CRYOGENIC ENGINEERING

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Lecture No - 36

Earlier Topics

- Introduction to Cryogenic Engineering
- Properties of Cryogenic Fluids
- Properties of Materials at Cryogenic Temperature
- Gas Liquefaction and Refrigeration Systems
- Gas Separation
- Cryocoolers
- Cryogenic Insulations

Current Topic

Topic : Vacuum Technology

- Need of Vacuum in Cryogenics
- Vacuum fundamentals
- Conductance and Electrical analogy
- Pumping Speed and Pump down time
- Vacuum Pumps

- The current topic will be covered in 3 lectures.

- Tutorials and assignments are also included.

Outline of the Lecture

Topic : Vacuum Technology

- Need of Vacuum in Cryogenics
- Vacuum Fundamentals
- Conductance and Electrical Analogy

Introduction

- The net heat in leak into a cryogenic vessel is
  - $Q_{Net} = Q_{Gas\ Cond.} + Q_{Conv.} + Q_{Solid\ Cond.} + Q_{Rad.}$

- Gas conduction and convection are minimized by having vacuum between two surfaces of different temperatures.

- Use of evacuated/opacified powders decreases $\mathbf{k_A}$. Also, MLI functions only in good vacuum.

- Therefore, vacuum technology forms a very important aspect in Cryogenics.

Vacuum

- The word **Vacuum** comes from the Latin roots. It means the **Empty** or the **Void**.
- A perfect vacuum can be defined as a space with no particles of any state (solid, liquid, gas etc.).
- It is important to note that the above definition is a theoretical understanding, although it is practically impossible to achieve perfect vacuum.
- The pressures in vacuum are lower than atmospheric pressures. The degree of vacuum is decided by mean free path (**λ**).

Mean Free Path

- Mean free path (**λ**) is defined as the average distance travelled by the molecules between the subsequent collisions.
- **λ** is given as

$$\lambda = \frac{\mu}{p}\left(\frac{\pi RT}{2}\right)^{0.5}$$

- Where,
  - **μ** – Viscosity of gas
  - **p** – Pressure of gas
  - **T** – Temperature of gas
  - **R** – Specific gas constant

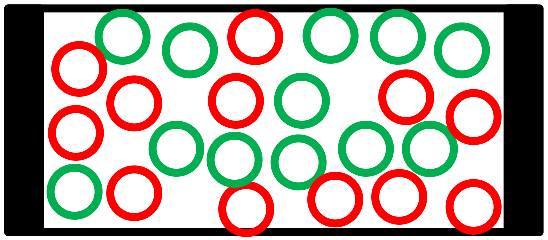
Mean Free Path

$$\lambda = \frac{\mu}{p}\left(\frac{\pi RT}{2}\right)^{0.5}$$

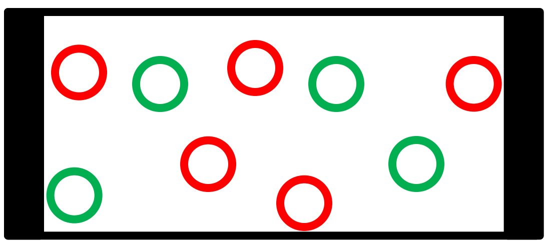
- It is clear that **λ**,
  - Increases with decrease in pressure
  - Increases with increase in temperature

- The value of mean free path (**λ**) plays an important role in deciding the flow regimes in vacuum.

Flow Regimes



High Pressure



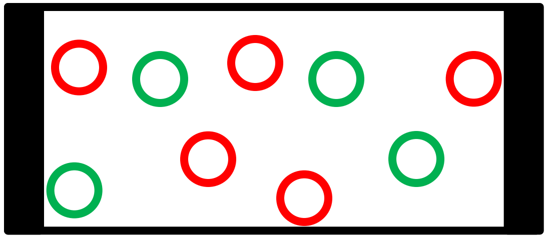
Low Pressure

- Consider a closed system as shown in the figure.
- With the lowering of pressure
  - The number of molecules are reduced
  - The residual molecules are pulled apart.
- As a result, mean free path (**λ**) of residual molecules becomes larger than the dimensions of the system.

Flow Regimes

- In such systems, the molecules collide only with the walls of the container.

- Such a flow of fluid is called as Free Molecular Flow.



Low Pressure

Flow Regimes

- If **λ** is much smaller than the characteristic lengths, such flows are called as continuum flows.

- In fluid mechanics, Reynold's Number (**Re**) is used to categorize the pipe flow regimes as shown above.

- In these flows, molecules collide with each other as well as physical boundaries, if any.

- Pressures are in the range of atmospheric values.

Flow Regimes in Vacuum

- Knudsen Number (**N_{Kn}**) is used to categorize the flow regimes in vacuum.
- This concept is analogous to Reynold's Number (**Re**) in fluid mechanics.
- Knudsen Number (**N_{Kn}**) is given as

$$N_{Kn} = \frac{\lambda}{D}$$

- Where,
  - **λ** – Mean free path
  - **D** – Characteristic diameter

Flow Regimes in Vacuum

- Based on the Knudsen Number ($\mathbf{N_{Kn}}$), the above figure characterizes the flow regimes in vacuum.

- Summarizing, we have
  - Continuum Flow for $\mathbf{N_{Kn}}$ less than 0.01.
  - Mixed Flow for $\mathbf{N_{Kn}}$ between 0.01 and 0.3.
  - Free Molecular Flow for $\mathbf{N_{Kn}}$ greater than 0.3.

Units of Pressure

- In an S. I. system, pressure is measured in **Pascal** or **N / m²**. Very often, **Bar** is also used for pressure measurement.

- For example, the standard atmospheric pressure can be expressed as
  - 1.013×10^5 **Pa**
  - 1.013×10^5 **N / m²**
  - 1 **bar**
  - **760 mm** of Hg column at standard sea level.

Units of Pressure

- In vacuum, normally unit for pressure is **Torr** or **milli bar**.
- This unit is named after **Evangelista Torricelli**, an Italian physicist, in the year 1644.
- **1 Torr** is defined as **1** mm of Hg column at standard sea level.
- Therefore, **1 Torr = 133.28 Pa = 133.28 N/m^2**.

Units of Pressure

- The conversion table for pressure is as shown below.

Conversion Table				
	Pa	Bar	atm	Torr
1 Pa	1	10^{-5}	9.8×10^{-6}	7.5×10^{-3}
1 Bar	10^{5}	1	0.98	750.06
1 atm	1.013×10^{5}	1.013	1	760
1 Torr	133.3	1.33×10^{-3}	1.31×10^{-3}	1

- **1 milli = 10^{-3}.**
- **1 Kilo = 10^{+3}.**

Degree of Vacuum

- As mentioned earlier, pressures are lower than atmospheric pressures in vacuum spaces.
- Depending upon the pressure in the system, the degree of the vacuum is categorized.
- The table on the next slide correlates the pressure and degree of the vacuum.

Degree of Vacuum

Degree of Vacuum	Pressure
Rough Vacuum	25 torr < **p** <760 torr
	3 kPa < **p** < 103 kPa
Medium Vacuum	0.001 torr < **p** < 25 torr
	0.1 Pa < **p** < 3000 Pa
High Vacuum	10^{-6} torr < **p** < 10^{-3} torr
	0.1 mPa < **p** < 100 mPa
Very High Vacuum	10^{-9} Torr < **p** < 10^{-6} Torr
	0.1 μPa < **p** < 100 μPa
Ultra High Vacuum	**p** < 10^{-9} torr

Pressure Drop

- Consider a fluid flowing across a pipe of constant cross sectional area as shown in the figure.

- For simplicity, let the flow regime be Continuum. As the fluid flows from point **1** to point **2**, there is a pressure drop due to viscosity. That is, $\mathbf{p_2 < p_1}$.

- The difference between inlet and exit pressures, ($\mathbf{p_1 - p_2}$), is called as pressure drop. Let it be denoted by $\mathbf{\Delta p}$. That is, $\mathbf{\Delta p = p_1 - p_2}$.

Pressure Drop

- Pressure drop for a laminar continuum flow is

$$\Delta p = \frac{128\mu L\dot{m}}{\pi D^4 \rho}$$

- It is called as Poisseuille's equation, which correlates pressure drop (**Δp**) and mass flow rate (**m**).

- Here,
  - **μ**, **ρ** – Viscosity and Density of fluid
  - **L**, **D** – Length and diameter of tube
  - **m** – Mass flow rate

Pressure Drop

- For a rough vacuum (N_{Kn}<0.01), as mentioned earlier, the operating pressures are in between **25** to **760** Torr.

- An ideal gas behaviour is assumed and hence, the corelation between average pressure and density is

$$\rho = \frac{\overline{p}M}{\Re T}$$

- Here,
  - **ρ**, **T** – Density and Temperature of gas
  - **M** – Molecular weight of gas
  - $\Re$ – Universal Gas constant
  - **p** – Average pressure

Pressure Drop

$$\Delta p = \frac{128\mu L\dot{m}}{\pi D^4 \rho}$$

$$\rho = \frac{\overline{p}M}{\Re T}$$

- Combining the above two equations, the pressure drop (**Δp**) for a continuum laminar flow is

$$\Delta p = \frac{128\mu L\dot{m}\Re T}{\pi D^4 \overline{p}M} \rightarrow \dot{m} = \frac{\pi D^4 \overline{p}M\Delta p}{128\mu L\Re T}$$

- From the above equation, it is clear that the mass flow rate (**m**) is
  - Directly proportional to pressure drop (**Δp**).
  - Directly proportional to **4th** power of diameter (**D**).

Pressure Drop

- With lowering of pressure in tube, ($0.01 < \mathbf{N_{Kn}} < 0.3$), an intermediate flow regime between the continuum and the free molecular flows exists.

- This regime is called as Mixed Flow or Slip Flow.

- In such conditions, the gas molecules close to the wall appear to slip past the wall with a finite velocity parallel to axis of tube, and hence the name **slip flow**.

Pressure Drop

- From the kinetic theory of gases, mass flow rate (**ṁ**) and pressure drop (**Δp**) for slip flow in a circular tube is given by

$$\dot{m} = \frac{\pi D^4 \overline{p} \Delta p}{128 \mu L \Re T}\left(1 + \frac{8\mu}{\overline{p} D}\left(\frac{\pi \Re T}{2M}\right)^{0.5}\right) \qquad \dot{m} = \frac{\pi D^4 \overline{p} M \Delta p}{128 \mu L \Re T}$$

- On comparison of above equation with mass flow rate (**ṁ**) for a continuum laminar flow,
  - The first term accounts for the internal laminar flow (away from walls).
  - The second term accounts for the finite velocity correction near the tube walls.

Pressure Drop

$$\dot{m} = \frac{\pi D^4 \bar{p} \Delta p}{128 \mu L \Re T}\left(1 + \frac{8\mu}{\bar{p} D}\left(\frac{\pi \Re T}{2M}\right)^{0.5}\right)$$

- From the above equation, the mass flow rate (**m**) is directly proportional to the pressure drop (**Δp**).

- However, the dependence of diameter (**D**) of the tube is more complex, as compared to **4th** power relationship in laminar continuum flow.

Pressure Drop

- With further lowering of pressure (**N_{Kn}**>0.3), the number of molecules are reduced as well as the residual gas molecules are pulled apart.

- This flow regime is called as Free Molecular Flow.

- In such conditions, mean free path (**λ**) of the molecules is larger than the diameter of the tube. The flow is limited due to collisions of molecules with the walls.

Pressure Drop

- The mass flow rate (**m**) and the pressure drop (**Δp**) in a free molecular flow are related by

$$\dot{m} = \frac{D^3 \Delta p}{L} \left(\frac{\pi M}{18 \Re T} \right)^{0.5}$$

- From the above equation, it is clear that the mass flow rate (**m**) is
  - Directly proportional to pressure drop (**Δp**).
  - Directly proportional to **3rd** power of diameter (**D**).

Throughput (Q)

- Apart from mass flow rate (**m**), the rate of fluid flow is often measured by a quantity called as Throughput (**Q**).

- Throughput is defined as a product of volumetric flow rate (**V**) and pressure (**p**), measured at the point where **V** is measured.

- Mathematically, we have $Q = p\dot{V}$

- The S. I. unit for Throughput is **Pa-m³/s**. Very often at low pressures, it is also expressed in **Torr-Lit/s** or **bar-Lit/s**.

Throughput (Q)

- Assuming an ideal gas behavior, the volumetric flow rate (**V**) is expressed using an ideal gas law as

$$\dot{V} = \frac{\dot{m}\Re T}{pM}$$

- From the definition of Throughput, we have $Q = p\dot{V}$

- Combining the above two equations, we get

$$Q = \frac{\dot{m}\Re T}{M}$$

- Here,
  - **m** – Mass flow rate, **M** – Molecular weight of gas
  - **T** – Temperature, $\Re$ – Universal Gas constant

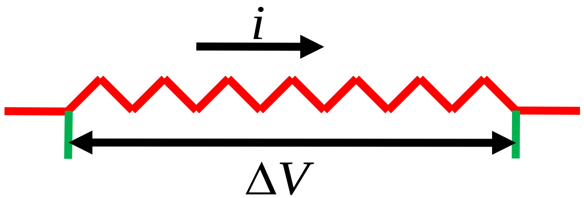
Electrical Analogy

- It is important to note that vacuum systems involve complex piping arrangements.

- In order to analyze these systems, a mathematical theory is developed based on an analogy between electrical circuits and piping systems.

- Linear transport laws like **Ohm's** law and **Fourier's** law are used in formulating the problem.

Electrical Analogy



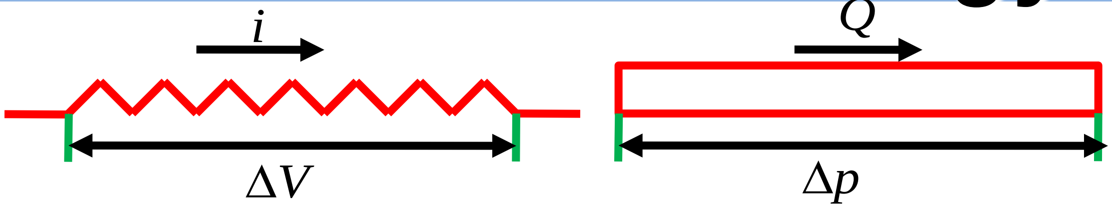
- Consider a small electric conductor as shown.

- When a current (**i**) flows across this conductor, there is a voltage drop (**ΔV**) due to the resistance (**R**) offered by the conductor.

- These quantities are mathematically related by **Ohm's** Law as given below.

$$\Delta V = iR$$

Electrical Analogy



- Similarly, consider a fluid flowing across a small pipe as shown above.

- For a throughput (**Q**), there is a pressure drop (**Δp**) due to conductance (**C**) offered by this pipe.

- Comparing the above figures, we have

ΔV analogues to **Δp**
i analogues to **Q**
R analogues to **1 / C**

$$\Delta V = iR \rightarrow \Delta p = \frac{Q}{C}$$

Conductance in Vacuum

$$\Delta p = \frac{Q}{C} \rightarrow Q = C(\Delta p)$$

- It is clear that for a given pressure drop (**Δp**) across a pipe, Throughput (**Q**) is directly proportional to conductance (**C**).

- For an ideal gas, the following equations hold true.

$$Q = \frac{\dot{m}\Re T}{M} \qquad \Delta p = \frac{\Delta\rho\Re T}{M}$$

- Substituting, we have

$$\frac{\dot{m}\Re T}{M} = C\frac{\Delta\rho\Re T}{M} \qquad C = \frac{\dot{m}}{\Delta\rho}$$

Conductance in Vacuum

- Conductance for a pipe for different flow regimes can be derived by rearranging the pressure drop – mass flow rate equations derived earlier.

- Continuum Flow (**N_{Kn}**<0.01) –

$$C = \frac{\pi D^4 \bar{p}}{128 \mu L}$$

- Mixed Flow – (0.01<**N_{Kn}**<0.3)

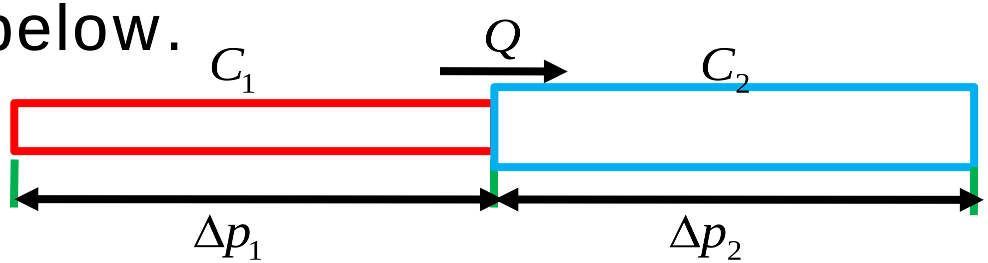
$$C = \frac{\pi D^4 \bar{p}}{128 \mu L}\left[1 + \frac{8\mu}{\bar{p} D}\left(\frac{\pi \Re T}{2M}\right)^{0.5}\right]$$

- Free Molecular Flow (**N_{Kn}**>0.3) –

$$C = \frac{D^3}{L}\left(\frac{\pi \Re T}{18M}\right)^{0.5}$$

Conductance in Vacuum

- Consider a series combination of two pipes with $\mathbf{C_1}$ and $\mathbf{C_2}$ as individual conductances respectively as shown below.

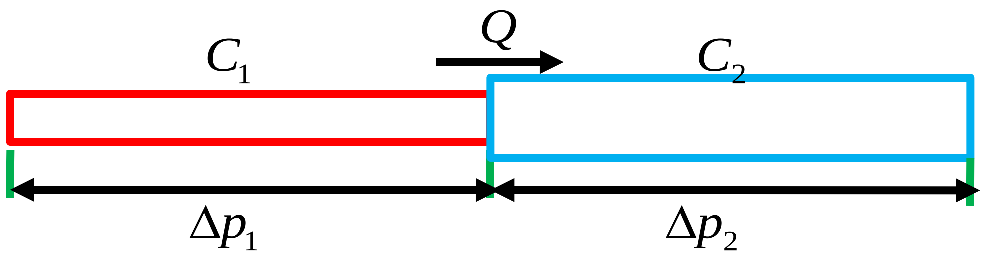


- Let **Q** be the Throughput for this system. It is clear that for a series combination, **Q** is same for each of the pipe.

- The pressure drops in each of the pipes are $\mathbf{\Delta p_1}$ and $\mathbf{\Delta p_2}$ respectively. That is,

$$\Delta p_1 = \frac{Q}{C_1} \qquad \Delta p_2 = \frac{Q}{C_2}$$

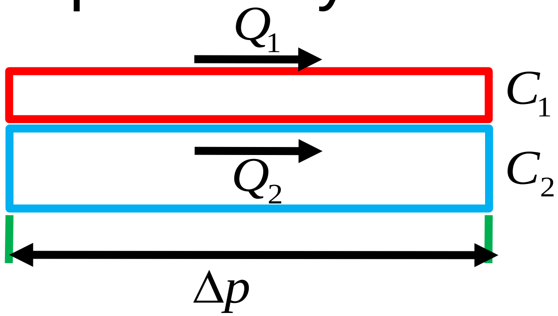
Conductance in Vacuum



- Let the overall conductance and the total pressure drop of the system be $\mathbf{C_o}$ and **Δp** respectively.
- Therefore, we have $\Delta p = \frac{Q}{C_o}$ $\Delta p_1 = \frac{Q}{C_1}$ $\Delta p_2 = \frac{Q}{C_2}$
- Using $\mathbf{\Delta p = \Delta p_1 + \Delta p_2}$, we get $\frac{1}{C_o} = \frac{1}{C_1} + \frac{1}{C_2}$
- Extending to **N** pipes in series, we have $\frac{1}{C_o} = \sum_i \frac{1}{C_i}$

Conductance in Vacuum

- Similarly, consider a parallel combination of two pipes with $\mathbf{C_1}$, $\mathbf{C_2}$ and $\mathbf{\Delta p}$ as conductance and pressure drop respectively.



- Let $\mathbf{C_o}$ and **Q** be given, we have

$$Q = C_o(\Delta p) \qquad Q_1 = C_1(\Delta p) \qquad Q_2 = C_2(\Delta p)$$

- Using $\mathbf{Q = Q_1 + Q_2}$, we get

$$C_o = C_1 + C_2$$

- Extending to **N** pipes in parallel, we have

$$C_o = \sum_i C_i$$

Summary

- Heat in leak is minimized by having vacuum between two surfaces of different temperatures.
- **λ** is defined as the average distance travelled by the molecules between the subsequent collisions.
- Based on Knudsen Number ($\mathbf{N_{Kn}}$), we have Continuum Flow ($\mathbf{N_{Kn}}<0.01$), Mixed Flow ($0.01<\mathbf{N_{Kn}}<0.3$), Free Molecular Flow ($\mathbf{N_{Kn}}>0.3$).
- Conductance

  Series : $$\frac{1}{C_o} = \sum_i \frac{1}{C_i}$$ Parallel : $$C_o = \sum_i C_i$$

- A self assessment exercise is given after this slide.

- Kindly asses yourself for this lecture.

Self Assessment

1. Gas conduction and convection are minimized by having ______.
2. The degree of vacuum is decided by ____.
3. In a _____ flow, mean free path (**λ**) is larger than the dimensions of the system.
4. ____ is used to categorize, pipe flow regimes in fluid mechanics.
5. ___ is used to categorize, flow regimes in vacuum.
6. 1 Torr = _____.
7. ___ equation correlates pressure drop and mass flow rate.
8. The intermediate flow regime between continuum and free molecular flows is _____.

Answers

1. Vacuum
2. Mean free path
3. Free Molecular Flow
4. Reynold's Number
5. Knudsen Number
6. 133.28 Pa
7. Poisseuille's
8. Slip Flow

Thank You!