CRYOGENIC ENGINEERING

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Lecture No - 32

Earlier Lecture

- In the earlier lecture, we have seen the phasor analysis of an Orifice Pulse Tube Cryocooler.
- There exists a phase angle between the mass flow rate at the cold end and the pressure vector.
- Various phase shifting mechanisms have been developed in order to optimize this phase angle.
- The phasor diagrams of Basic, Orifice and Double Inlet Pulse Tube Cryocoolers are discussed.

Earlier Lecture

- Heat lifted at the cold end (**Q_c**) is dependent on **$|m_c|$**, **p_1 / p_0**, **T_c**, phase angle.

$$\dot{Q}_c = \left(\frac{\Re T_c p_1 |\dot{m}_c|}{2 p_0} \right) \cos\theta$$

- In the phasor analysis, an adiabatic process is assumed in PT and an isothermal process is assumed in other elements like connecting tubes, **AC**, **CHX** and **HHX**.

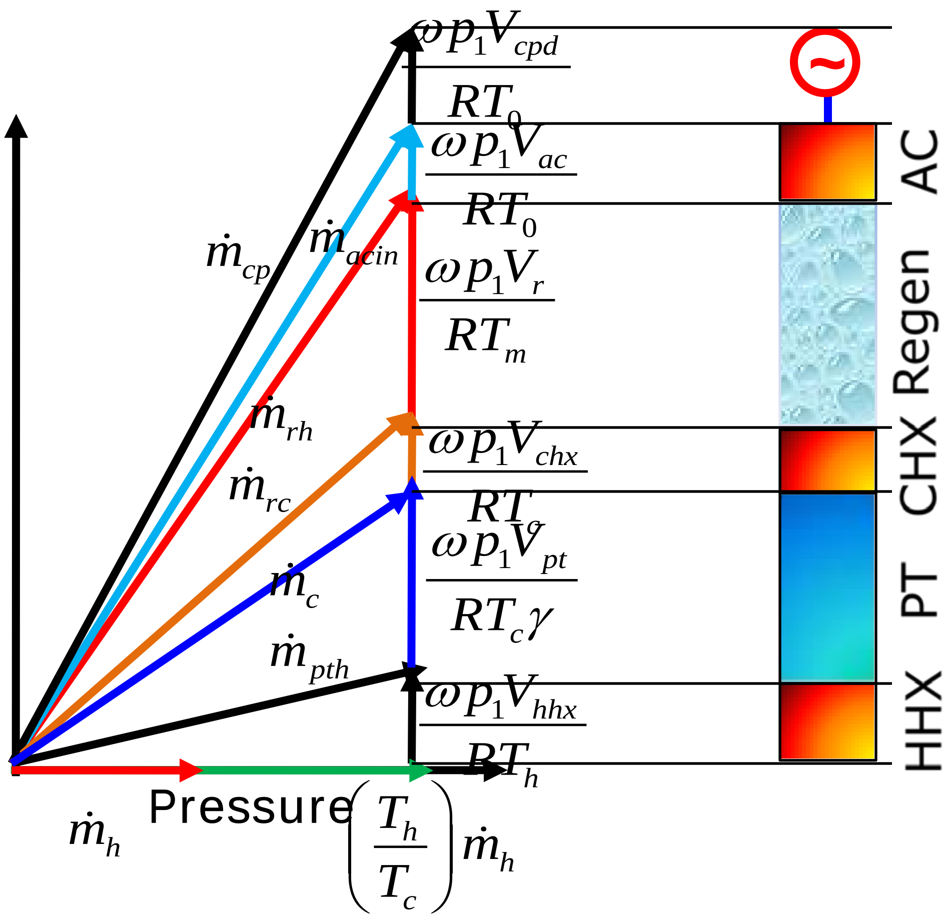
- In the phasor diagram, the relative length of the vectors indicate the mass flow rate in those parts.

Outline of the Lecture

Topic : Cryocoolers

- Phasor Analysis (contd)
- Electric Analogy
- Tutorial
- PTC research at IIT Bombay
- Conclusion

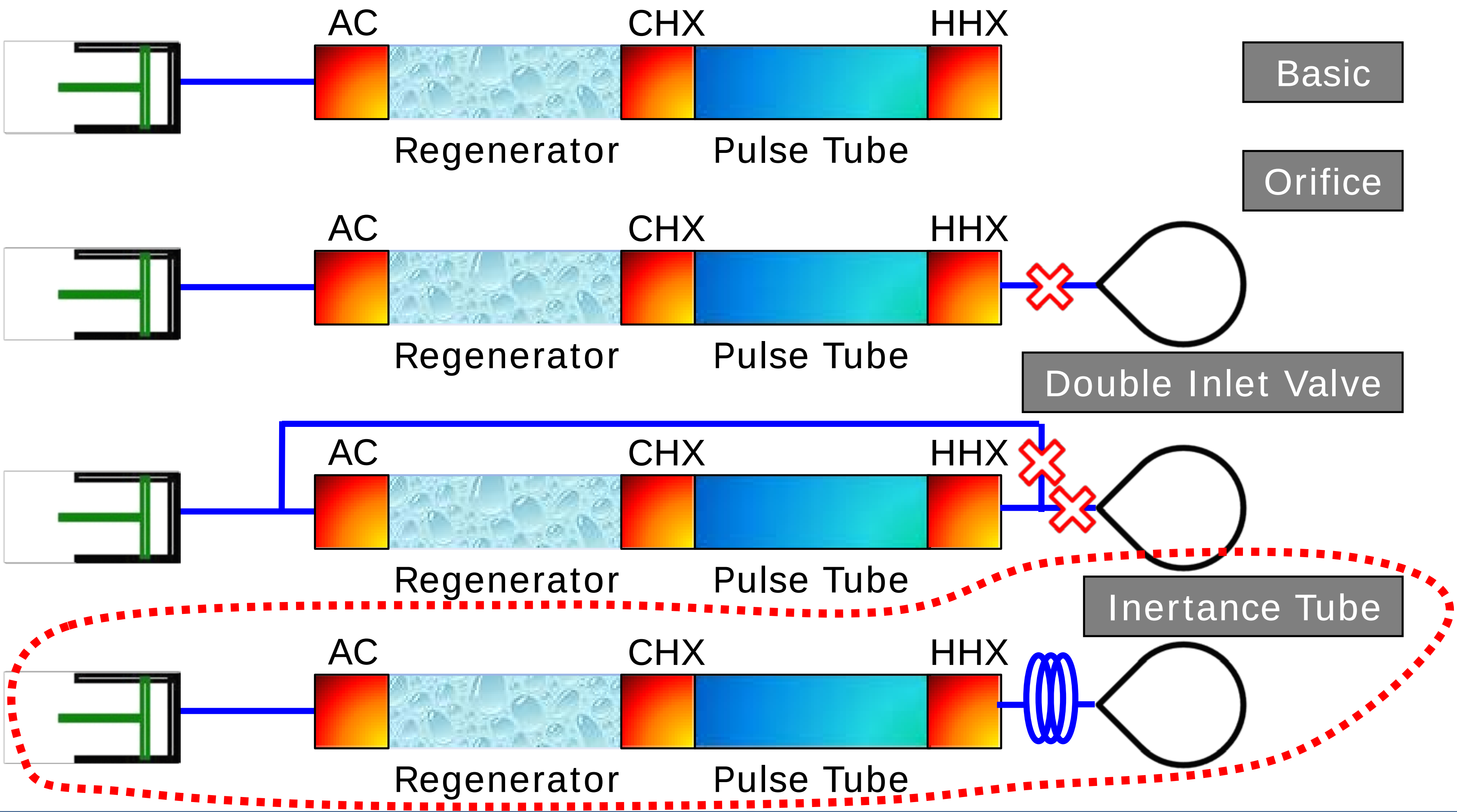
Phasor Analysis



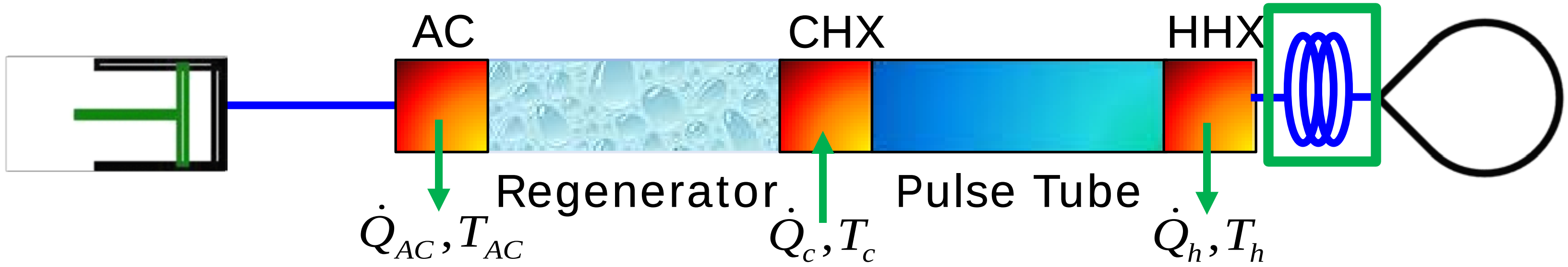
- As seen the earlier lecture, the phasor diagram of an OPTC is as shown in the figure.

- In the phasor diagram, the relative lengths of the vectors indicate the mass flow rate in those parts.

PT Classification



Phasor Diagram – ITPTC

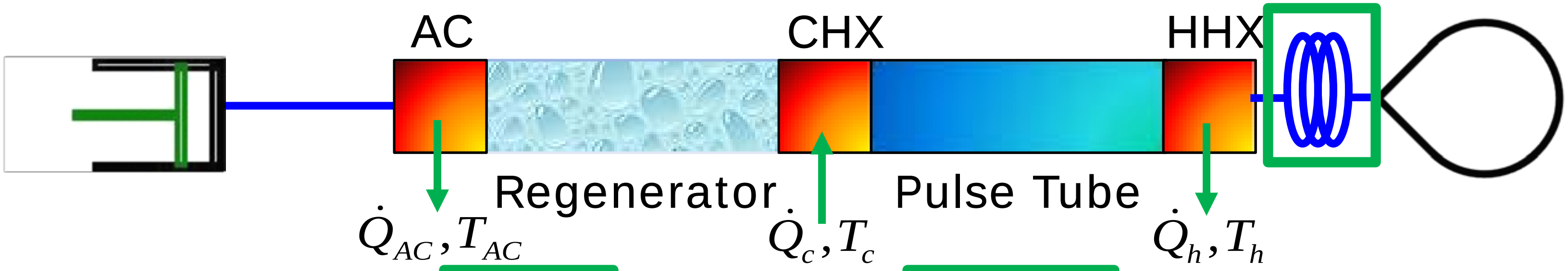


- The figure shows the schematic of an Inertance Tube Pulse Tube Cryocooler (ITPTC).

- An orifice is replaced with a very long tube of a small diameter. This tube is called as **Inertance Tube** (IT).

- The word **inertance** comes from the words Inertia and Inductance.

Phasor Diagram – ITPTC

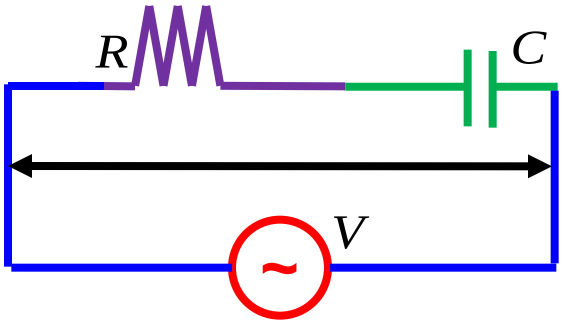


- Inertance = Inerta + Inductance.

- For the sake of understanding, let us study a RL circuit and thereby the electrical analogy between the RL circuit and PTC's.

- Let **f** be the frequency and **j** represent the imaginary part.

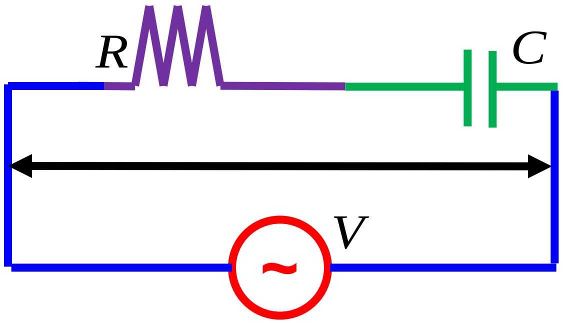
Electrical Analogy – RC



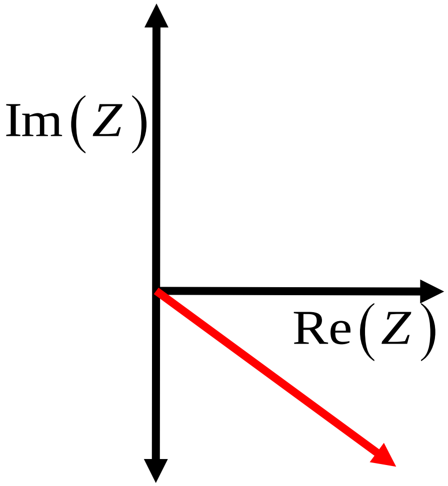
- The schematic of a series RC circuit is as shown.
- The impedance is given by $Z = \frac{V}{i} = R - j\left(\frac{1}{\omega C}\right)$
- The angle between the current and the voltage is

$$\theta = \tan^{-1}\left(-\frac{1}{\omega RC}\right)$$

Electrical Analogy – RC



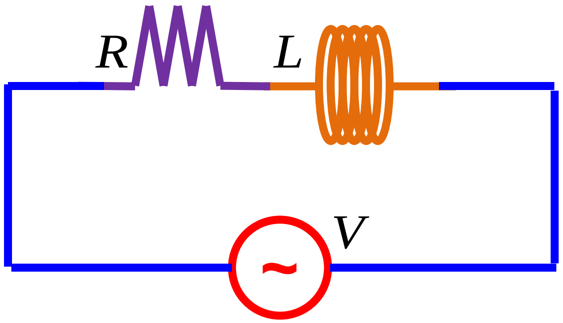
$$\theta = \tan^{-1}\left(-\frac{1}{\omega RC}\right)$$



- From the above equation, it is clear that the angle is always negative. Hence, the current always leads the voltage.

- Magnitude of the impedance is $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

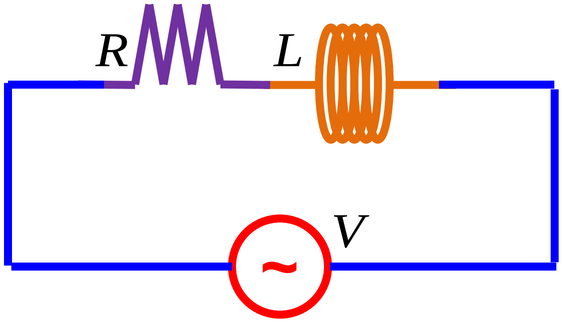
Electrical Analogy – RL



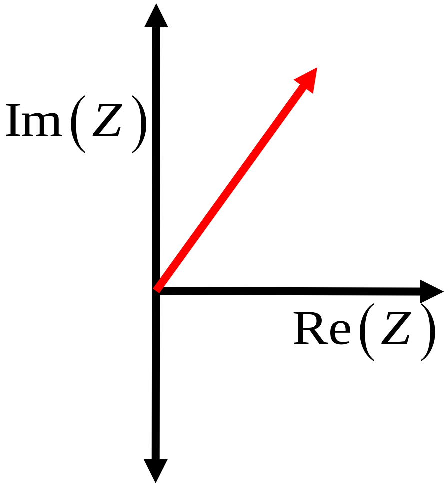
- The schematic of a series RL circuit is as shown.
- The impedance is given by $Z = \frac{V}{i} = R + j\left(\omega L\right)$
- The angle between the current and the voltage is

$$\theta = \tan^{-1}\left(+\frac{\omega L}{R}\right)$$

Electrical Analogy – RL



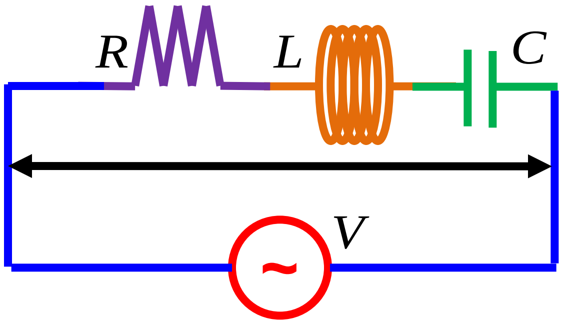
$$\theta = \tan^{-1}\left(+\frac{\omega L}{R}\right)$$



- From the above equation, it is clear that the angle is always positive. Hence, the current always lags the voltage.

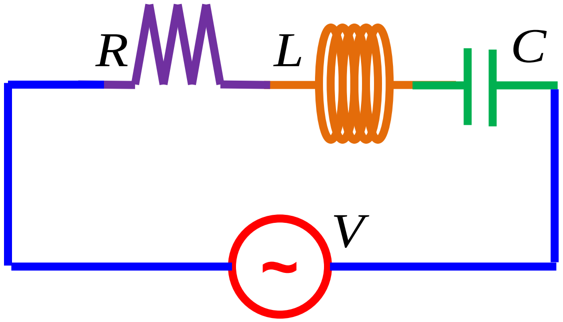
- Magnitude of the impedance is $Z = \sqrt{R^2 + (\omega L)^2}$

Electrical Analogy – RLC



- The schematic of a series RLC circuit is as shown.
- It is a series combination of a Resistance (R), an Inductance (L), a Capacitance (C) and a sinusoidal voltage source (V).
- In this circuit, both **L** and **C** have a collective effect on the performance of the circuit.

Electrical Analogy – RLC



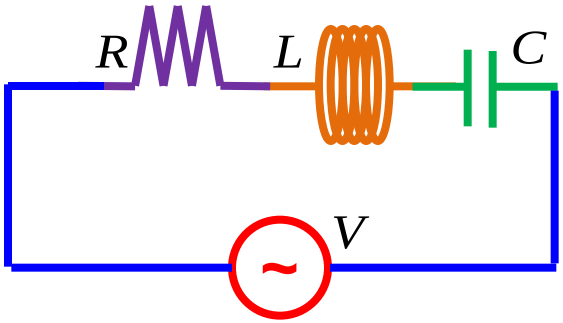
- The impedance is given by

$$Z = \frac{V}{i} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

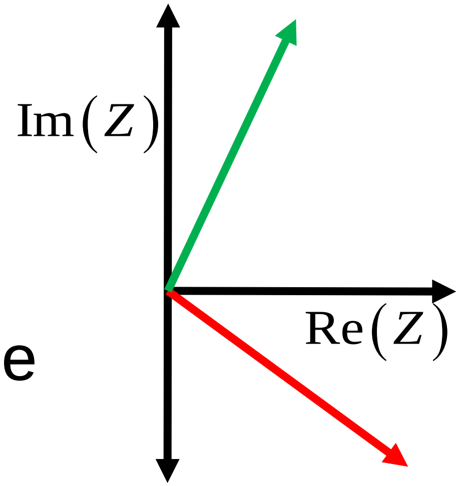
- The angle between the current and the voltage is

$$\theta = \tan^{-1}\left(\frac{\omega L - (\omega C)^{-1}}{R}\right)$$

Electrical Analogy – RLC



$$\theta = \tan^{-1}\left(\frac{\omega L - (\omega C)^{-1}}{R}\right)$$



- It is clear that depending upon the value of **L**, **C** and **f**, the angle can either be positive, negative or zero.

- In addition to L and C, frequency plays a major role.

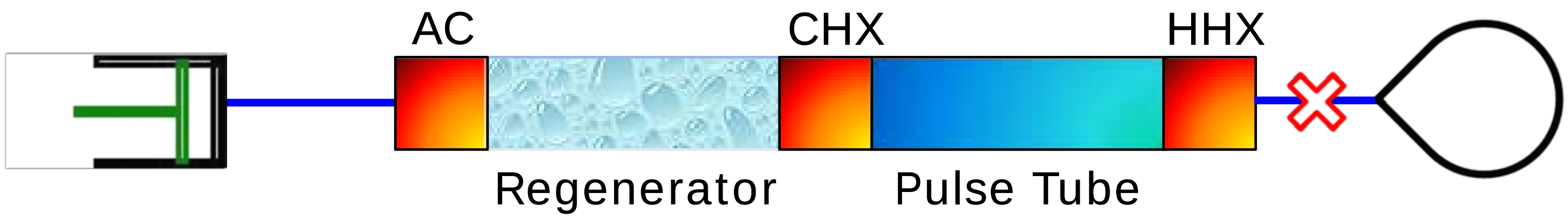
Electrical Analogy

- In a lumped electric model, we have the following analogies.
  - Oscillating Pressure → Voltage.
  - Mass flow rate → Current.
  - Reservoir → Capacitance (Compliance).
  - Orifice → Resistance.
  - Inertance Tube → Inductance.

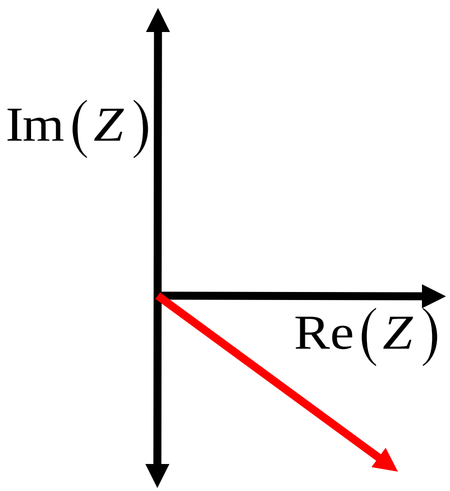
- The inductance of an IT is given by $$L = \frac{\rho L}{A}$$

- The compliance of a reservoir is given by $$C = \frac{V_{res}}{\gamma p_m}$$

Electrical Analogy

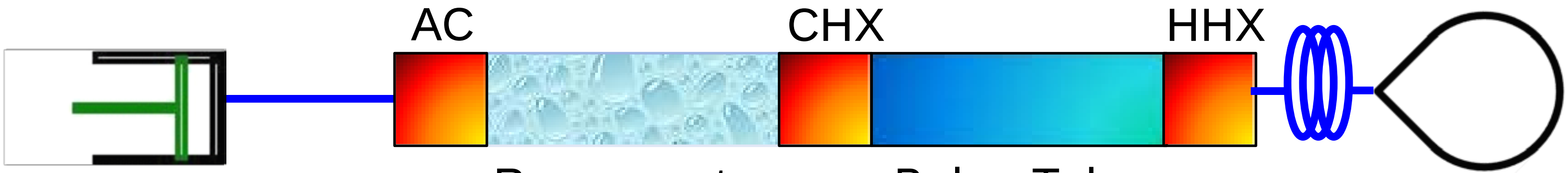


$$\theta = \tan^{-1}\left(-\frac{1}{\omega RC}\right)$$

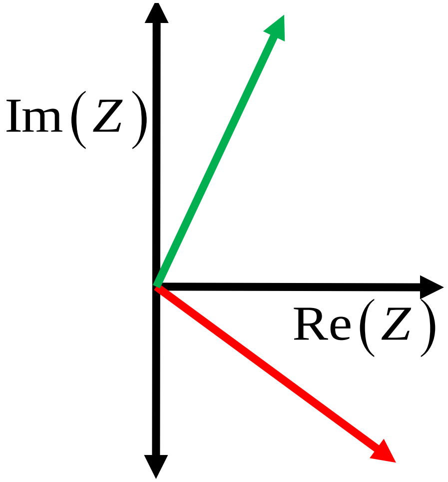


- An orifice in an OPTC is analogues to resistance.
- In an OPTC, the impedance phasor lies in the fourth quadrant.
- Hence, the mass flow rate leads the pressure vector.

Electrical Analogy



$$\theta = \tan^{-1}\left(\frac{\omega L - (\omega C)^{-1}}{R}\right)$$



- An orifice together with an IT is analogues to resistance and inductance in series.
- Depending upon the values of **L**, **C** and **f**, the angle can be through a large range.
- The impedance may lie in **1st** or **4th** quadrant.

Electrical Analogy

- From the electrical analogy, following things are well understood.

- In an OPTC, the mass flow rate always leads the pressure pulse.

- In a high frequency ITPTC, depending on the inductance of the inertance tube, the phase angle can be changed through a large value.

- IT is significant only for high frequency PTC, while for low frequency PTC, double inlet phase mechanism is used.

Tutorial

- Draw a phasor diagram for a 50 K OPTC with Helium as working fluid. The other operating parameters are as given below.

Parameters
Frequency: 30 Hz
Charge pressure: 20 bar (abs)
Dynamic pressure: 4 bar (abs)
PT volume: 8 cc
Regenerator Volume : 20 cc
Compressor dead volume: 20 cc
Heat exchanger volume: 2 cc
Temperature: 300 K
Orifice mass flow rate: 2 gm/s

Tutorial

Given	
$f = 30$ Hz	$V_{PT} = 8 \times 10^{-6}$
$p_o = 20$ bar	$V_{Regen} = 20 \times 10^{-6}$
$p_1 = 4$ bar	$V_{HX} = 2 \times 10^{-6}$
$T_o = 300$ K	$V_{CP} = 20 \times 10^{-6}$
$T_c = 50$ K	$m_o = 2 \times 10^{-3}$

Required
Phasor Diagram
Phase angle $\mathbf{m_c}$ and pressure vector
Phase angle $\mathbf{m_{cp}}$ and pressure vector

Tutorial

- $\mathbf{m_o = m_h}$ = 0.002 Kg/s.
- Pressure Vector:

$$m_h\left(\frac{T_h}{T_c}\right) = 0.002\left(\frac{300}{50}\right)$$

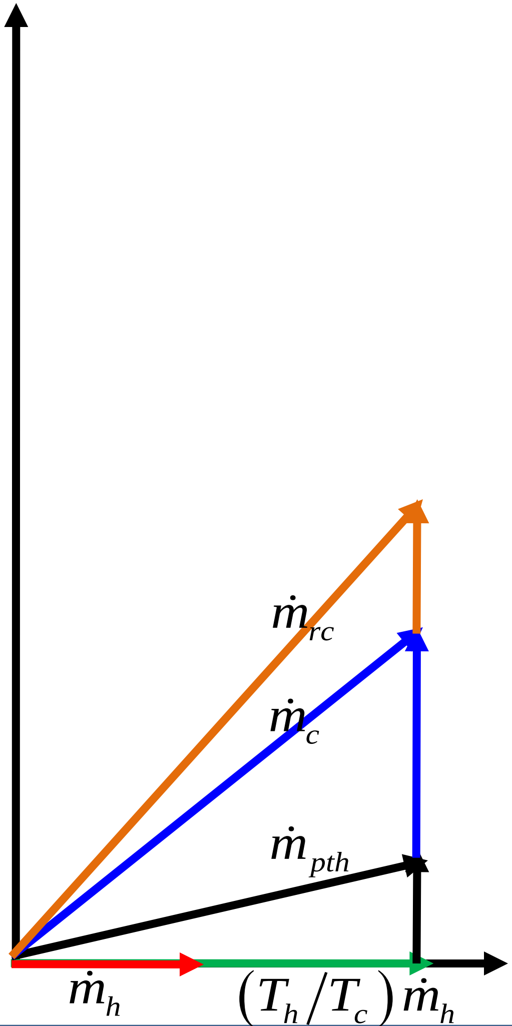
$$= 0.012$$

- Mass flow rate at Hot end (kg/s).

$$\frac{\omega p_1 V_{hhx}}{RT_h} = \frac{2\pi(30)(4)\left(10^{5}\right)(2)\left(10^{-6}\right)}{(2078.5)(300)}$$

$$= 0.2418\left(10^{-3}\right)$$

Tutorial



- Mass flow rate in Pulse Tube (kg/s).

$$\frac{\omega p_1 V_{pt}}{\gamma R T_c} = \frac{2\pi(30)(4)(10^5)(8)(10^{-6})}{(1.67)(2078.5)(300)}$$

$$= 3.482(10^{-3})$$

- Mass flow rate in Cold end (kg/s).

$$\frac{\omega p_1 V_{chx}}{R T_c} = \frac{2\pi(30)(4)(10^5)(2)(10^{-6})}{(2078.5)(50)}$$

$$= 1.451(10^{-3})$$

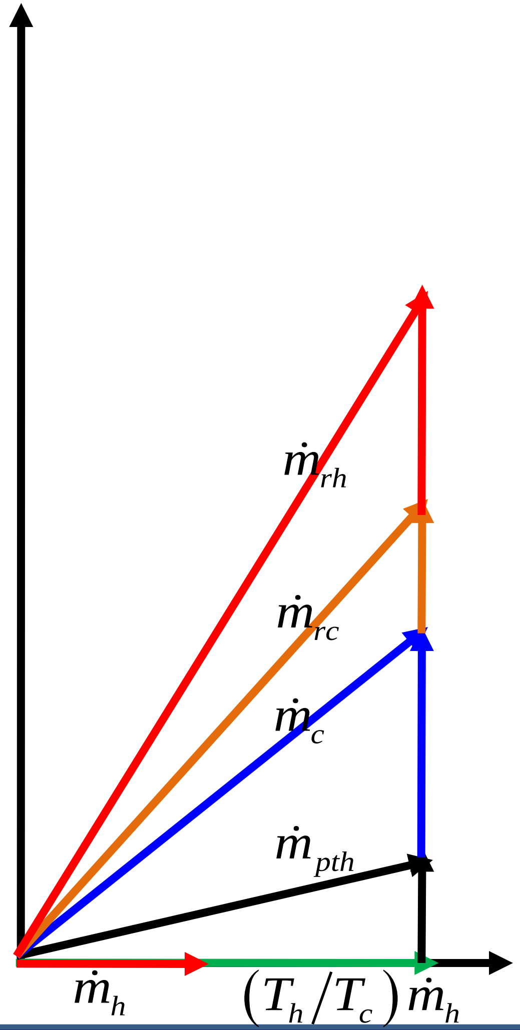
Tutorial

- Mass flow rate in Regenerator (kg/s).

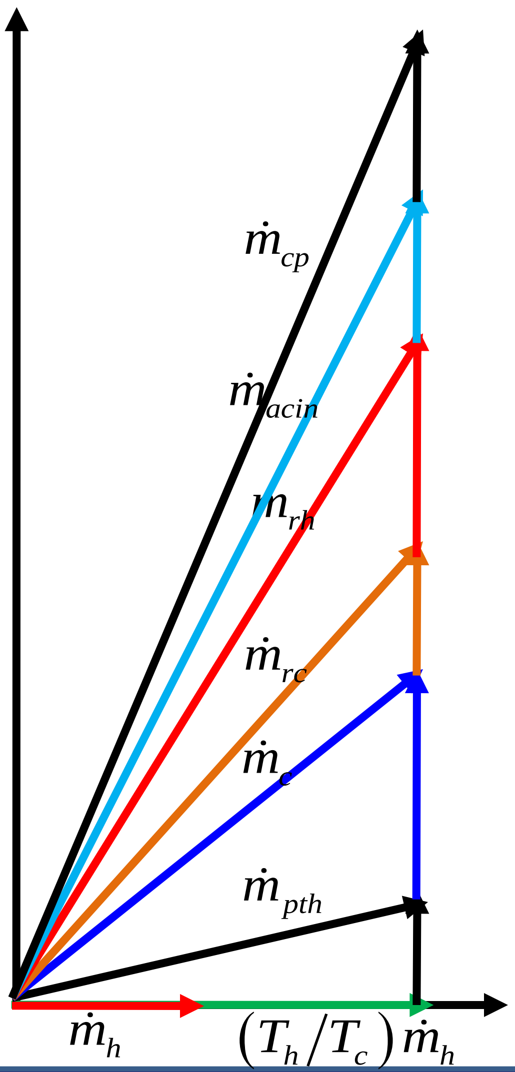
$$T_m = \frac{T_h - T_c}{\ln(T_h/T_c)} = \frac{300-50}{\ln(300/50)} = 139.5$$

$$\frac{\omega p_1 V_{regen}}{RT_m} = \frac{2\pi(30)(4)(10^5)(20)(10^{-6})}{(2078.5)(139.5)}$$

$$= 5.2(10^{-3})$$



Tutorial



- Mass flow rate in After cooler (kg/s).

$$\frac{\omega p_1 V_{hhx}}{RT_h} = \frac{2\pi(30)(4)\left(10^5\right)(2)\left(10^{-6}\right)}{(2078.5)(300)}$$

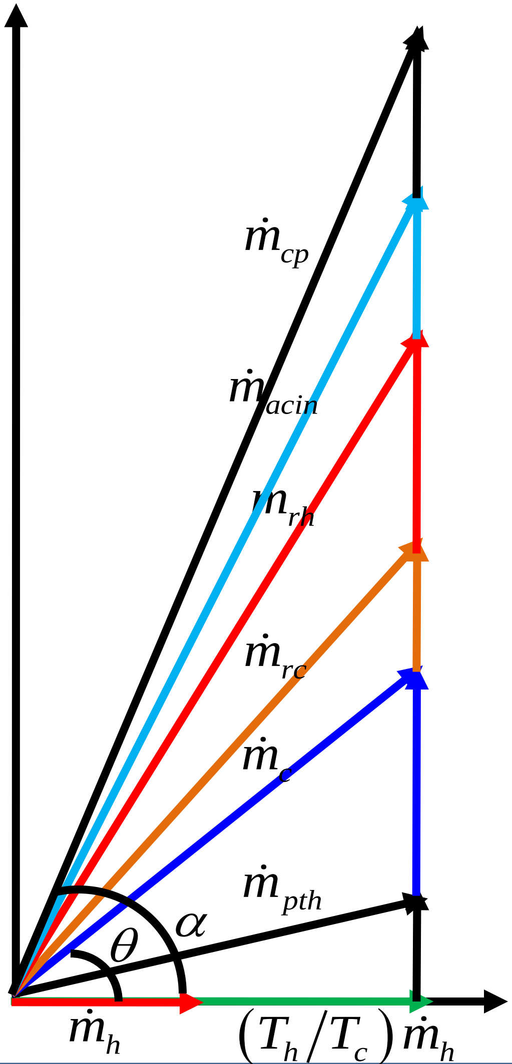
$$= 0.2418\left(10^{-3}\right)$$

- Mass flow rate in Compressor (kg/s).

$$\frac{\omega p_1 V_{cpd}}{RT_0} = \frac{2\pi(30)(4)\left(10^5\right)(20)\left(10^{-6}\right)}{(2078.5)(300)}$$

$$= 2.418\left(10^{-3}\right)$$

Tutorial



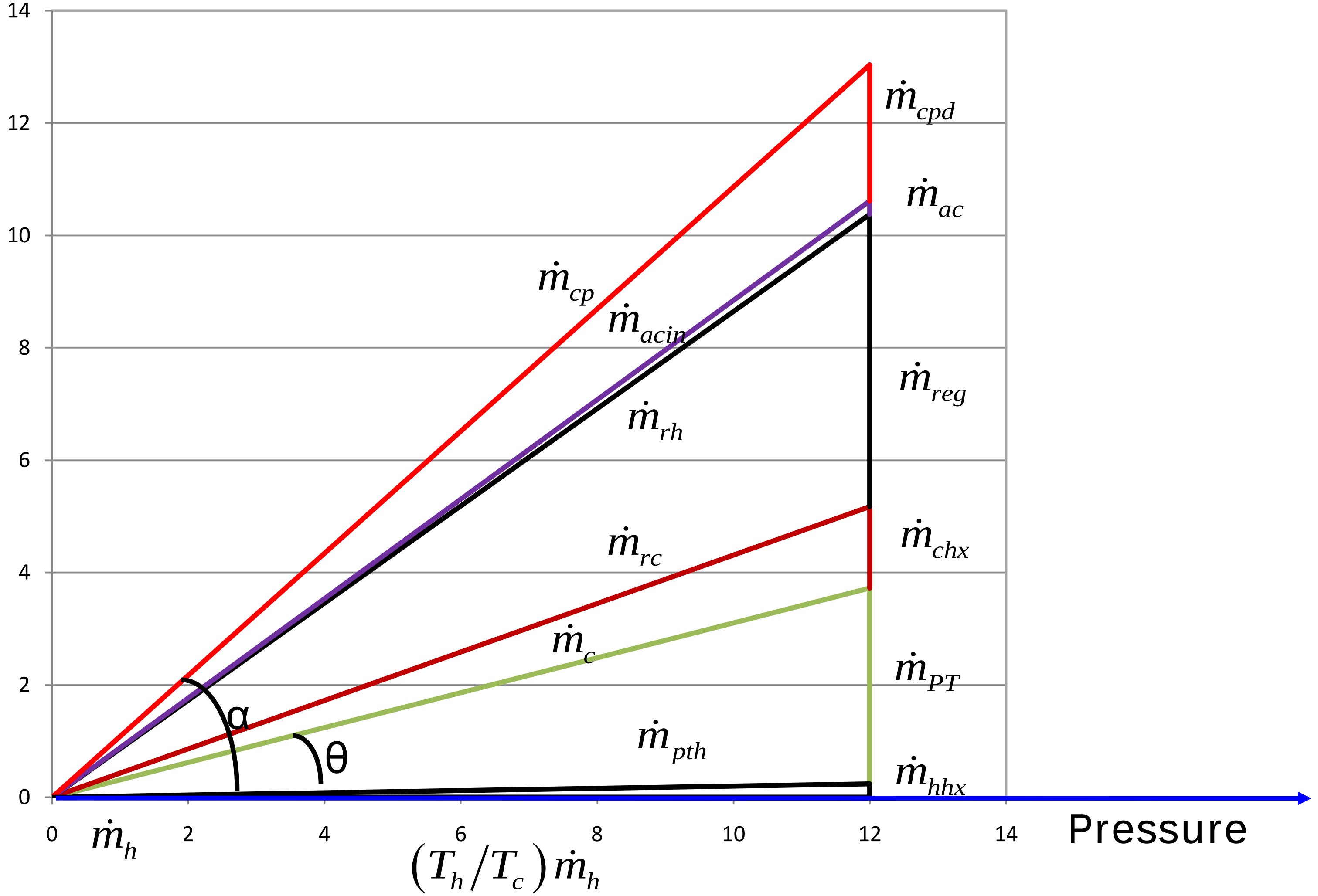
- The phase angle between the mass flow rate at the cold end and the pressure vector is

$m_c = 13.1\, g/s$ $\theta = 23.3^o$

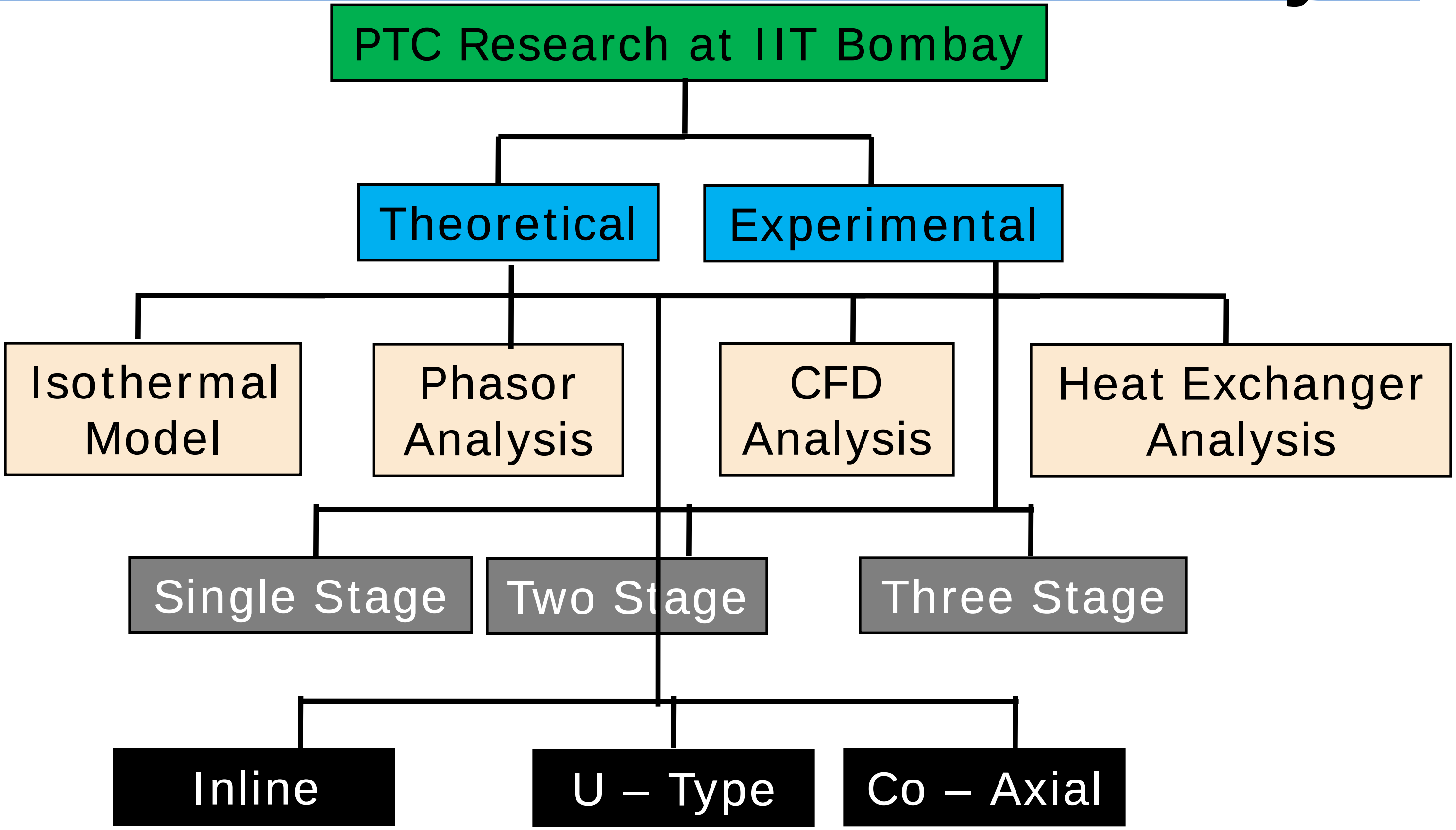
- The phase angle between the mass flow rate in the compressor and the pressure vector is

$m_{cp} = 16.0\, g/s$ $\alpha = 47.4$

Tutorial


14
12
10
8
6
4
2
0
0
2
4
6
8
10
12
14
$\dot{m}_{cpd}$
$\dot{m}_{ac}$
$\dot{m}_{cp}$
$\dot{m}_{acin}$
$\dot{m}_{reg}$
$\dot{m}_{rh}$
$\dot{m}_{rc}$
$\dot{m}_{chx}$
$\dot{m}_{c}$
$\dot{m}_{PT}$
α
θ
$\dot{m}_{pth}$
$\dot{m}_{hhx}$
$\dot{m}_h$
$(T_h/T_c)\,\dot{m}_h$
Pressure

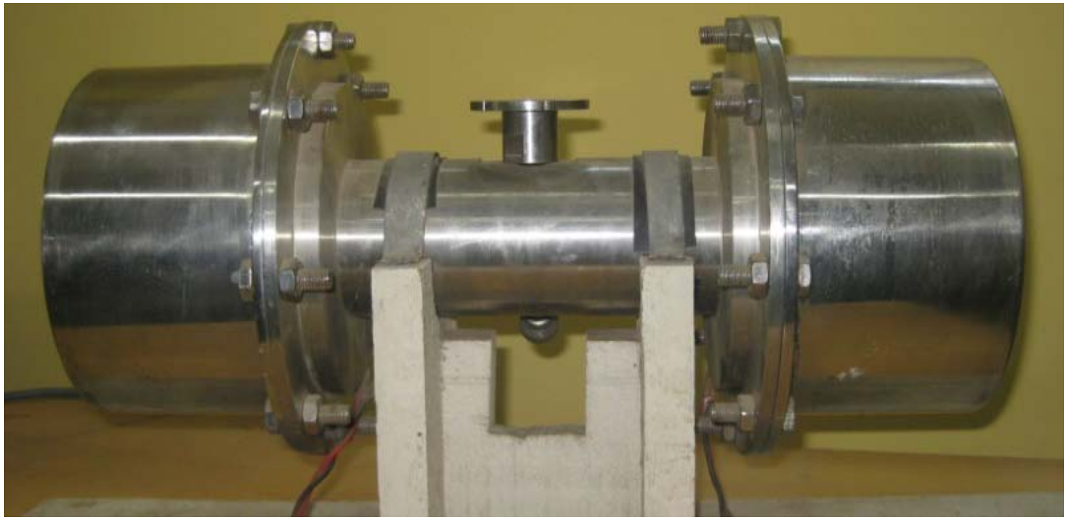
PTC Research at IIT Bombay



PTC Research at IIT Bombay

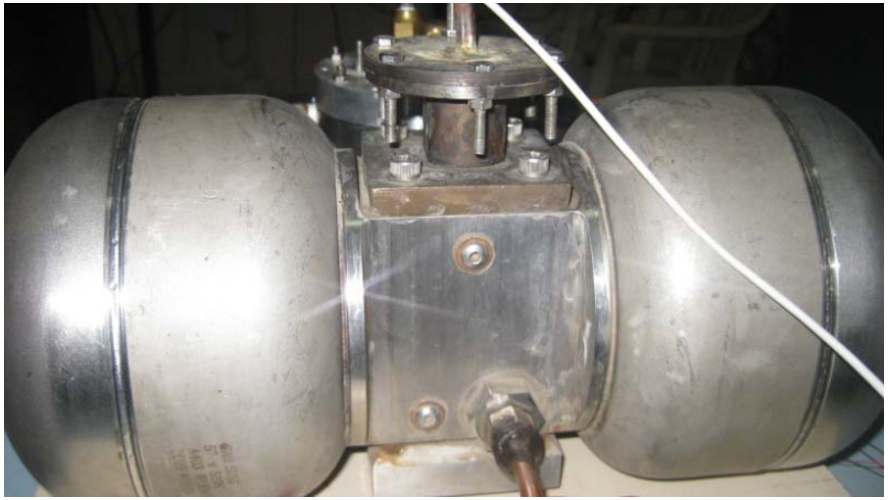
- Development of Theoretical Models
- Development of single stage Inline PTC.
- Development of single stage U – type PTC.
- Development of single stage Co – axial PTC.

Linear Compressor (PWG)

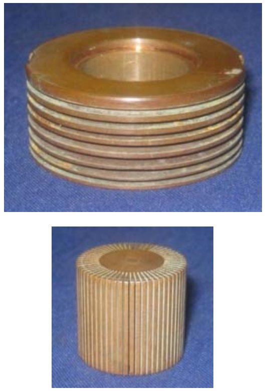


Linear Compressor developed at IITB

CFIC make Linear Compressor



Fabricated Parts at IITB



Aftercooler

PT-I

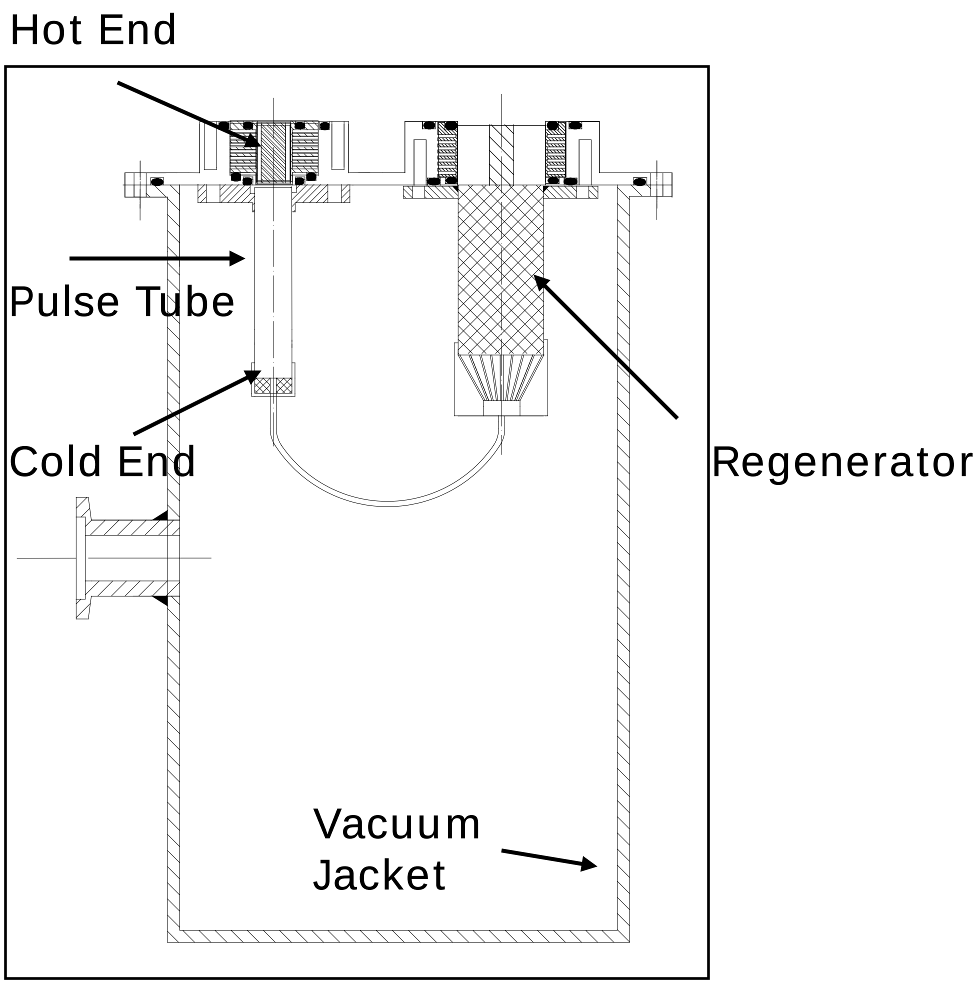
Hot End HX

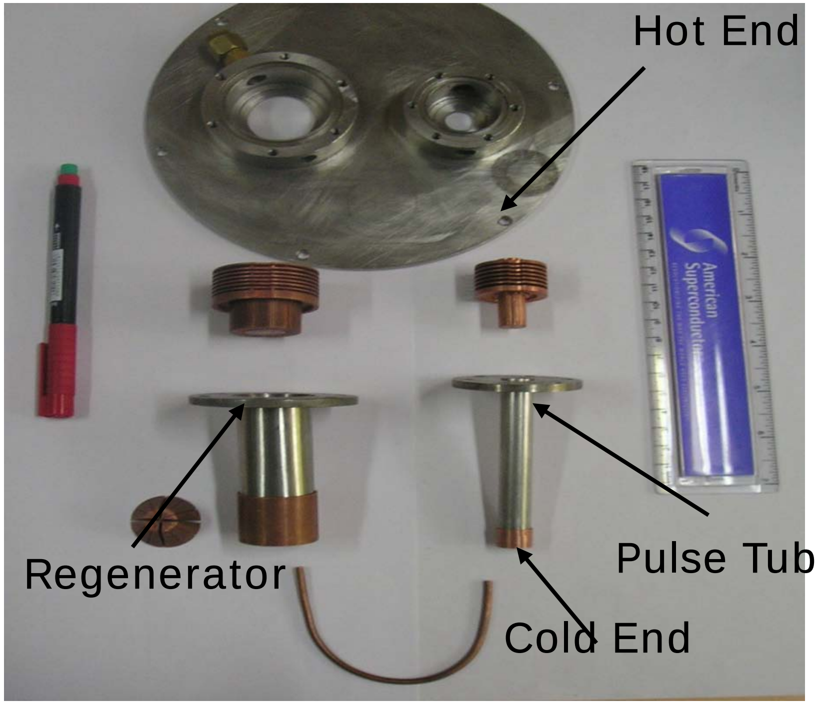
Regenerator

Cold End HX

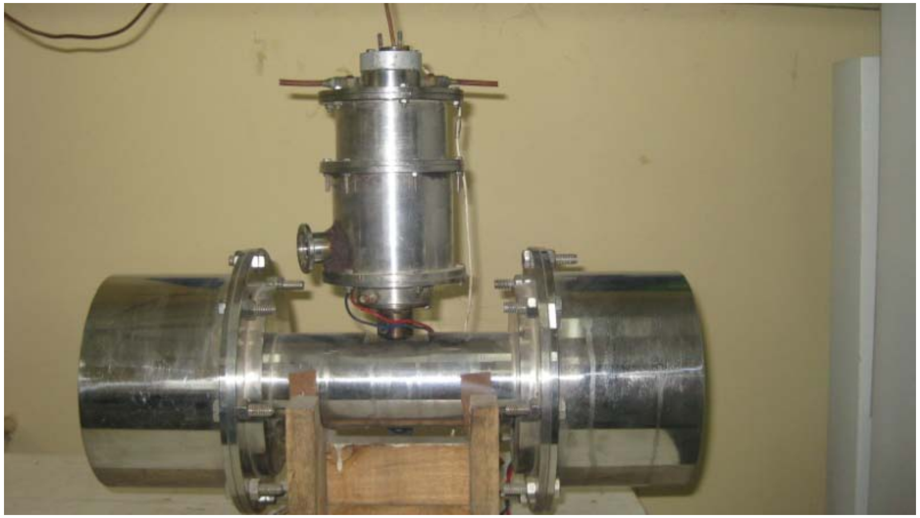
Vacuum Jacket

Typical U type PTC





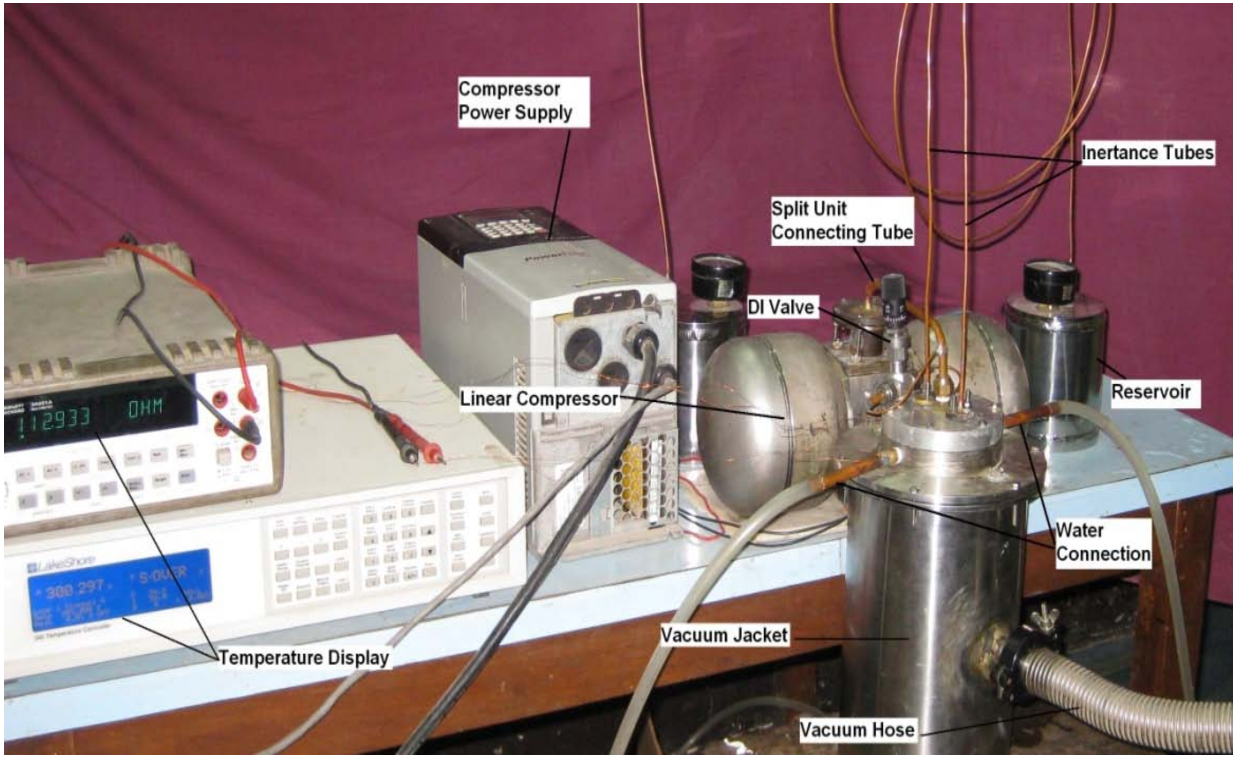
Experimental setup



U type PTC
Split Unit

In-line PTC
Integral Unit

Experimental setup



Linear Compressor:
CFIC, Model 2S132W
Max. Power 350 W

DI Valve:
Swagelok
Model SS-4MG-MH

Pr. Measurement:
ENDEVCO
Piezo-resistive sensors

Temp. Measurement:
Silicon Diodes

PTC Research at IIT Bombay

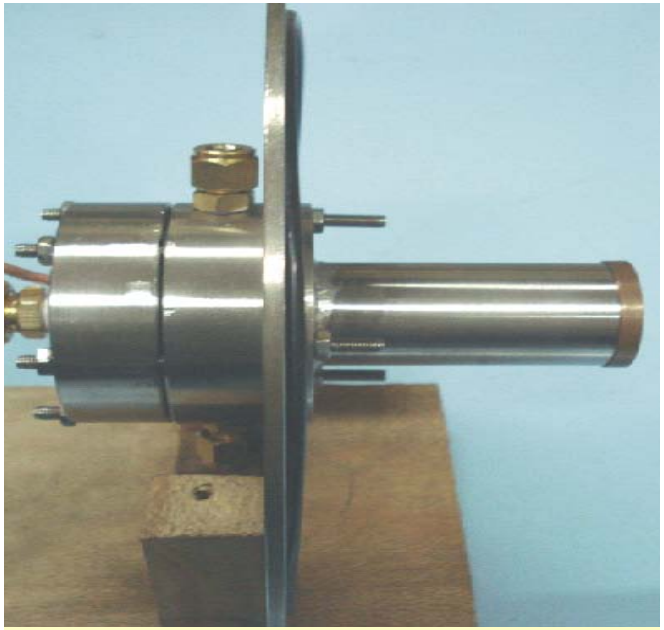
Single-Stage PTC :

Inline

Min. Temp : 50 K

'U' type

Min. Temp : 54 K

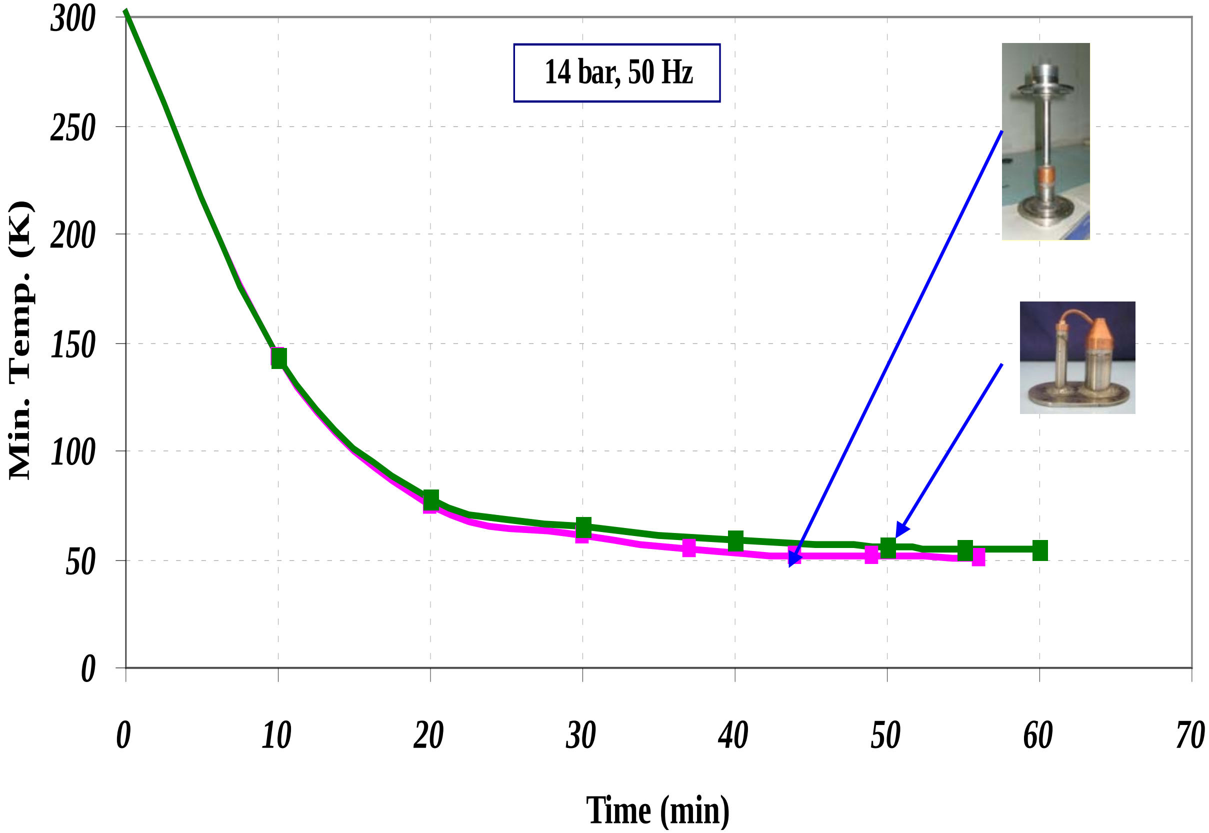


Coaxial

Min. Temp : 61 K

Experimental Results

Comparison of Single-Stage Inline Vs. U :

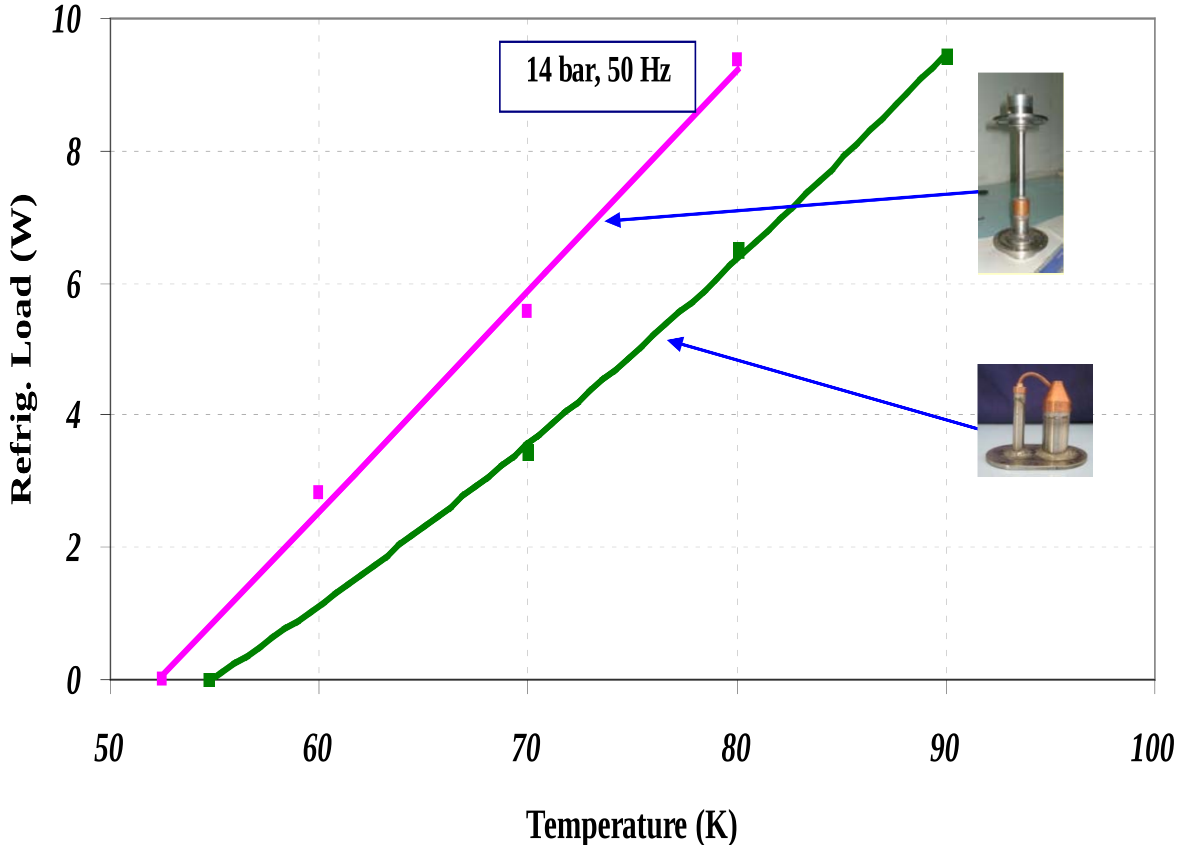


50 K…Inline
54 K…'U' type

Cooldown Curve

Experimental Results

Comparison of Single-Stage Inline Vs. U :

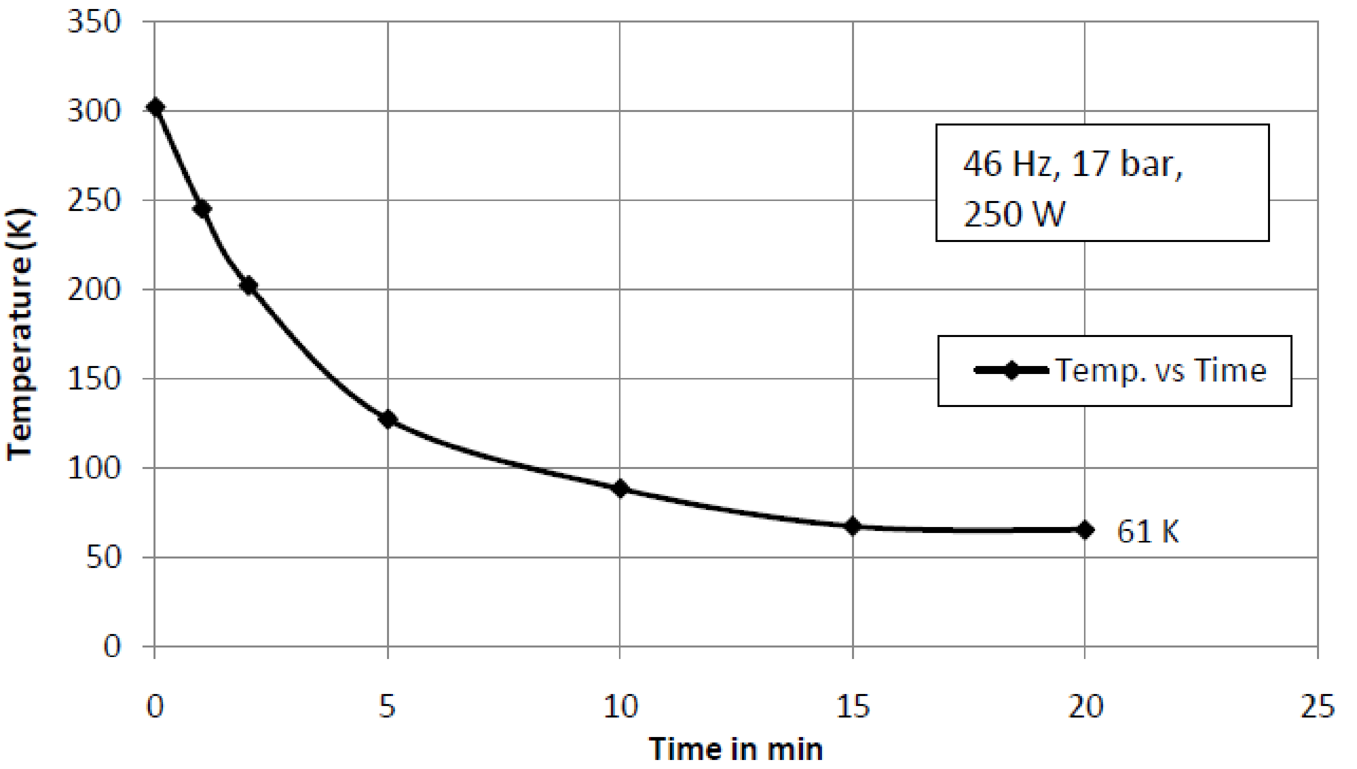


Refrigeration Load

9.4 W@ 80 K…Inline
6.5 W @ 80 K…'U' type

Experimental Results

Experimental results of Coaxial Unit:



Cooldown Curve

Min. Temp. 61 K

Summary

- A Cryocooler is a mechanical device which generates low temperature due to compression and expansion of gas.
- Example of recuperative cryocoolers are J – T, Brayton and Claude Cryocoolers.
- Examples of regenerative cryocoolers are Stirling, Gifford – McMahon, Pulse Tube Cryocoolers.
- A Stirling Cycle was first conceived by Robert Stirling in the year 1815.

Summary

- Depending upon the relative arrangements of piston and displacer/piston, **α**, **β**, **γ** are the different types of Stirling cryocooler.
- For an optimum design of a cryocooler, a compromise between the operating and the design parameters may be sought.
- A combined effect of parameters on performance of system as a whole, is given in **Walker's optimization charts**.

Summary

- In a GM system, for an optimum performance, the relation between the pressure pulse generated by valve mechanism and expander motion is vital.

- A GM system can reach much lower temperatures as compared to a Stirling system.

- Single stage (**~ 30 K**), **SS** mesh.
- 2 – stage (**~ 10 K**), 1st stage: **SS** mesh, 2nd stage: **Lead** balls.
- 2 – stage (**~ 4.2 K**), 1st stage: **SS** mesh **+ Lead** balls, 2nd stage: **Lead** balls **+** $\mathbf{Er_3Ni}$ balls.

Summary

- In a Pulse Tube cryocooler, the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling.
- PT systems can be classified based on the
  - Stirling type or GM type
  - Geometry and Operating frequency
  - Phase shift mechanism
- There exists a phase angle between mass flow rate at the cold end and pressure vector.

Summary

- Heat lifted at the cold end (**Q_c**) is dependent on **$|m_c|$**, **p_1/p_0**, **T_c**, phase angle.
- An orifice in an OPTC is analogues to resistance.
- An orifice, together with an inertance tube is analogues to resistance and inductance connected in series.
- We have seen the fabricated components of PTC at IIT Bombay.

Publication on PTC

1. Tendolkar M. V., Narayankhedkar K. G., Atrey M. D., Experimental Investigations on 20 Stirling-Type Two- Stage Pulse Tube Cryocooler with Inline Configuration, Paper presented at 16th International Cryocooler Conference pp 309-315, 2010.

2. Badgujar, A. D., M. D. Atrey, Theoretical and Experimental Investigations on Flow Straighteners in U-type Pulse Tube Cryocooler, Paper presented at 16th International Cryocooler Conference pp 211-217, 2010

3. Lokanath Mohanta, M. D. Atrey, Phasor Analysis of Pulse Tube Refrigerator, Paper presented at 16th International Cryocooler Conference, pp 299-308, 2010.

4. Tendolkar M. V., Narayankhedkar K. G. and Atrey M. D., Performance Comparison of Stirling Type Single Stage Pulse Tube Refrigerator for Inline and 'U' Configurations, Cryocooler-15, pp. 209-215, (2008).

5. Lokanath Mohanta, M. D. Atrey "Experimental Investigation on Single Stage Inline Stirling Type Pulse Tube Refrigerator", Cryocooler-15, pp 185-189, (2008).

6. Tendolkar M. V., Narayankhedkar K. G. and Atrey M. D., Performance Comparison of Stirling Type Single Stage Pulse Tube Refrigerator for Inline and 'U' Configurations, Proceeding 15th International Cryocooler Conference, pp 209-215, (2008).

7. Mohanta L. and Atrey M. D., Experimental Investigation on Single Stage Inline Stirling Type Pulse Tube Refrigerator, Proceeding 15th International Cryocooler Conference, pp185-189, (2008).

8. C. V. Thaokar, M. D. Atrey, High Frequency Pulse Tube Refrigerator for 100 K, Indian Journal of Cryogenics, Vol. 34, No.1-4, pp 158-163, (2010).

Publication on PTC

9. Mridul Sarkar, M. D. Atrey, Experimental Investigations on 80 K Stirling Type Coaxial Pulse Tube Refrigerator, Indian Journal of Cryogenics, Vol. 35, No.1-4, pp 327-332, (2010). [Won best paper award].

10. Gawali, S., Atrey, M.D., Narayankhedkar, K.G., Performance Prediction and Experimental Investigation on Orifice Pulse Tube Cryocooler', ICEC 19, pp 391-394, (2002).

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Thank You!