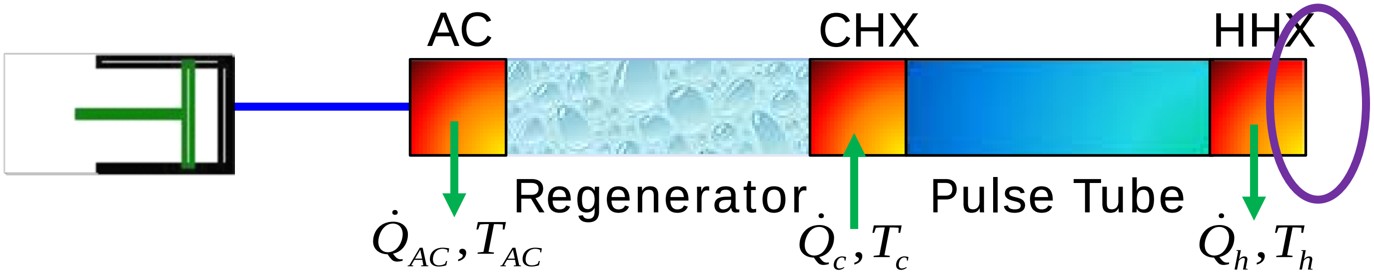
PT Classification



PT Classification



Phasor Diagram – BPTC

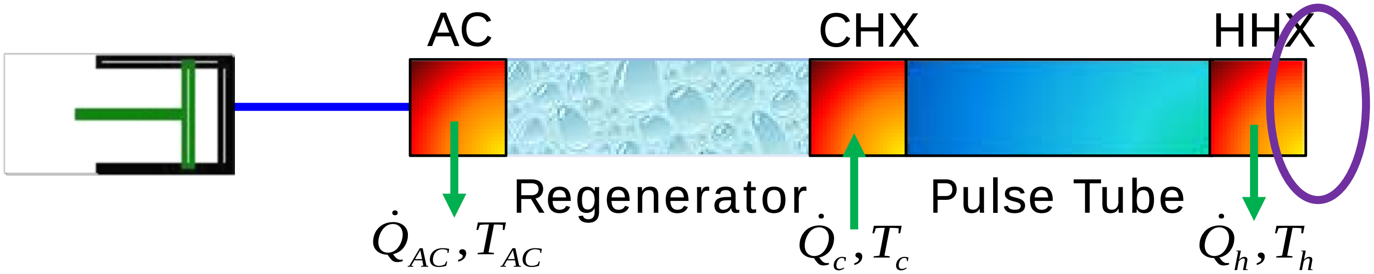


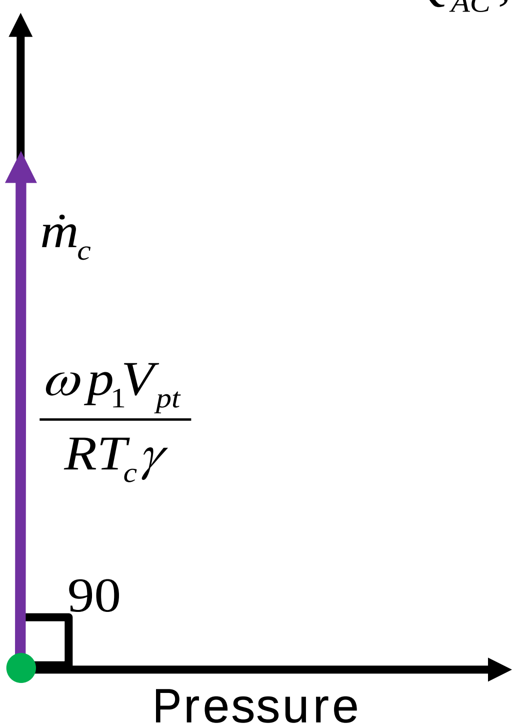
- The schematic of a Basic Pulse Tube Cryocooler (BPTC) is as shown above.

$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} \cancel{\dot{m}_h}$$

- The mass flow rate at the Hot end heat exchanger is zero.

Phasor Diagram – BPTC



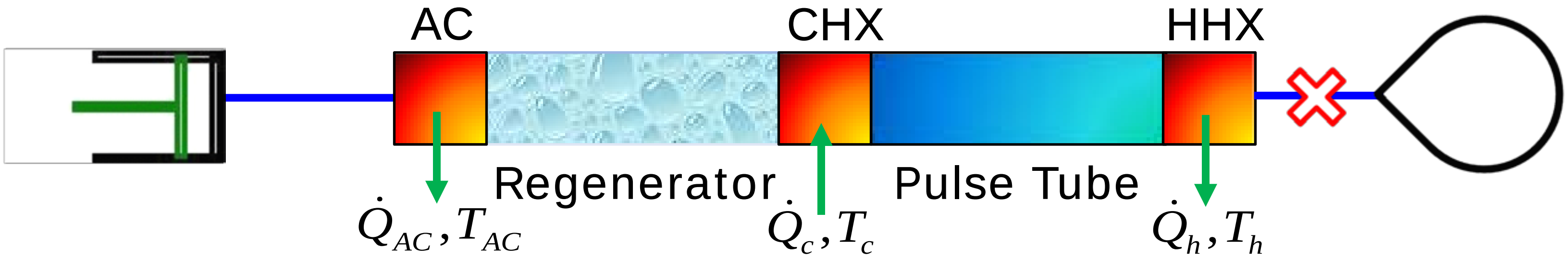


- The Phasor diagram for a BPTC is as shown in the figure.

$$\dot{Q}_c = \frac{1}{2} \Re T_c \left(\frac{p_1}{p_0} \right) \left| \dot{m}_c \right| \cancel{\cos} \theta$$

- It is clear that the phase angle is **90º**, rendering the net heat lifted at the cold end as zero.

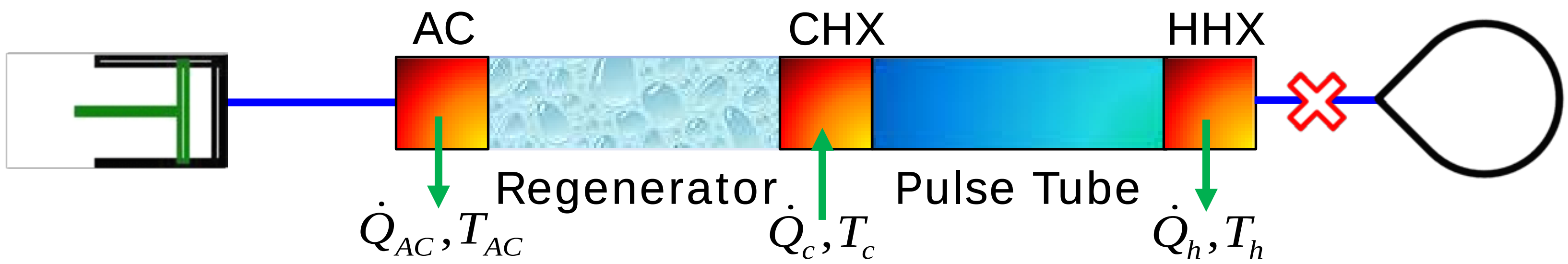
Phasor Diagram – OPTC

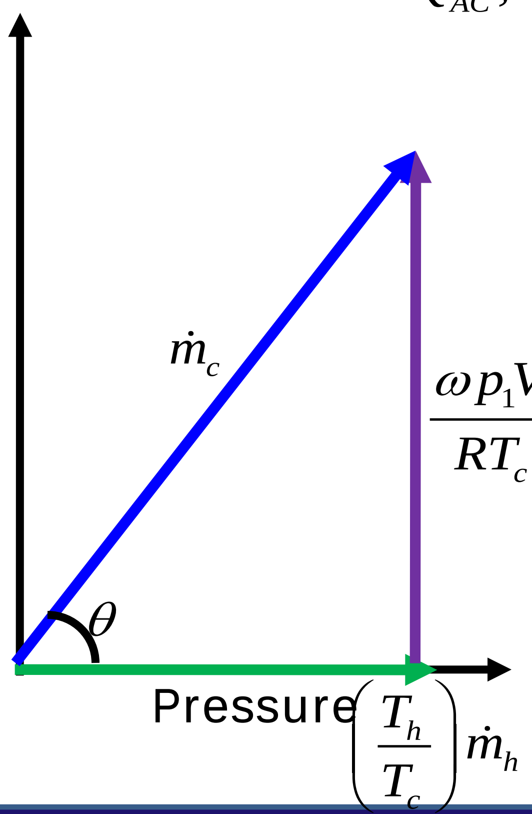


- As seen earlier, in an Orifice Pulse Tube Cryocooler (OPTC), an orifice and a compliance volume is used as phase shift mechanism.

- There exists a finite mass flow rate at the Hot end.

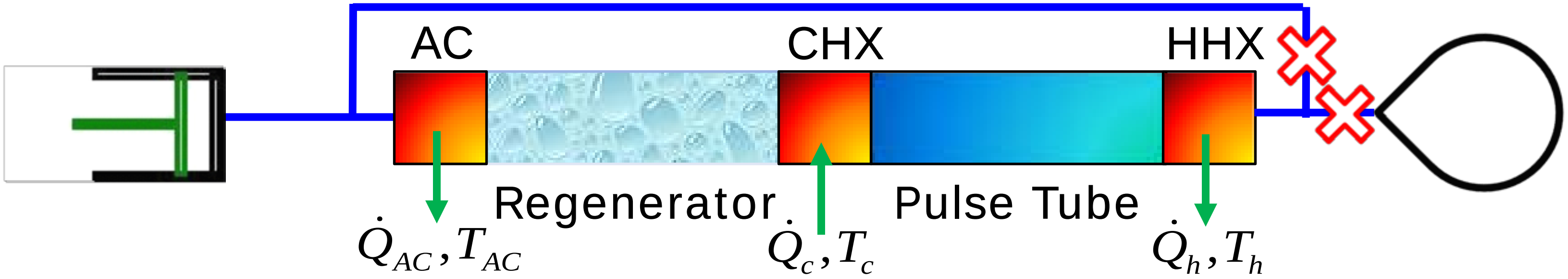
Phasor Diagram – OPTC





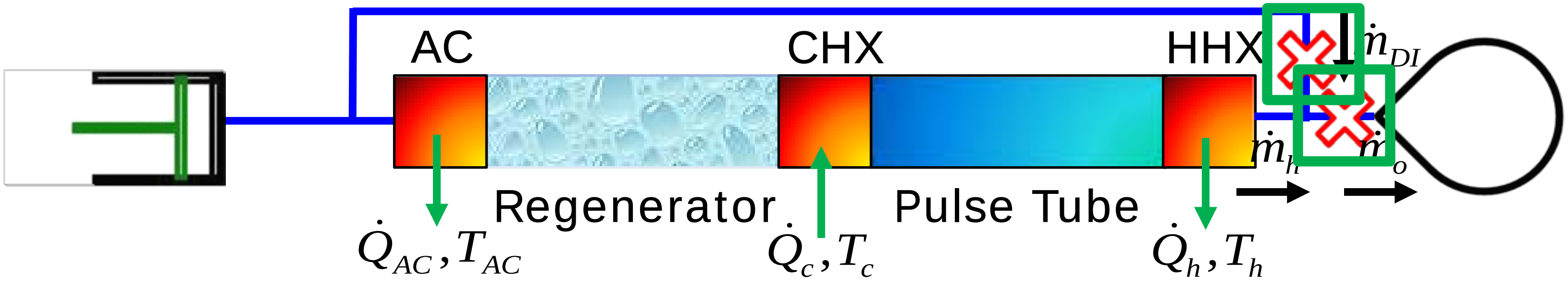
- The Phasor diagram for an OPTC is as shown in the figure.
- The relative opening or closing of the orifice alters $|\mathbf{m_c}|\cos\boldsymbol{\theta}$.
- For an optimum orifice opening, the product $|\mathbf{m_c}|\cos\boldsymbol{\theta}$ reaches a maximum to maximize $\mathbf{Q_c}$.

Phasor Diagram – DIPTC



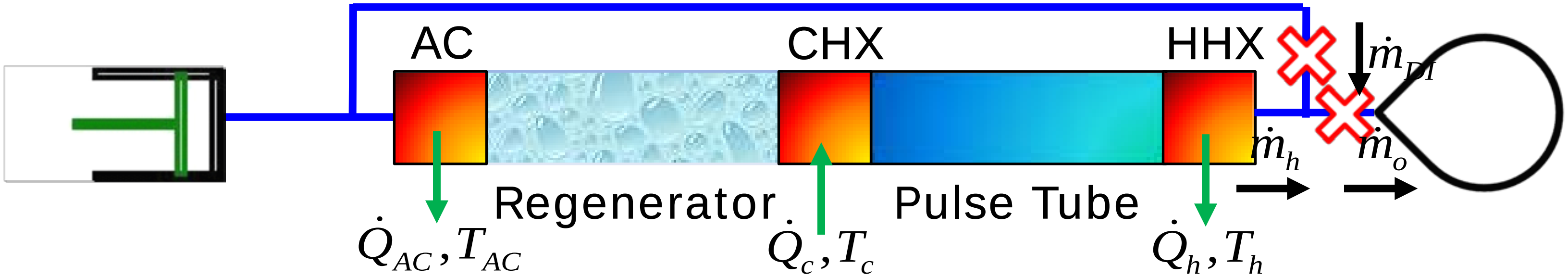
- The schematic of a Double Inlet Pulse Tube Cryocooler (DIPTC) is as shown in the figure.
- Some portion of gas is bypassed from the After Cooler and is fed at the Hot end Heat Exchanger.
- This is called as Double Inlet Line.

Phasor Diagram – DIPTC



- On the double inlet line, an orifice is used to control/regulate the flow of working fluid.
- This double inlet orifice together with an Hot end orifice alter the phase angle.
- Mass flow rates are as shown.

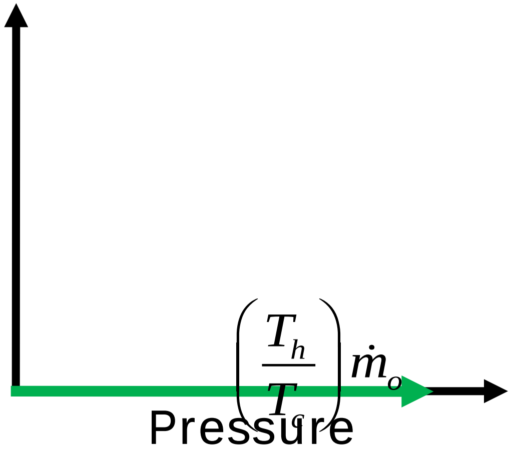
Phasor Diagram – DIPTC



- For the mass flow rates, we have

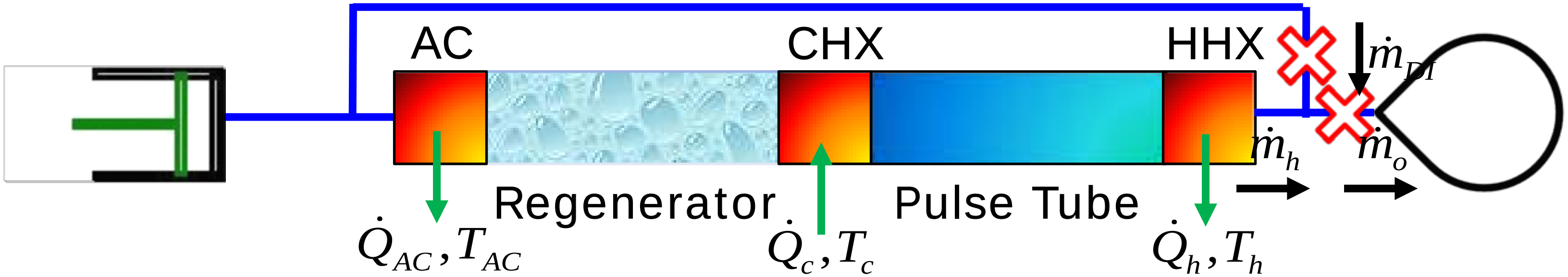
$$\dot{m}_o = \dot{m}_h + \dot{m}_{DI}$$

$$\left(\frac{T_h}{T_c}\right)\dot{m}_o = \left(\frac{T_h}{T_c}\right)\dot{m}_h + \left(\frac{T_h}{T_c}\right)\dot{m}_{DI}$$

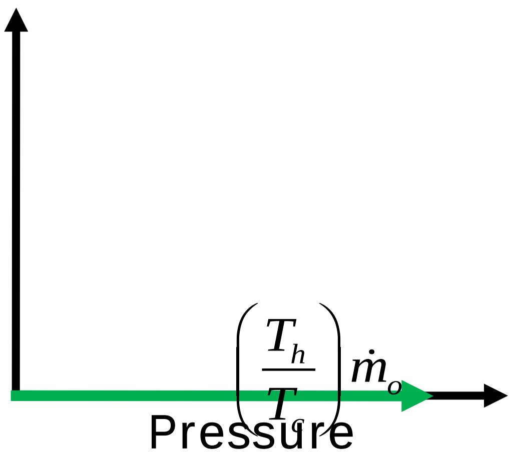


- It implies that **m$_o$** is the vector sum of **m$_h$** and **m$_{DI}$**.

Phasor Diagram – DIPTC

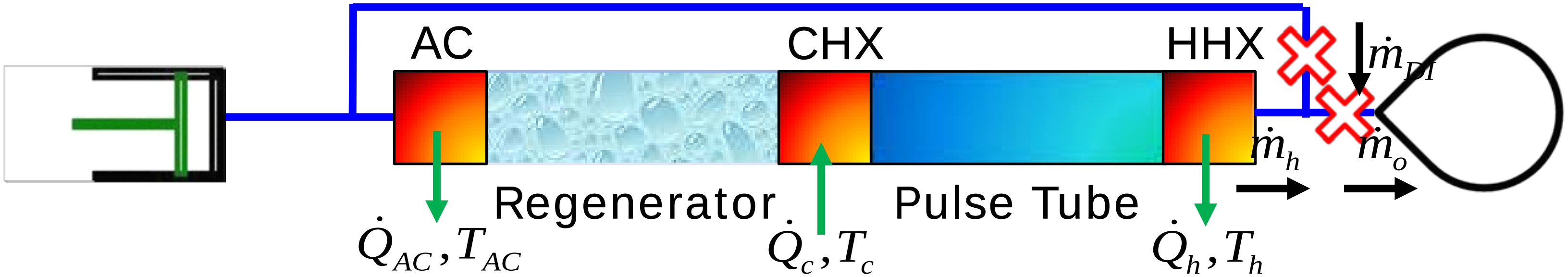


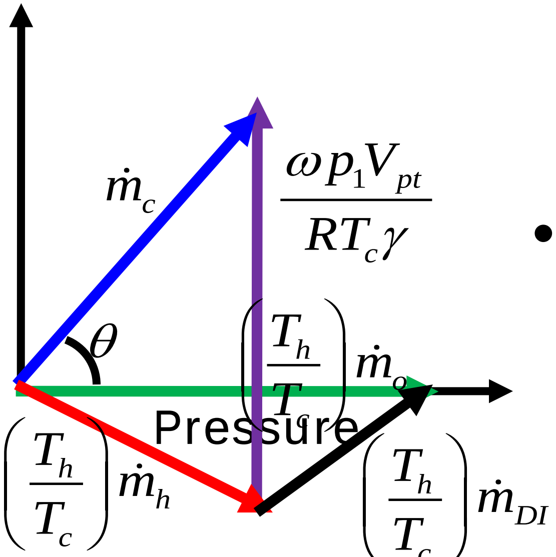
$$\left(\frac{T_h}{T_c}\right)\dot{m}_o = \left(\frac{T_h}{T_c}\right)\dot{m}_h + \left(\frac{T_h}{T_c}\right)\dot{m}_{DI}$$



- We know that **m$_o$** is always in phase with the pressure vector.
- The phasor diagram for this vector is as shown in the figure.

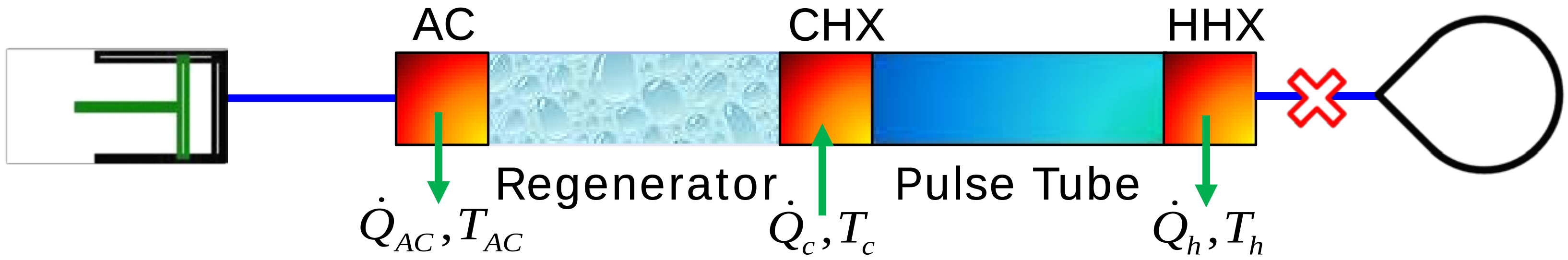
Phasor Diagram – DIPTC





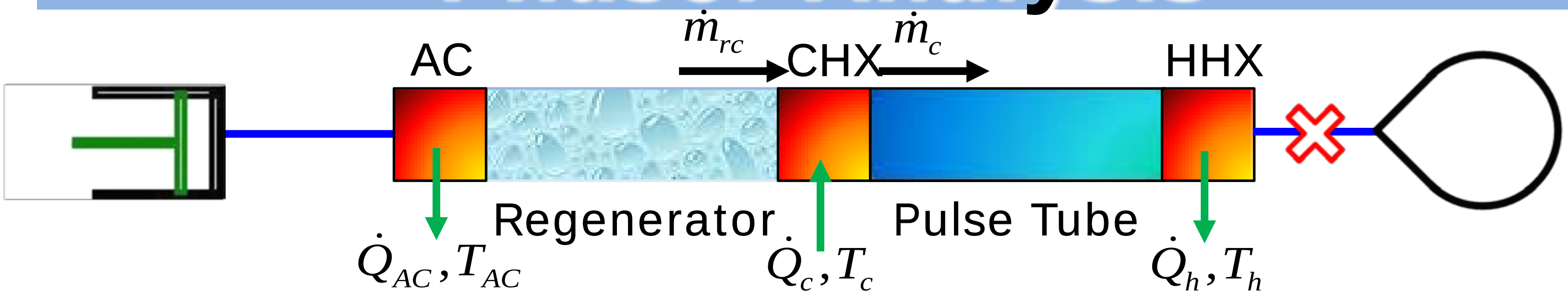
- **m_h** and **m_{DI}** are vectorially added to yield **m_o** as shown in the figure.
- From the equation of **m_c**, we have the other sides of the triangle, as shown in the figure.
- It is clear that the phase angle is reduced due to the modification.

Phasor Analysis



- In the phasor analysis, we have assumed an adiabatic process in the Pulse Tube.
- In the other elements, for example in connecting tubes, **AC**, **CHX**, **HHX**, an isothermal process is assumed.
- For the sake of understanding, let us analyze the Cold End Heat Exchanger in an OPTC.

Phasor Analysis



- As done before, **p** and **T** are assumed as follows.

$$p = p_0 + p_1 \cos(\omega t) \qquad T = T_0$$

- Let **m$_{rc}$** and **m$_{c}$** be the mass flow rates at the inlet and outlet as shown. We have $\dot{m}_{chx} = \dot{m}_c - \dot{m}_{rc}$

- Following the earlier derivation, we have

$$\dot{m}_{rc} = -\frac{\omega p_1 V_{chx}}{R T_c} \sin(\omega t) + \dot{m}_c$$

Phasor Analysis

$$\dot{m}_{rc} = \frac{\omega p_1 V_{chx}}{RT_c}\cos\left(\omega t + \frac{\pi}{2}\right) + \dot{m}_c$$

$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma RT_c}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c}\dot{m}_h$$

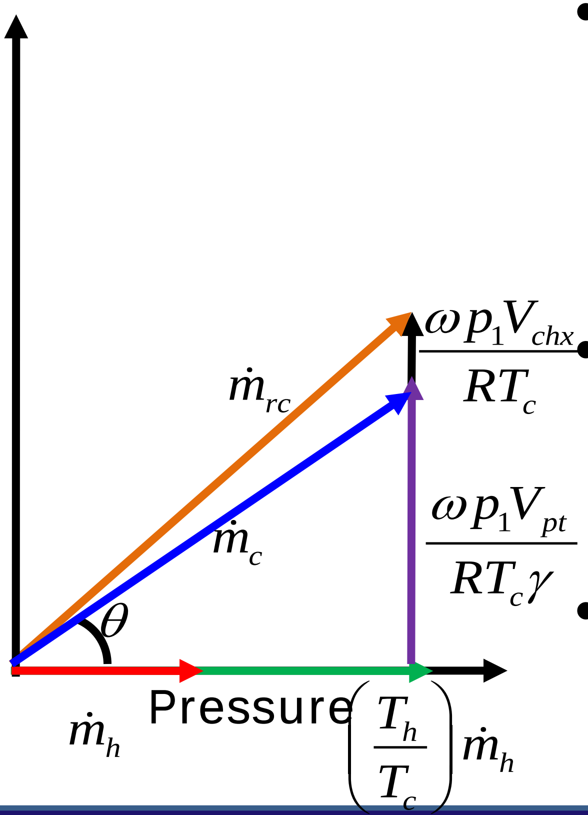
- Combining the above equations, we have

$$\dot{m}_{rc} = \left(\frac{\omega p_1 V_{chx}}{RT_c} + \frac{\omega p_1 V_{pt}}{\gamma RT_c}\right)\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c}\dot{m}_h$$

- Therefore, the mass flow rate (**$\mathbf{m_{rc}}$**) is the sum of two vectors which are at **90^{o}** to each other.

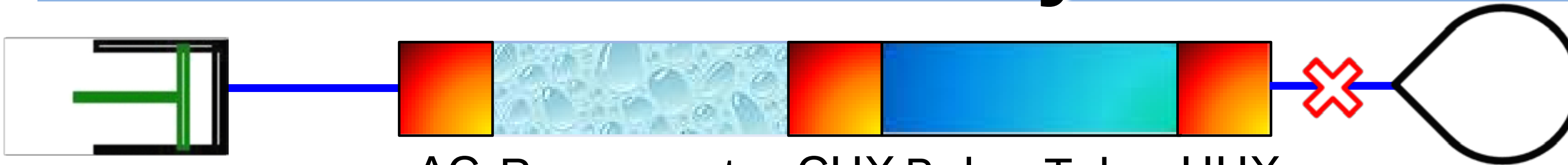
Phasor Analysis

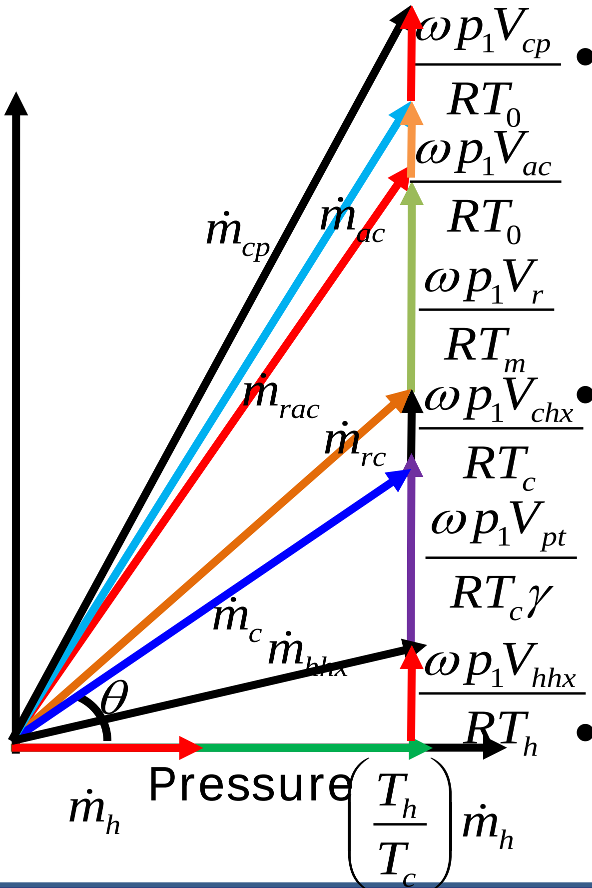
$$\dot{m}_{rc} = \left(\boxed{\frac{\omega p_1 V_{chx}}{RT_c} + \frac{\omega p_1 V_{pt}}{\gamma RT_c}} \right) \cos\left(\omega t + \frac{\pi}{2} \right) + \boxed{\frac{T_h}{T_c}} \dot{m}_h$$



- From the above equation, it is clear that the first term is at **90º** to the second term.
- Plotting these two vectors, we have the figure as shown.
- $\mathbf{m_{rc}}$ lies above the $\mathbf{m_c}$ vector. This is due to the vector addition as shown in the figure.

Phasor Analysis





- Following a similar procedure for the rest of the elements, we have a phasor diagram as shown.
- From the figure, it is clear that $\mathbf{m_{cp}}$ vector is almost double the length of $\mathbf{m_c}$ vector.
- Therefore, for a given $\mathbf{m_c}$, we need a very large $\mathbf{m_{cp}}$.

Summary

- There exists a phase angle between mass flow rate at the cold end and pressure vector.

$$\dot{Q}_c = \left(\frac{\Re T_c p_1 |\dot{m}_c|}{2 p_0} \right) \cos\theta$$

- Heat lifted at the cold end (**Q_c**) is dependent on **$|m_c|$**, **p_1 / p_0**, **T_c**, phase angle.

- In the phasor analysis, adiabatic process is assumed in PT and isothermal process is assumed in all other parts. The relative length of the vectors indicate the mass flow rate in those parts.

- A self assessment exercise is given after this slide.

- Kindly asses yourself for this lecture.

Self Assessment

1. In a perfect regeneration, the net heat energy stored by regenerator is ____.
2. For a given design and operating parameters, the $\mathbf{Q_c}$ is maximum when phase angle is ____.
3. The heat exchange during pressurization and depressurization is ____.
4. Phase shifting mechanisms are used to ____ the phase angle.
5. In a Basic PT Cryocooler, the phase angle is ___.
6. In Phasor Analysis, PT undergoes _______ process.
7. Apart from PT, all other parts undergo ___ process.
8. In ____, double inlet orifice and hot end orifice alter the phase angle.

Answers

1. Zero
2. Minimum
3. Same
4. Minimize
5. 90^o
6. Adiabatic
7. Isothermal
8. DIPTC

Thank You!