CRYOGENIC ENGINEERING

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Lecture No - 34

Earlier Lecture

- Cryogenic vessels use insulation to minimize all modes of heat transfer.
- Apparent thermal conductivity ($\mathbf{k_A}$) is calculated based on all possible modes of heat transfer.
- Expanded foam is a low density, cellular structure. A gas filled powder or a fibrous insulation reduces the gas convection due to the small size of voids.
- Radiation heat transfer is reduced by using radiation shields.

Outline of the Lecture

Topic : Cryogenic Insulation (contd)

- Vacuum
- Evacuated Powders
- Opacified Powders
- Tutorial

Types of Insulation

- Expanded Foam – Mass
- Gas Filled Powders & Fibrous Materials – Mass
- Vacuum alone – Vacuum
- Evacuated Powders – Mass + Vacuum
- Opacified Powders – Mass + Vacuum + Reflective
- Multilayer Insulation – Vacuum + Reflective

Introduction

- As seen earlier, the different modes of heat transfer are Conduction, Convection and Radiation.
- If the physical matter between the hot and the cold surfaces is removed, that is, by maintaining a perfect vacuum, Conduction and Convection are eliminated.
- However, Radiation heat transfer does not require any medium and in such cases, it is the only mode of heat transfer.

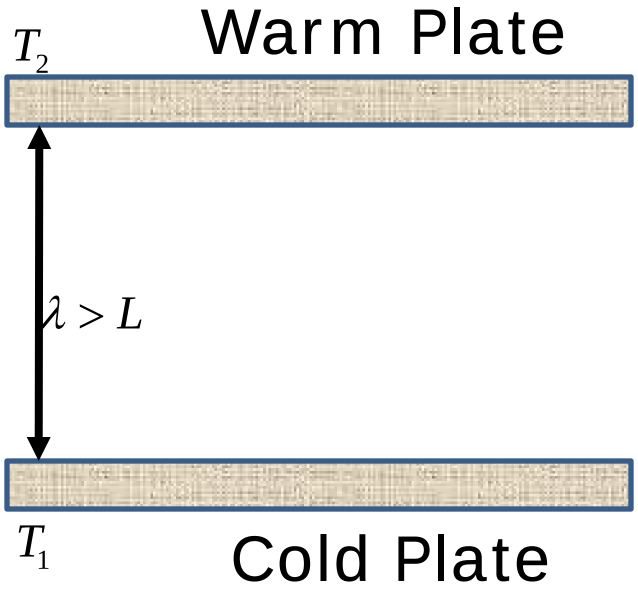
Vacuum

- It is important to note that even in vacuum, there is some residual gas.
- These gas molecules contribute to the heat transfer by gaseous conduction.
- As the vacuum improves, this gas conduction decreases.
- In an ordinary conduction, a linear temperature gradient is built up. The molecules exchange heat with each other and as well as with the surfaces.

Vacuum

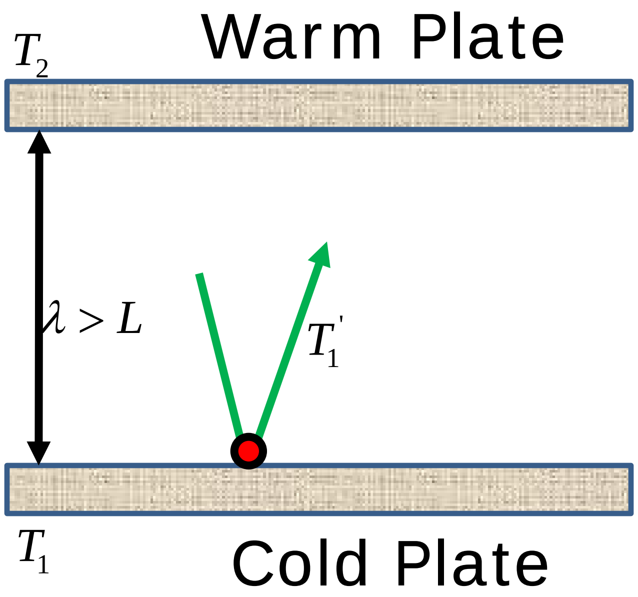
- But in vacuum, the mean free path (**λ**) of the molecules is more than the distance between the surfaces; the molecules rarely collide with each other.
- The energy is exchanged only between the surface and the colliding molecules.
- This type of heat transfer is called as free molecular conduction or residual gas conduction.
- This exists only at very low pressures or at very good vacuum.

Vacuum



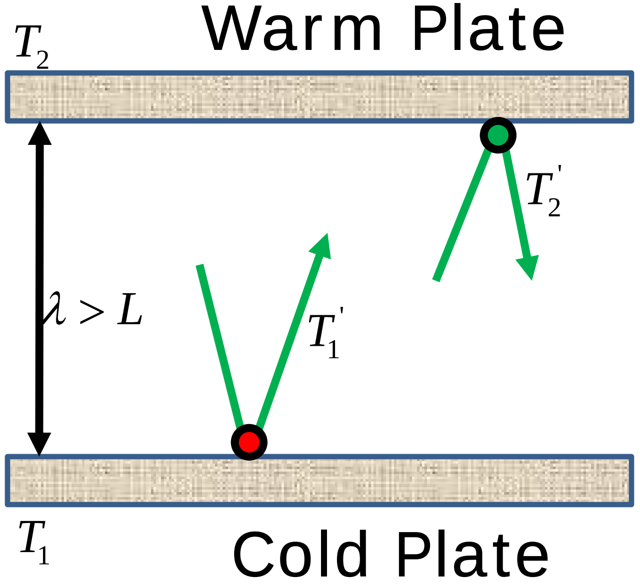
- For the sake of understanding, consider two plates with temperatures $\mathbf{T_1}$ and $\mathbf{T_2}$, ($\mathbf{T_2 > T_1}$) as shown.
- The gas pressure is very low in order to ensure that the mean free path (**λ**) of the molecules is greater than **L**.
- In such situations, the gas molecules collide only with the surfaces and exchange energy.

Vacuum



- Consider a molecule colliding with bottom plate and leaving towards upper plate.
- The gas molecule collides with this surface at $\mathbf{T_1}$ and it transfers some energy to the surface.
- It leaves the cold surface with a kinetic energy corresponding to a temperature $\mathbf{T'_1}$, higher than $\mathbf{T_1}$.

Vacuum



- Again, consider a molecule colliding with upper plate and leaving towards bottom plate.
- This gas molecule collides with surface at $\mathbf{T_2}$ and leaves at a temperature $\mathbf{T'_2}$, lower than $\mathbf{T_2}$.
- It is clear that, in both these impacts, thermal equilibrium is not attained. This process is repeated and contributes to free molecular conduction.

Vacuum

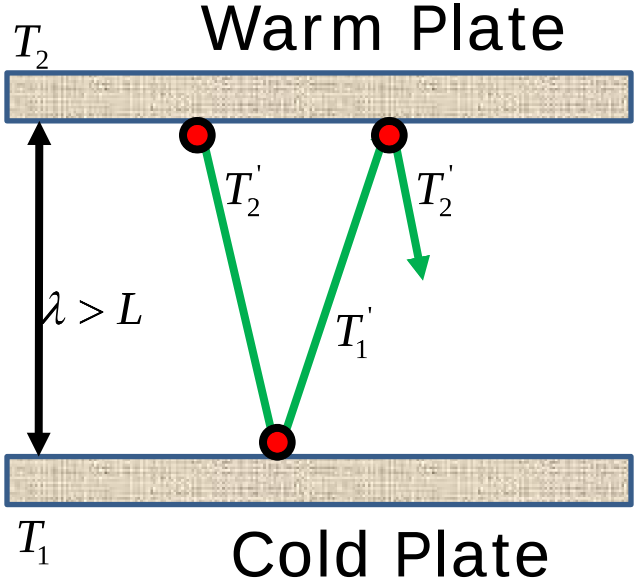
- In order to measure the degree of thermal equilibrium between the molecule and the surface, we define Accommodation Coefficient (**a**).

- It is a ratio of actual energy transfer to the maximum possible energy transfer.

- Mathematically, $$a = \frac{Actual\ Heat\ Transfer}{Max\ Heat\ Transfer}$$

- Its value depends on the gas – surface interaction and the temperature of the surface.

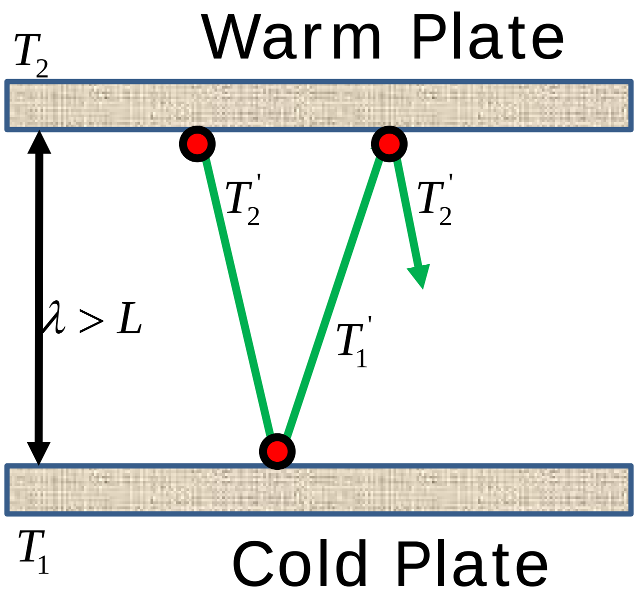
Vacuum



- From the figure, for the cold surface, the actual temperature change is **(T'_2 – T'_1)**.
- But, the maximum possible temperature change is **(T'_2 – T_1)**.
- By definition, the accommodation coefficient for cold plate is

$$a_1 = \frac{T_2^{'} - T_1^{'}}{T_2^{'} - T_1}$$

Vacuum



- Similarly, for the hot surface, the actual temperature change is **(T'_2 – T'_1)**.
- But, the maximum possible temperature change is **(T_2 – T'_1)**.
- Therefore, the accommodation coefficient for the hot surface is given by

$$a_2 = \frac{T_2' - T_1'}{T_2 - T_1'}$$

Vacuum

- From the earlier slides, the accommodation coefficients are

$$a_1 = \frac{T_2' - T_1'}{T_2' - T_1} \qquad a_2 = \frac{T_2' - T_1'}{T_2 - T_1'}$$

- Rearranging the above equations, we have

$$T_1 = T_2' - \frac{T_2' - T_1'}{a_1} \qquad T_2 = \frac{T_2' - T_1'}{a_2} + T_1'$$

$$T_2 - T_1 = \left(T_2' - T_1'\right)\left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)$$

Vacuum

$$T_2 - T_1 = \left(T_2^{'} - T_1^{'}\right)\left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)$$

- Similar to an emissivity factor, we define a term accommodation factor $\mathbf{F_a}$, which is given by

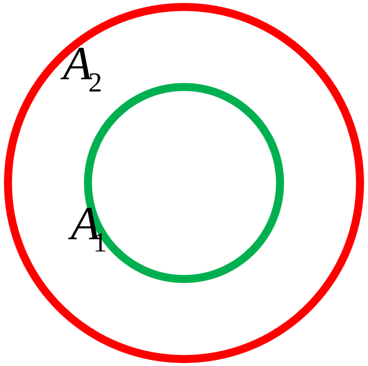
$$\frac{1}{F_a} = \left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)$$

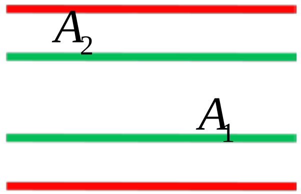
$$T_2 - T_1 = \left(T_2^{'} - T_1^{'}\right)\frac{1}{F_a}$$

$$F_a = \frac{T_2^{'} - T_1^{'}}{T_2 - T_1}$$

Vacuum

- The approximate accommodation coefficients for concentric sphere and concentric cylinder geometries are as tabulated below.





Temp (K)	He	H_2	Ne	Air
300	0.29	0.29	0.66	0.8-0.9
78	0.42	0.53	0.83	1.0
20	0.59	0.97	1.0	1.0

- The subscript **1** denotes the enclosed surface and subscript **2** denotes the enclosure.

Vacuum

Temp (K)	He	H_2	Ne	Air
300	0.29	0.29	0.66	0.8-0.9
78	0.42	0.53	0.83	1.0
20	0.59	0.97	1.0	1.0

- At a given temperature, the accommodation coefficient increases with the increase in the molecular weight of the gas.

- For a given gas, the accommodation coefficient increases with the decrease in the temperature, due to better heat transfer at lower temperatures.

Vacuum

- From the kinetic theory of gases, the total energy of a molecule is the sum of internal energy and kinetic energy.

- Mathematically, $e = U + KE$

$$e = \left(c_v + \frac{R}{2}\right)T$$

$$\Delta e = \left(c_v + \frac{R}{2}\right)\Delta T$$

- where,
  - R – Specific gas constant
  - c_v = **R / (γ-1)** – Specific heat of gas
  - ΔT = **(T'_2 – T'_1)** – Change in temperature

Vacuum

$$\Delta e = \left(c_v + \frac{R}{2} \right) \Delta T$$

- The definition of $\mathbf{C_v}$ and $\mathbf{F_a}$ are as given below.

$$c_v = \frac{R}{\gamma - 1}$$

$$T_2' - T_1' = F_a \left(T_2 - T_1 \right)$$

- Substituting, we have

$$\Delta e = \left(\frac{R}{\gamma - 1} + \frac{R}{2} \right) \left(T_2 - T_1 \right) F_a$$

$$\Delta e = \frac{F_a R}{2} \left(T_2 - T_1 \right) \left(\frac{\gamma + 1}{\gamma - 1} \right)$$

Vacuum

- The mass flux per unit time is given by

$$\frac{\dot{m}}{A} = \frac{\rho \overline{\upsilon}}{4}$$

- where,
  - ρ – Density, $\overline{\upsilon}$ – Average velocity

- From Kinetic theory, average velocity is $\overline{\upsilon} = \left(\frac{8RT}{\pi}\right)^{0.5}$

- Combining the above, together with equation of state, we have

$$\frac{\dot{m}}{A} = \frac{1}{4}\left(\frac{p}{RT}\right)\left(\frac{8RT}{\pi}\right)^{0.5}$$

$$\frac{\dot{m}}{A} = p\left(\frac{1}{2\pi RT}\right)^{0.5}$$

Vacuum

- The total energy transfer per unit area owing to the molecular conduction is as given below.

$$\frac{\dot{Q}}{A} = \frac{\dot{m}}{A}(\Delta e)$$

$$\frac{\dot{m}}{A} = p\left(\frac{1}{2\pi RT}\right)^{0.5}$$

$$\Delta e = \frac{F_a R}{2}(T_2 - T_1)\left(\frac{\gamma+1}{\gamma-1}\right)$$

$$\frac{\dot{Q}}{A} = p\left(\frac{1}{2\pi RT}\right)^{0.5}\left(\frac{F_a R}{2}(T_2 - T_1)\left(\frac{\gamma+1}{\gamma-1}\right)\right)$$

$$\frac{\dot{Q}}{A} = \left(\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{R}{8\pi T}\right)^{0.5} F_a\right)p(T_2 - T_1)$$

- **T** is the temperature of the pressure gauge measuring the gas pressure.

Vacuum

$$\frac{\dot{Q}}{A} = \left(\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{R}{8\pi T}\right)^{0.5} F_a\right) p(T_2 - T_1)$$

- In the above equation, let us denote the term in the parenthesis by **G**. We have,

$$\dot{Q} = G \; p \; A \, (T_2 - T_1)$$

- **Q** is valid only when the distance (**L**) between the plates is less than the mean free path (**λ**). Mathematically,

$$L < \lambda = \frac{\mu}{p}\left(\frac{\pi R T}{2}\right)^{0.5}$$

Vacuum

$$\dot{Q} = GpA(T_2 - T_1)$$

$$\lambda = \frac{\mu}{p}\left(\frac{\pi RT}{2}\right)^{0.5}$$

- From the above two equations, it is clear that the
  - The free molecular regime can be achieved by achieving very good vacuum.
  - The free molecular conduction heat transfer can be made negligible compared to other modes, by lowering the pressure, decreasing F_a, decreasing $(\mathbf{T_2} - \mathbf{T_1})$.

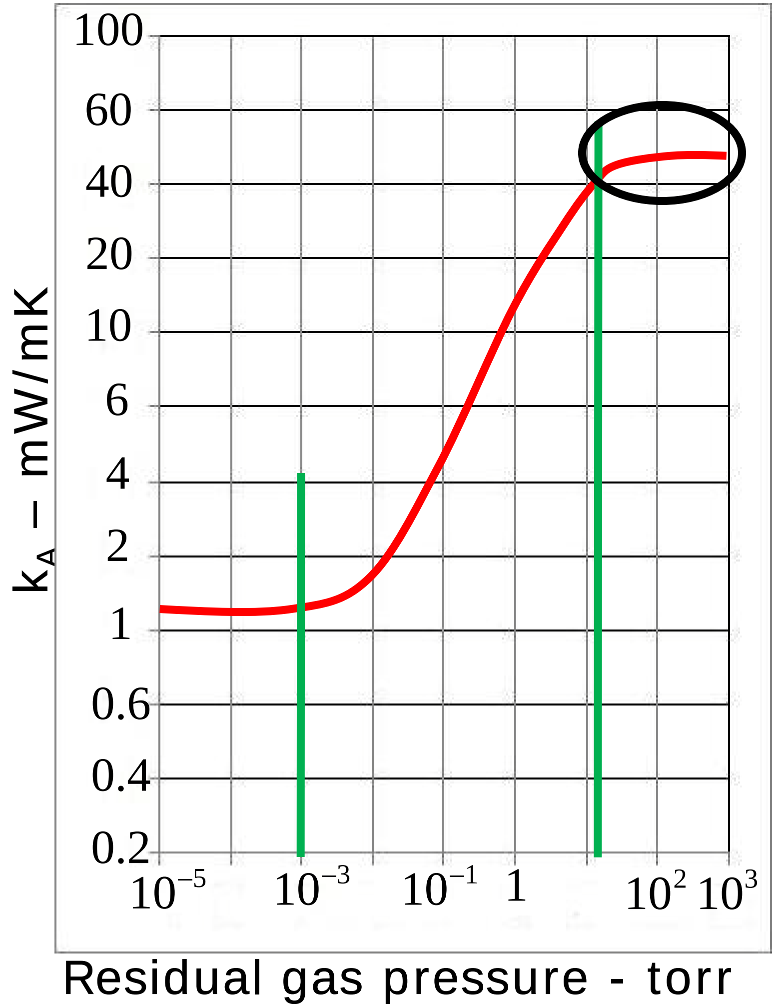
Evacuated Powder

- Gas conduction is the primary and the dominant mode of heat transfer in a gas filled powder and fibrous insulations.

- One of the obvious ways to reduce this heat transfer is to evacuate the powder and the fibrous insulations.

- Usually, the vacuum that is commonly maintained in these insulations is in the range of $\mathbf{10^{3}}$ to $\mathbf{10^{-5}}$ torr. **1** torr = 1 mm of Hg.

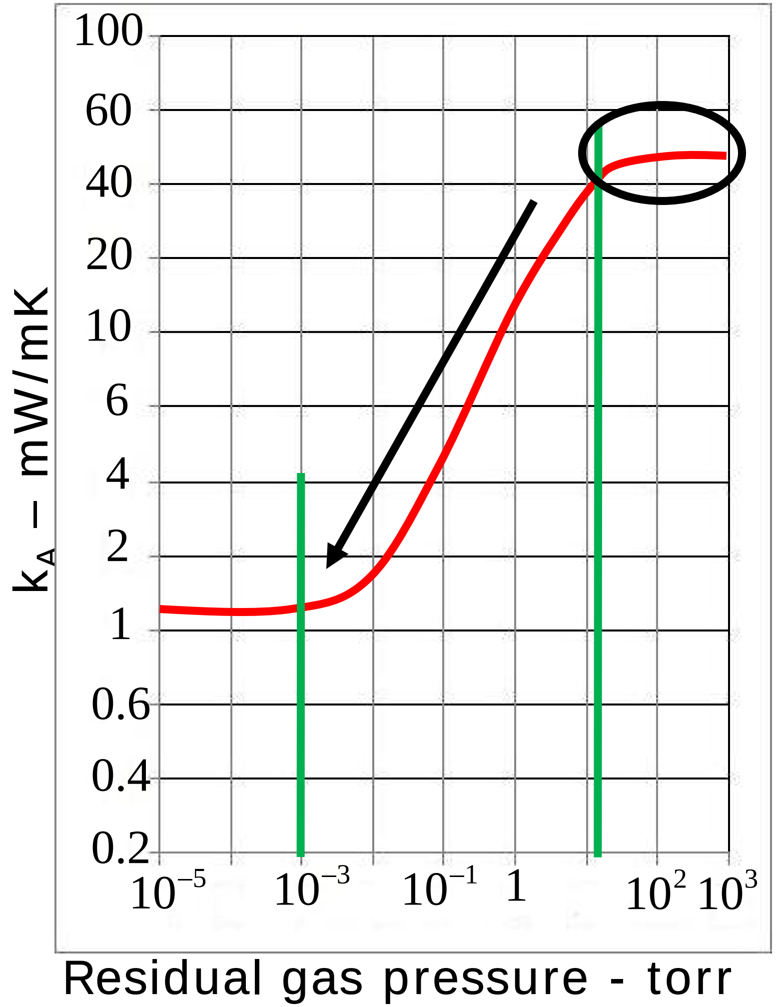
Evacuated Powder



- The adjacent figure shows the variation of $\mathbf{k_A}$ with the residual gas pressure inside an evacuated powder insulation.

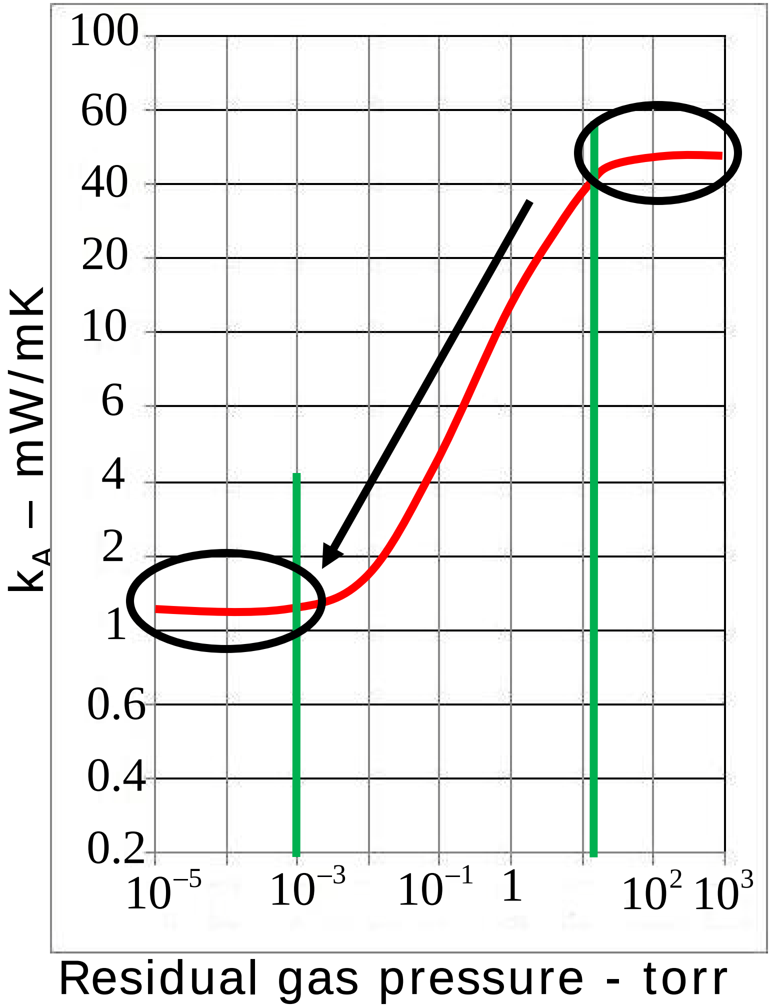
- $\mathbf{k_A}$ is independent of residual gas pressures lying between atmospheric and 15 torr.

Evacuated Powder



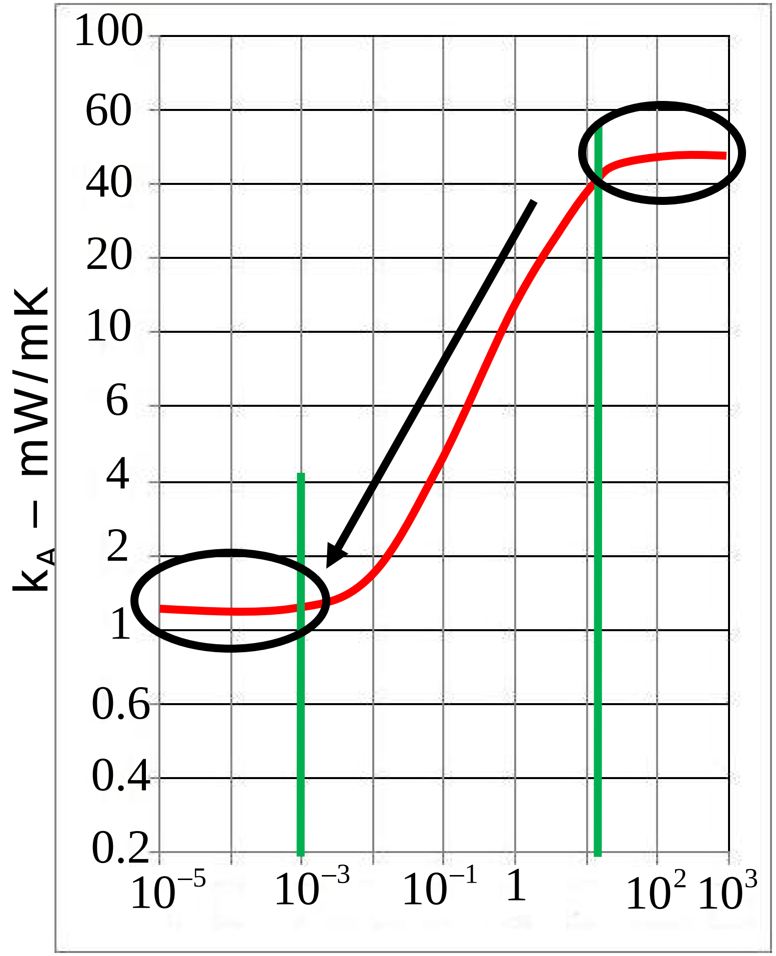
- With the lowering of pressure, 15 torr to 10^{-3} torr, $\mathbf{k_A}$ becomes directly proportional to the pressure.
- It varies almost linearly on a logarithmic chart as shown.
- Here, the modes of heat transfer are due to radiation, solid conduction and free molecular conduction (dominant).

Evacuated Powder



- With the further lowering of pressure, below **10^{-3}** torr, the variation of **k_A** is almost null.
- The mode of heat transfer is primarily due to solid conduction and radiation.
- Evacuated powders are superior in performance than vacuum alone in **300-77 K**, as the radiation heat transfer is comparatively less.

Evacuated Powder



- At low pressures and temperatures, the solid conduction in evacuated powder dominates the radiant heat transfer.
- Hence, it is more advantageous to use vacuum alone in **77 K** to **4 K**. From Fourier's Law, we have

$$\dot{Q} = \frac{k_A A_m (T_h - T_c)}{\Delta x}$$

Evacuated Powder

$$\dot{Q} = \frac{k_A A_m (T_h - T_c)}{\Delta x}$$

- where,
  - k_A = Apparent thermal conductivity
  - $T_h - T_c$ = Temperature difference
  - Δx = Distance
  - A_m = Mean area of insulation. $\mathbf{A_m}$ for concentric cylinders and concentric spheres is as given below.

$$A_{m,cyl} = \frac{A_2 - A_1}{\ln A_2 / A_1}$$

$$A_{m,sph} = \left(A_1 A_2\right)^{\frac{1}{2}}$$

Evacuated Powder

- The apparent thermal conductivity and density of few commonly used evacuated powder insulations are as shown.

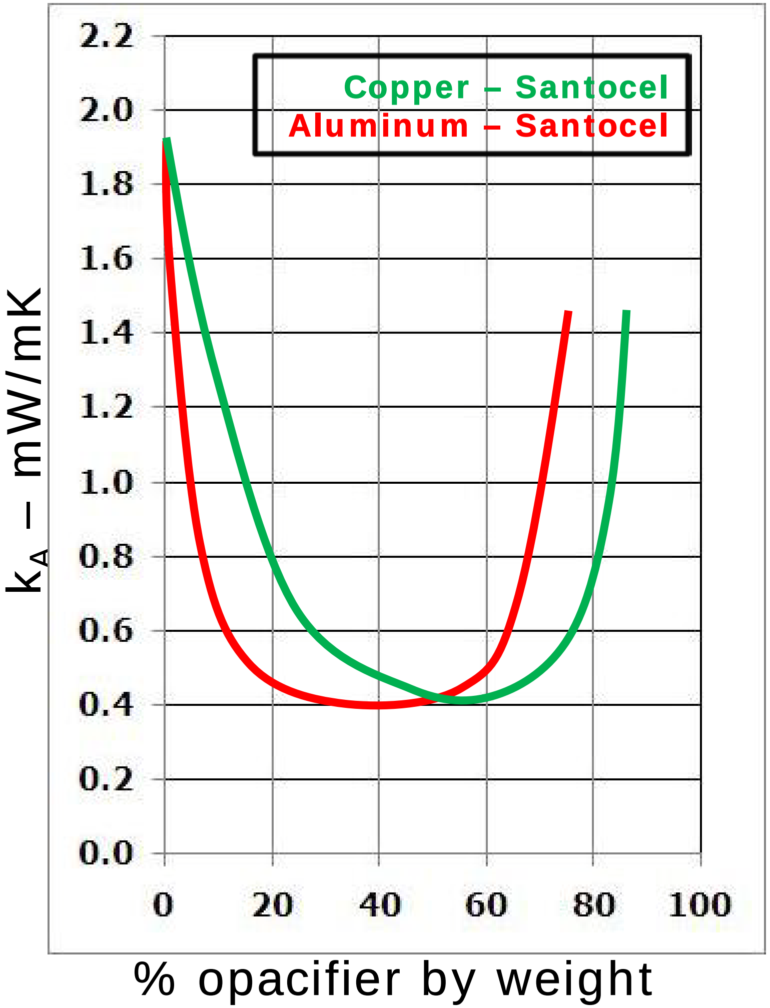
- The residual gas pressure is less than 10^{-3} torr for temperatures between **77 K** to **300 K**.

Powder	ρ (kg / m³)	k (mW / mK)
Fine Pertile	180	0.95
Coarse Perlite	64	1.90
Lampblack	200	1.20
Fiberglass	50	1.70

Opacified Powder Insulation

- Radiation heat transfer still contributes to the heat in leak in **300 K** to **77 K** temperature range in case of evacuated powders.
- In the year 1960, **Riede** and **Wang**, **Hunter** *et. al*. minimized this radiant heat transfer by addition of reflective flakes made of **Al** or **Cu** to the evacuated powder.
- These flakes act like radiant shields in the tiny heat transfer paths that are formed in the interstices of the evacuated powder.

Opacified Powder Insulation

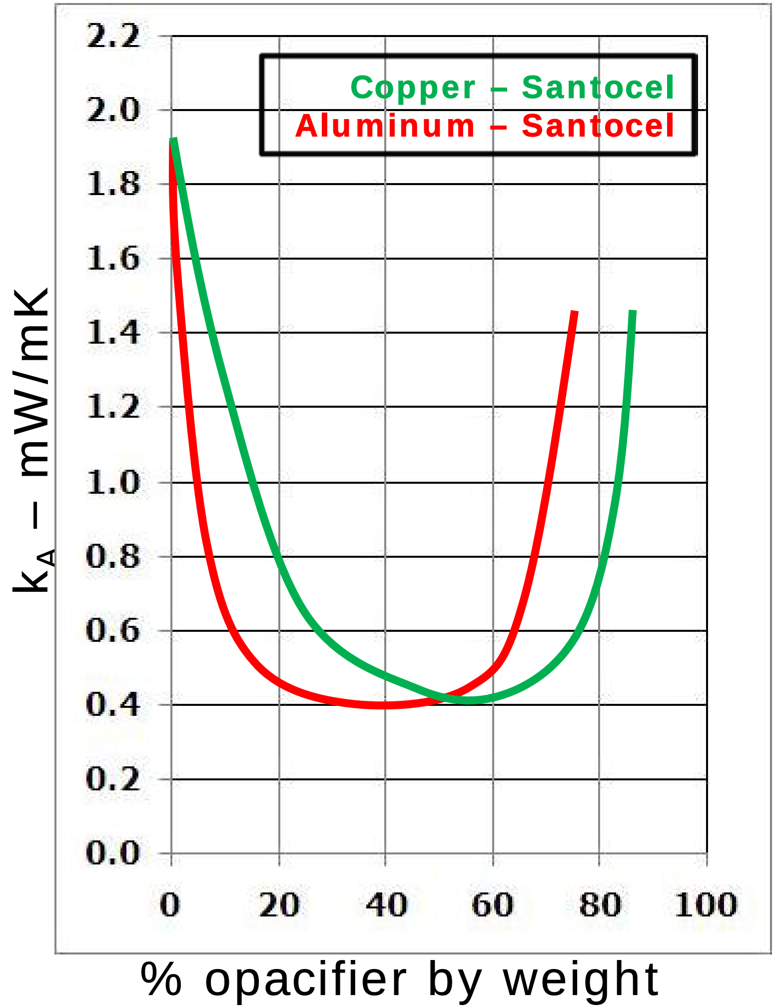


- The figure shows the variation of % opacifier with thermal conductivity for **Cu – santocel** and **Al – santocel**.

- There exists an optimum operating point for each of these insulations.

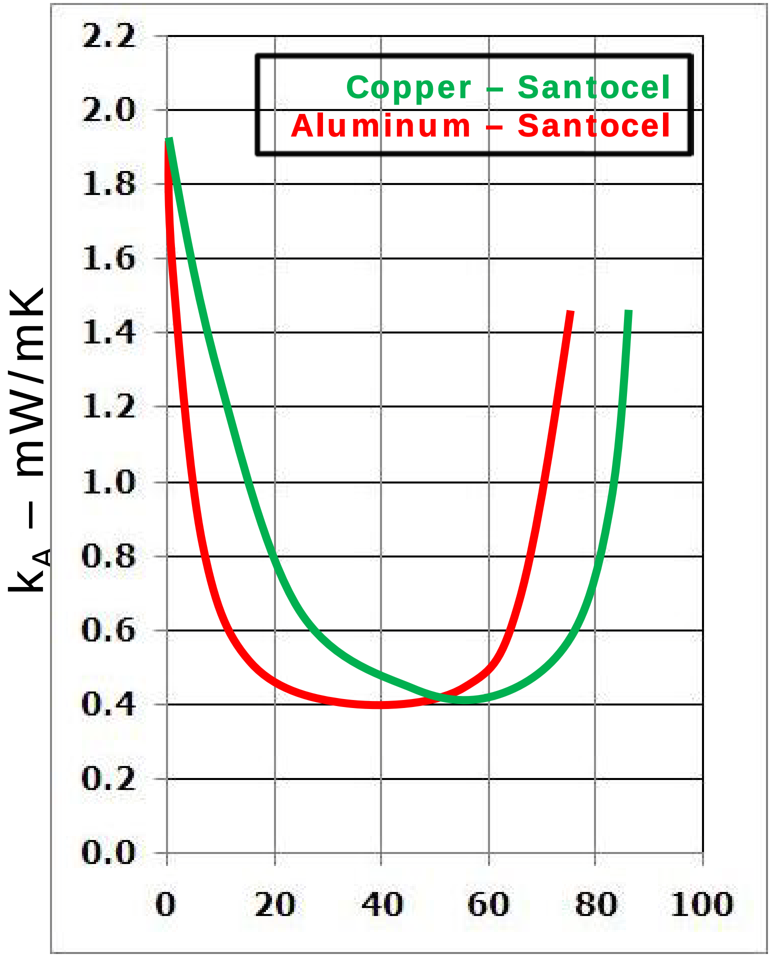
- It has been observed that, with these additions, $\mathbf{k_A}$ can be reduced by 5 times.

Opacified Powder Insulation



- **Cu** flakes are more preferred as compared to **Al** flakes.
- The **Al** flakes have large heat of combustion.
- These together with $\mathbf{O_2}$ can lead to accidents when used on **LOX** containers.

Opacified Powder Insulation



% opacifier by weight

- Another disadvantage of this insulation is that the vibrations tend to pack the flakes together.
- This, not only increases the thermal conductivity but also short circuits the conduction heat transfer.

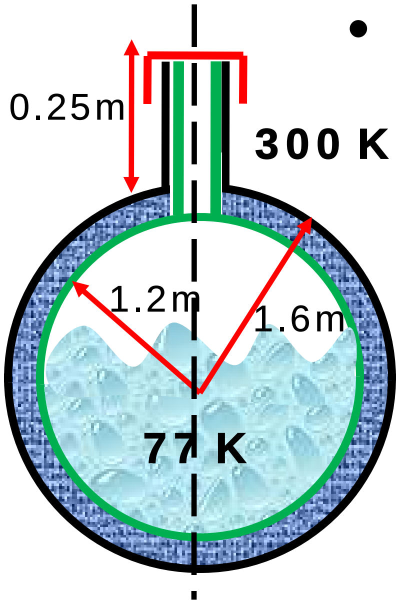
Opacified Powder Insulation

- The apparent thermal conductivity (mW/mK) and density (kg/m^3) of few commonly used opacified powder insulations are as shown.

- The residual gas pressure is less than 10^{-3} torr for temperatures between **77 K** to **300 K**.

Powder	ρ (kg/m^3)	k (mW/mK)
50/50 Cu – Santocel	180	0.33
40/60 Al – Santocel	160	0.35
50/50 Bronze – Santocel	179	0.58
Silica – Carbon	80	0.48

Tutorial



- A spherical **LN2** vessel (**e**=0.8) is as shown. The inner and outer radii are 1.2m and 1.6m respectively. Compare and comment on the heat in leak for the following cases.

  - Perlite (26 mW/mK)
  - Less Vacuum (1.5mPa)
  - Vacuum alone
  - Vacuum + 10 shields (e_s=0.05)
  - Evacuated Fine Perlite (0.95 mW/mK)
  - 50/50 Cu – Santocel (0.33 mW/mK)

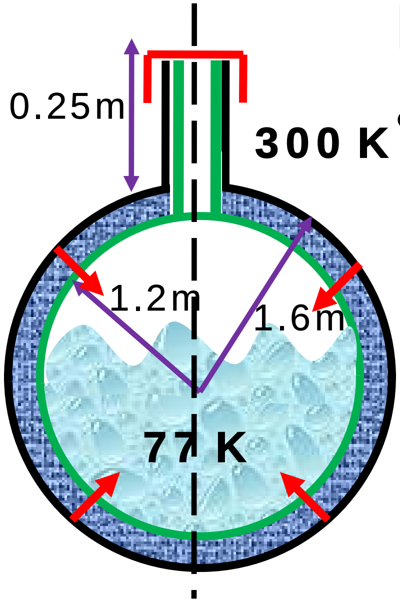
Tutorial

Given
Apparatus : Spherical vessel (**e**=0.8)
Working Fluid : Liquid Nitrogen
Temperature : 77 K (inner), 300 K (outer)

Calculate heat in leak	
1	Perlite (26 mW/mK)
2	Less Vacuum (1.5mPa)
3	Vacuum alone
4	Vacuum + 10 shields
5	Evacuated Fine Perlite (0.95 mW/mK)
6	50/50 Cu – Santocel (0.33 mW/mK)

- The shape factor between the two containers is assumed to be 1.

Tutorial



Perlite (k_A = 26mW/m-K)

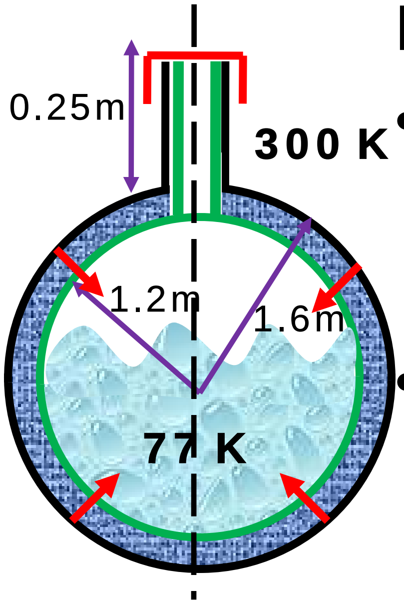
- **Sphere** - R_1=1.6m, R_2=1.2m, k_A, ΔT=(300-77)=223.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{\left(R_2 - R_1\right)}$$

$$Q = \frac{4\pi\left(26\right)\left(10^{-3}\right)\left(1.6\right)\left(1.2\right)\left(223\right)}{\left(1.6-1.2\right)}$$

$$Q = 349.7W$$

Tutorial



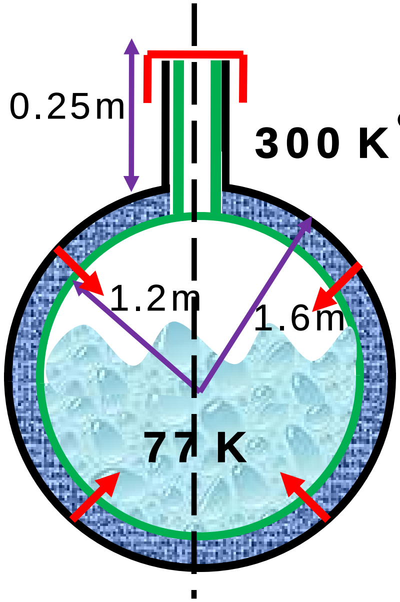
Less Vacuum (1.5mPa)

- **Sphere** - $R_1=1.6$m, $R_2=1.2$m, $e_1=e_2=0.8$, $T_1=77$ K, $T_2=300$ K.
- The net heat transfer is due to both radiation and residual gas conduction.

$$F_e=\left(\frac{1}{e_1}+\left(\frac{A_1}{A_2}\right)\left(\frac{1}{e_2}-1\right)\right)^{-1}$$

$$F_e=\left(\frac{1}{0.8}+\left(\frac{1.2}{1.6}\right)^2\left(\frac{1}{0.8}-1\right)\right)^{-1}=0.72$$

Tutorial



Less Vacuum (1.5mPa)

- **Sphere** - R_1=1.6m, R_2=1.2m, e_1=e_2=0.8, T_1=77 K, T_2=300 K.

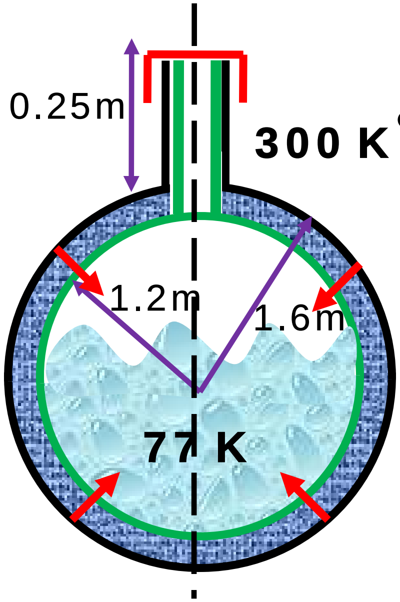
$$Q = F_e F_{1\rightarrow 2} \sigma A_1 \left(T_2^4 - T_1^4\right)$$

$$F_e = 0.72$$

$$Q = (0.72)(1)(5.67)\left(10^{-8}\right)\pi\left(1.6^2\right)\left(300^4 - 77^4\right)$$

$$Q_r = 2648W$$

Tutorial



Less Vacuum (1.5mPa)

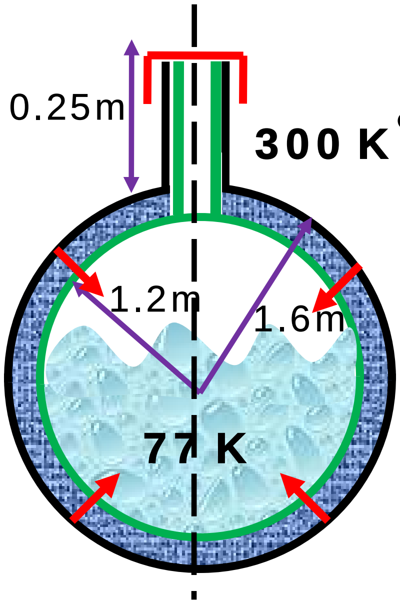
- **Sphere** - $R_1=1.6$m, $R_2=1.2$m, $T_1=77$ K, $T_2=300$ K, p=1.5 mPa.

$$\lambda = \frac{\mu}{p}\left(\frac{\pi RT}{2}\right)^{0.5}$$

$$\lambda = \frac{(18.47)(10^{-6})}{(1.5)(10^{-3})}\left(\frac{\pi(287.6)(300)}{2}\right)^{0.5} = 4.53$$

- It is clear that the mean free path (**λ**) is greater than distance between the surfaces (0.4m).

Tutorial



Less Vacuum (1.5mPa)

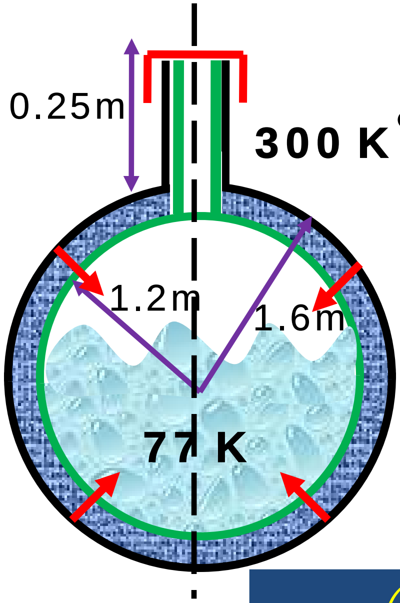
- **Sphere** - $R_1=1.6$m, $R_2=1.2$m, $T_1=77$ K, $T_2=300$ K, p=1.5 mPa.

$$F_a=\left(\frac{1}{\alpha_1}+\left(\frac{A_1}{A_2}\right)\left(\frac{1}{\alpha_2}-1\right)\right)^{-1}$$

$$F_e=\left(\frac{1}{1}+\left(\frac{1.2}{1.6}\right)^2\left(\frac{1}{0.85}-1\right)\right)^{-1}=0.91$$

T (K)	Air
300	0.8-0.9
78	1.0
20	1.0

Tutorial



Less Vacuum (1.5mPa)

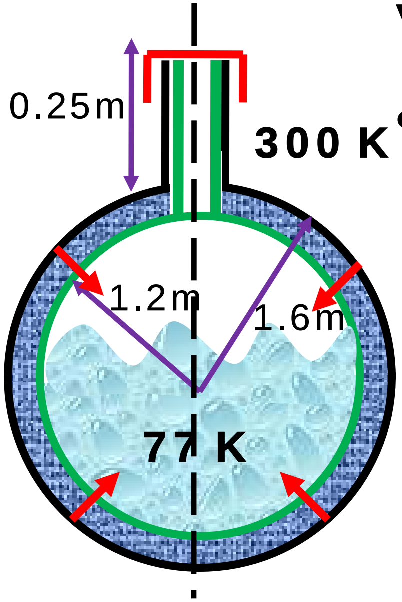
- **Sphere** - R_1=1.6m, R_2=1.2m, T_1=77 K, T_2=300 K, p=1.5 mPa.

$$\dot{Q} = \left(\left(\frac{\gamma+1}{\gamma-1} \right) \left(\frac{R}{8\pi T} \right)^{0.5} F_a \right) pA(T_2 - T_1)$$

$$\dot{Q} = \left(\left(\frac{1.4+1}{1.4-1} \right) \left(\frac{287.6}{8\pi(300)} \right)^{0.5} (0.91) \right) (1.5)(10^{-3})(300-77)$$

$$Q_{gc} = 0.356W$$

Tutorial



Vacuum alone

- **Sphere** - R_1=1.6m, R_2=1.2m, k_A, T_1=77K, T_2=300K, e_1, e_2=0.8, $F_{1\rightarrow2}$=1.

$$Q = F_e F_{1\rightarrow2} \sigma A_1 \left(T_2^4 - T_1^4\right)$$

$$F_e = 0.72$$

$$Q = (0.667)(1)(5.67)\left(10^{-8}\right)\pi\left(1.6^2\right)\left(300^4 - 77^4\right)$$

$$Q = 2648W$$

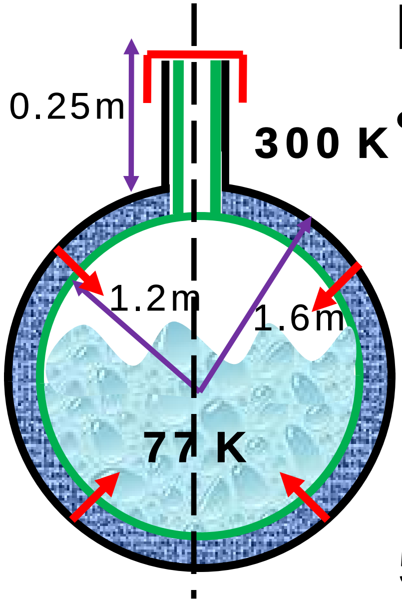
Vacuum + 10 shields

- e_1, e_2=0.8, e_s=0.05.

$$F_e = 0.003$$

$$Q = 11.02W$$

Tutorial



Evacuated Fine Perlite (k_A = 0.95mW/mK)

- **Sphere** - R_1=1.6m, R_2=1.2m, k_A, ΔT=(300-77)=223.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{\left(R_2 - R_1\right)}$$

$$Q = 12.7W$$

50/50 Cu – Santocel (k_A = 0.33mW/m-K)

- **Sphere** - R_1=1.6m, R_2=1.2m, k_A, ΔT=(300-77)=223.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{\left(R_2 - R_1\right)}$$

$$Q = 4.41W$$

Tutorial

Heat in leak (Q)	
Perlite	349.7 W
Less Vacuum (1.5mPa)	Q_r=2648 W Q_{gc}=0.356 W
Vacuum alone	2648 W
Vacuum + 10 shields	11.02 W
Evacuated Fine Perlite	12.7 W
50/50 Cu – Santocel	4.41 W

Summary

- In vacuum, the radiation is the dominant mode of heat transfer.
- Evacuated powders are superior in performance than vacuum alone in **300-77 K**, as the radiation heat transfer is comparatively less.
- At low pressures and temperatures, the solid conduction in evacuated powder dominates the radiant heat transfer.
- In an opacified powder, the radiation heat transfer is minimized by addition of reflective flakes.

Thank You!