CRYOGENIC ENGINEERING

Prof. Milind D. Atrey

Department of Mechanical Engineering,
IIT Bombay

Lecture No - 37

Earlier Lecture

- Heat in leak is minimized by having vacuum between two surfaces of different temperatures.
- **λ** is defined as the average distance travelled by the molecules between the subsequent collisions.
- Based on Knudsen Number ($\mathbf{N_{Kn}}$), we have Continuum Flow ($\mathbf{N_{Kn}}<0.01$), Mixed Flow ($0.01<\mathbf{N_{Kn}}<0.3$), Free Molecular Flow ($\mathbf{N_{Kn}}>0.3$).
- Conductance

  Series : $$\frac{1}{C_o} = \sum_i \frac{1}{C_i}$$ Parallel : $$C_o = \sum_i C_i$$

Outline of the Lecture

Topic : Vacuum Technology

- Conductance (contd..)
- Pumping Speed and Pump Down Time
- Tutorials

Introduction

- In the earlier lecture, we have seen the importance of vacuum in Cryogenics.
- The mean free path (**λ**) and the degree of vacuum decide the fluid flow regime.
- The conductance for a circular pipe for Continuum, Mixed and Free Molecular flow regimes, respectively, were derived as shown below.

$$C = \frac{\pi D^4 \overline{p}}{128 \mu L}$$

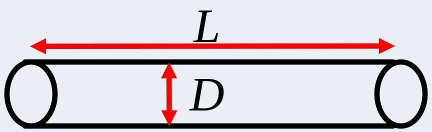
$$C = \frac{\pi D^4 \overline{p}}{128 \mu L}\left[1 + \frac{8\mu}{\overline{p} D}\left(\frac{\pi \Re T}{2M}\right)^{0.5}\right]$$

$$C = \frac{D^3}{L}\left(\frac{\pi \Re T}{18M}\right)^{0.5}$$

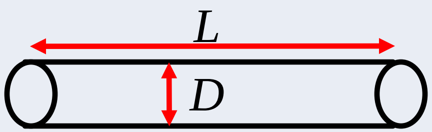
Conductance in Vacuum

- As mentioned in the earlier lecture, vacuum systems involve complex piping arrangements.
- These piping arrangements may involve circular straight tubes, rectangular straight tubes, 90^o elbow joints etc.
- The table on the next slide gives the conductance equations for some commonly used pipes and pipe joints.

Conductance in Vacuum

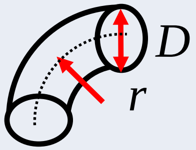
Element	Flow Regime	
Long Tube (L/D>30) L–Length D–Diameter	Continuum	$C = \dfrac{\pi D^4 \overline{p}}{128 \mu L}$ 
	Free Molecular	$C = \dfrac{D^3}{L}\sqrt{\left(\dfrac{\pi \Re T}{18 M}\right)}$

Conductance in Vacuum

Element	Flow Regime	
Long Tube (L/D>30) L–Length D–Diameter	Continuum	$C = \dfrac{\pi D^4 \bar{p}}{128 \mu L}$ 
	Free Molecular	$C = \dfrac{D^3}{L}\sqrt{\left(\dfrac{\pi \Re T}{18 M}\right)}$
Short Tube (L/D<30) D_1–Large Dia. D_2–Small Dia.	Continuum	$C = \dfrac{\pi D^4 \bar{p}}{128 \mu L}\left(1 + \dfrac{\dot{m}}{22 \mu L}\right)$
	Free Molecular	$C = \dfrac{D^2 \sqrt{\left(\pi \Re T / 18 M\right)}}{L/D + \left(4/3\right)\left(1 - \left(D_2/D_1\right)^2\right)}$

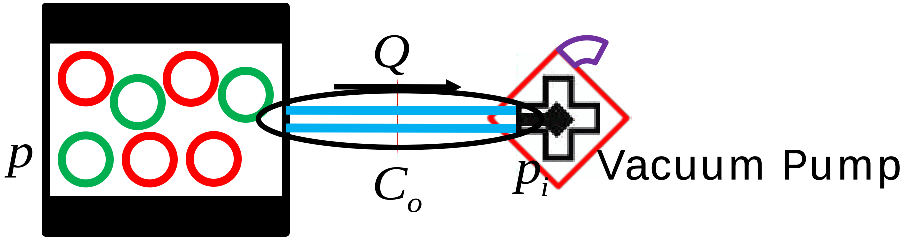
Conductance in Vacuum

Element	Flow Regime	
90° Elbow r-Mean radius D–Diameter	Continuum	$C=\frac{\pi K D^3 \overline{p}}{128\mu}$
	Free Molecular	$C=\frac{D^3}{r}\sqrt{\left(\frac{2\Re T}{9\pi M}\right)}$



K values					
r/D	K	r/D	K	r/D	K
0	0.017	4	0.073	10	0.034
1	0.05	6	0.056	12	0.029
2	0.083	8	0.042	14	0.026

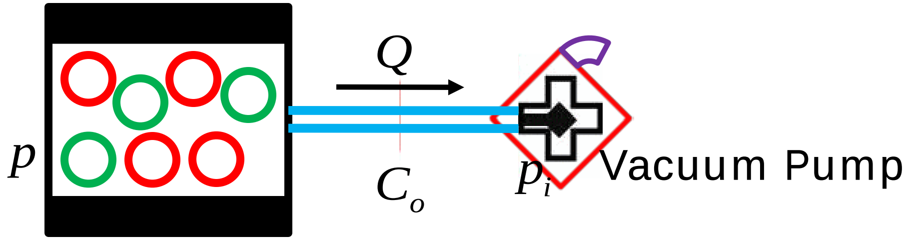
Pumping Speed



- Consider a closed cavity – vacuum pump system as shown in the figure.

- For the above system, let us assume the following parameters.
  - **p_i** – Pressure at the inlet to vacuum pump
  - **p** – Pressure in the cavity
  - **Q** – Throughput of the pump
  - **C_o** – Overall conductance of piping

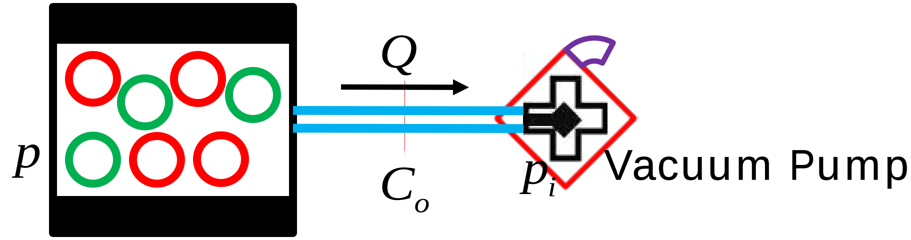
Pumping Speed



- In order to analyze the above system, the following quantities are defined.
- The capacity of a vacuum pump is denoted in terms of Pump Speed (**S_p**). It is the ratio of throughput (**Q**) to pressure at the inlet to vacuum pump (**p_i**).
- Mathematically, we have

$$S_p = \frac{Q}{p_i} \qquad m^3/\text{sec}$$

Pumping Speed

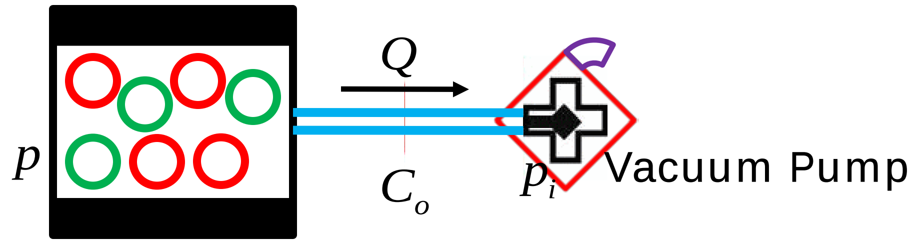


- On the similar lines, System Pumping Speed (**S_s**) is defined as the ratio of throughput (**Q**) to pressure in the cavity or the vacuumed space (**p**).

- Mathematically, we have $$S_s = \frac{Q}{p} \quad m^3/\sec$$

- Also, conductance (**C_o**) is $$C_o = \frac{Q}{(p - p_i)} \quad m^3/\sec$$

Pumping Speed



$$S_p = \frac{Q}{p_i} \qquad S_s = \frac{Q}{p} \qquad C_o = \frac{Q}{(p - p_i)}$$

$$p_i = \frac{Q}{S_p} \qquad p = \frac{Q}{S_s} \qquad p - p_i = \frac{Q}{C_o}$$

- Eliminating the pressures (**p$_i$** and **p**) from the above equations, we get

$$\frac{Q}{S_s} = \frac{Q}{S_p} + \frac{Q}{C_o} \qquad \frac{1}{S_s} = \frac{1}{S_p} + \frac{1}{C_o}$$

Pumping Speed

$$\frac{1}{S_s} = \frac{1}{S_p} + \frac{1}{C_o}$$

- From the above equation, it is clear that, $\mathbf{S_s}$ is lower than the minimum of $\mathbf{S_p}$ and $\mathbf{C_o}$.

- $\mathbf{S_p}$ depends on vacuum pump and therefore, in order to maximize $\mathbf{S_s}$, $\mathbf{C_o}$ should be maximum.

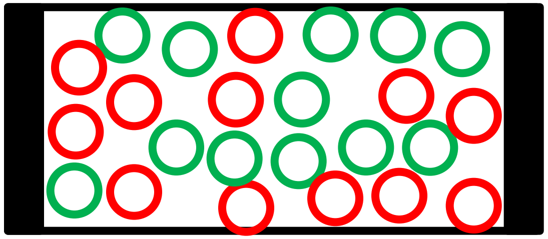
- In principle, $\mathbf{S_s}$ can be maximum when $\mathbf{C_o}$ is infinite.

Pumping Speed

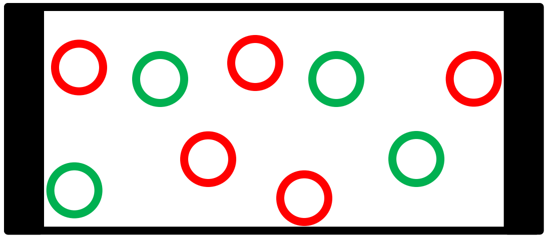
$$\frac{1}{S_s}=\frac{1}{S_p}+\frac{1}{C_o}$$

- That is, if $\mathbf{C_o} = \boldsymbol{\infty}$, then we have $\mathbf{S_s} = \mathbf{S_p}$.
- For a given connecting pipe, conductance increases with decrease in length and increase in diameter.
- Similarly, if $\mathbf{C_o} = \mathbf{S_p}$, then we have $\mathbf{S_s} = \mathbf{S_p / 2}$.

Pump Down Time



High Pressure - p_i



Low Pressure - p_f

- Consider a closed system as shown in the figure.
- Let the initial pressure in the system be $\mathbf{p_i}$.
- After vacuuming, let the final pressure in the system be $\mathbf{p_f}$.
- The amount of time taken by a vacuum pump to reduce the pressure from $\mathbf{p_i}$ to $\mathbf{p_f}$ is called as **Pump Down Time**.

Pump Down Time

- The application decides the degree of vacuum.
- Depending upon the application, the required pump down time is determined.
- Pump down time helps in selection of vacuum pump.
- Hence, there is a need to study the pump down time of vacuum system.

Gas Leaks

- Apart from heat in leak, gas in leak and out gassing are the major problems posed by a Cryogenic system.
- These leakage paths have to be considered for calculation of pump down time and selection of pump.
- The leakage paths increase the pump down time.

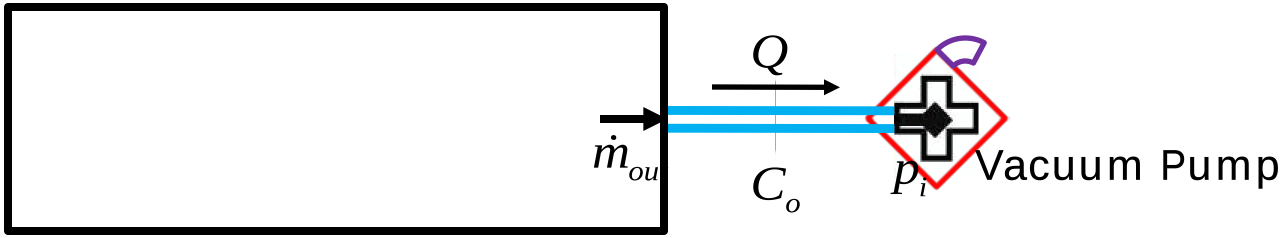
Gas Leaks

- Gas enters the system due to
  - Actual leak through the vessel walls, joints.
  - Trapped gas released from the pockets within the system, also called as Virtual leaks.
  - Out gassing of the metal walls or seals.

- Out gassing is the release of adsorbed gases either from surface or interior or both, when exposed to vacuum.

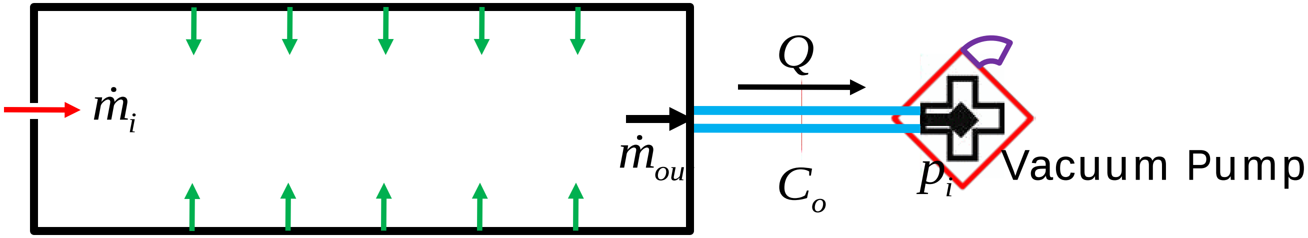
- The major contribution to mass in leak is the out gassing of the metal walls.

Pump Down Time



- Consider a closed cavity – vacuum pump system as shown above.
- Let the mass flow rate, leaving the system be $\mathbf{m_{out}}$.
- Mathematically, $\dot{m}_{out} = \rho S_s$
- Where, **ρ** and $\mathbf{S_s}$ are the density and system pumping speed respectively.

Pump Down Time



- Let the total inflow due to gas leak and out gassing be **m_i**. It can also be written as $\dot{m}_i = \dfrac{Q_i}{RT}$
- Applying mass conservation to this system, we have

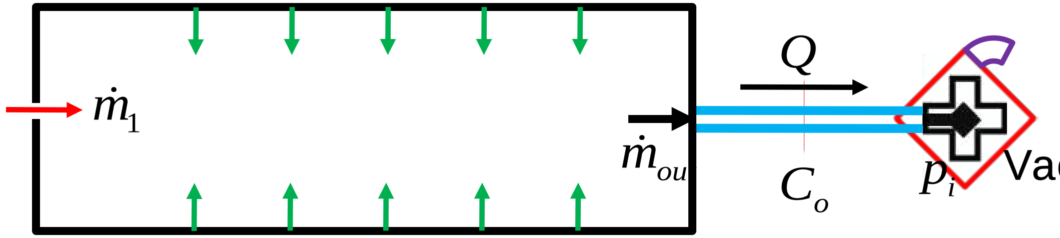
$$\dot{m}_i - \dot{m}_{out} = \frac{dm}{dt}$$

- From the definition of density, we have

$$m = \rho V$$

$$\dot{m}_i - \dot{m}_{out} = V\frac{d\rho}{dt}$$

Pump Down Time



$$\dot{m}_i - \dot{m}_{out} = V\frac{d\rho}{dt}$$

$$\rho = \frac{p}{RT}$$

- Using the ideal gas law, we have

$$\dot{m}_i - \dot{m}_{out} = \frac{V}{RT}\frac{dp}{dt}$$

- Combining the following equations, we have

$$\dot{m}_i = \frac{Q_i}{RT}$$

$$\dot{m}_{out} = \rho S_s$$

$$\frac{Q_i}{RT} - \frac{pS_s}{RT} = \frac{V}{RT}\frac{dp}{dt}$$

$$\frac{dp}{dt} = \frac{Q_i}{V} - \frac{S_s p}{V}$$

Pump Down Time

$$\frac{dp}{dt} = \frac{Q_i}{V} - \frac{S_s p}{V}$$

- This equation is valid for any vacuum system in general.
- It is important to note that both $\mathbf{Q_i}$ and $\mathbf{S_s}$ are time dependent functions, that is, $\mathbf{Q_i = f_1(t)}$, $\mathbf{S_s = f_2(t)}$.
- The equation can be integrated, analytically or numerically, if the transient variation of $\mathbf{Q_i}$ and $\mathbf{S_s}$ are known.

Pump Down Time

$$\frac{dp}{dt} = \frac{Q_i}{V} - \frac{S_s p}{V}$$

- At steady state or after a long time, the changes in pressure with time are negligibly small.
- Mathematically, we have **dp/dt=0**.
- The pressure at steady state is called as **Ultimate Pressure** (**p_u**). It is the minimum possible pressure that can be achieved using a certain pump.

$$p = p_u$$

- Therefore, we get $$p_u = Q_i / S_s$$

Pump Down Time

- It is important to note that, for most of the pumps, $\mathbf{S_p}$ is constant in its operating pressure range.

- Also, in a free molecular region, conductance (**C**) is independent of pressure.

$$\frac{1}{S_s} = \frac{1}{S_p} + \frac{1}{C_o}$$

- With $\mathbf{S_p}$ and $\mathbf{C_o}$ being constants, from the above equation, it is clear that $\mathbf{S_s}$ is also a constant or it is independent of pressure.

Pump Down Time

- Therefore, for a constant $\mathbf{S_s}$, the equation can be integrated with the following limits.

$$\frac{dp}{dt} = \frac{Q_i}{V} - \frac{S_s p}{V}$$

Limits	
t=0	$p=p_1$
$t=t_p$	$p=p_2$

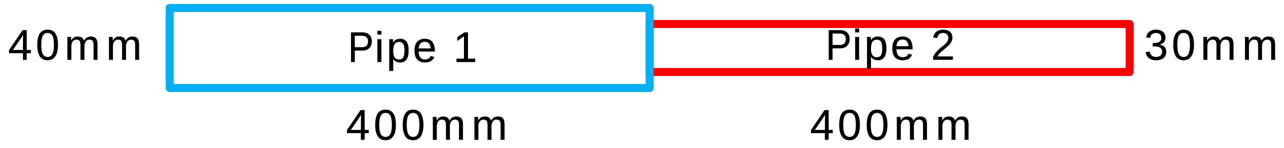
$$\int_{p_1}^{p_2} \frac{Vdp}{Q_i - S_s p} = \int_0^{t_p} dt$$

$$p_u = \frac{Q_i}{S_s}$$

$$-\frac{V}{S_s} \ln\left(Q_i - S_s p\right)\Big|_{p_1}^{p_2} = t\Big|_0^{t_p}$$

$$t_p = \frac{V}{S_s} \ln\left(\frac{p_1 - p_u}{p_2 - p_u}\right)$$

Tutorial – 1



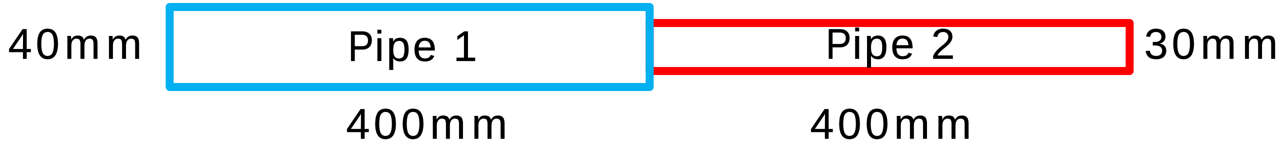
- Calculate the overall conductance of the pipe assembly shown above. The pressure on the right end of the 40 mm tube is 150 mPa, while the pressure on the left end of 30mm pipe is 10 mPa. The ambient temperature is 300 K. The molecular weight and viscosity of air are 28.95 g/mol and 18.47 μPa-s.

Tutorial – 1

Given	
Apparatus : Series Combination of pipes	
Working Fluid : Air (mol. wt. 28.95 g/mol)	
Temperature : 300 K	
Dimensions	Pipe 1 – 40mm dia., 400mm Length Pipe 2 – 30mm dia., 400mm Length

Calculate
Overall Conductance (**C_o**)

Tutorial – 1



Calculation of Flow Regime

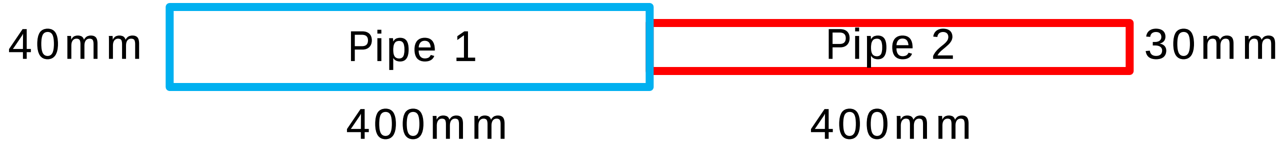
- N_{Kn} for pipe 1: D=0.04m, L=0.4m, T=300 K, R=8314/28.95, μ=18.47 μPa-s, p=0.15Pa.

$$N_{Kn} = \frac{\lambda}{D} = \frac{\mu}{Dp}\left(\frac{\pi RT}{2}\right)^{0.5}$$

$$N_{Kn} = \frac{18.47\left(10^{-6}\right)}{(0.04)(0.15)}\left(\frac{\pi(287.14)(300)}{2}\right)^{0.5}$$

$$N_{Kn} = 1.132$$

Tutorial – 1



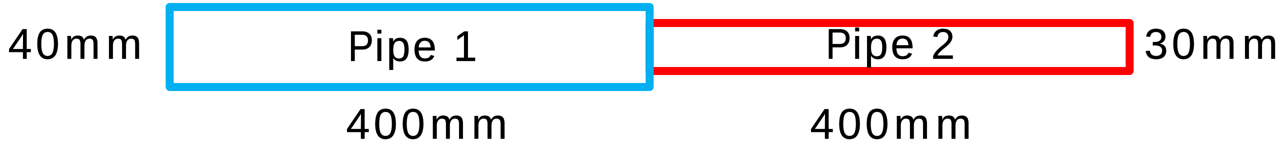
Similarly, calculating $\mathbf{N_{Kn}}$ for Pipe 2, we have

- $\mathbf{N_{Kn}}$ for pipe 1: 1.132
- $\mathbf{N_{Kn}}$ for pipe 2: 22.65

- The Knudsen numbers for both the pipes are greater than 0.3. Therefore, the flow is free molecular through out the series combination.

- The **L / D** ratios of each of these pipes being less than 30, these are classified as Short Pipes.

Tutorial – 1



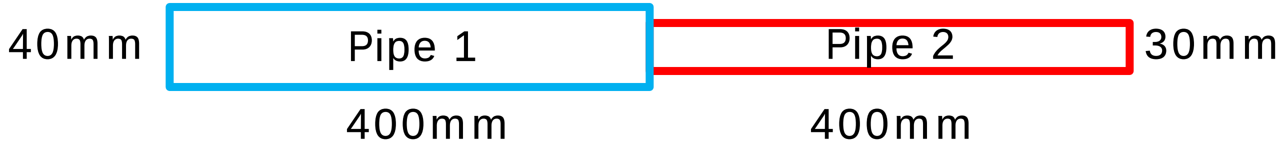
Conductance for pipe 1: D_1=0.04m, D_2=0.04m, L=0.4m, T=300 K, $\mathfrak{R}$=8314, M=28.95 gm/mol.

$$C_1 = \frac{D_1^2 \sqrt{\left(\pi \mathfrak{R} T / 18M\right)}}{L/D_1 + (4/3)\left(1 - \left(D_1/D_2\right)^2\right)}$$

$$C_1 = \frac{(0.04)^2 \sqrt{\left(\pi \mathfrak{R}(300)/18(28.95)\right)}}{0.4/0.04 + (4/3)\left(1 - (0.04/0.04)^2\right)}$$

$$C_1 = 0.0173 m^3 / s$$

Tutorial – 1



Similarly, calculating conductance for Pipe 2, we have

- Pipe 1 (**C_1**) : 0.0173
- Pipe 2 (**C_2**) : 0.0079

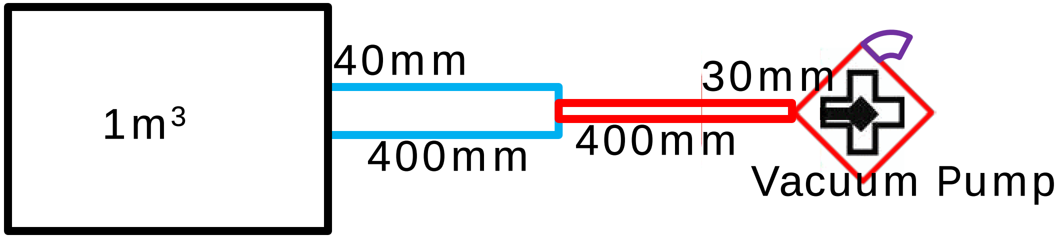
- The Overall Conductance (**C_o**) for a series combination is given by

$$\frac{1}{C_o} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_o} = \frac{1}{0.0173} + \frac{1}{0.0079}$$

$$C_o = 0.00542 m^3 / s$$

Tutorial – 2



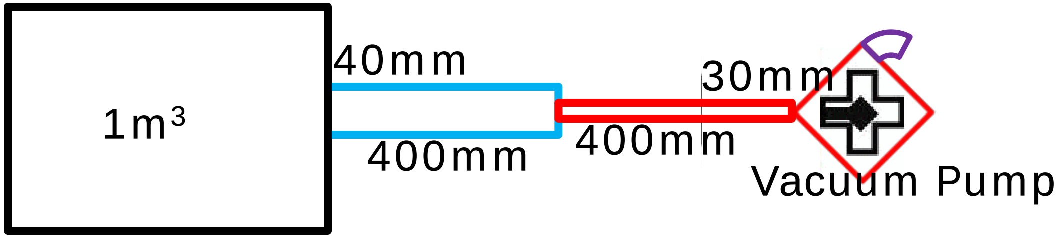
- Consider a vacuum vessel of $1m^3$ with an initial pressure of 1 atm at 300 K. It is connected to a vacuum pump via a connecting pipe as shown above. The ultimate pressure of the system is 0.1 mPa. Determine the system pumping speed, if the required vacuum in the cavity is 1 kPa in 1 hour.

Tutorial – 2

Given	
Apparatus	Vacuum Pump
Working Fluid	Air at 1 atm
Vacuum	1 kPa
Temperature	300 K
Connecting Pipe	Pipe 1 : 40mm (D), 400mm (L) Pipe 2 : 30mm (D), 400mm (L)
Time	1 Hour
Volume	1 m^3
Ultimate Pr.	0.1 mPa

Calculate
System Pumping Speed (S_p)

Tutorial – 2



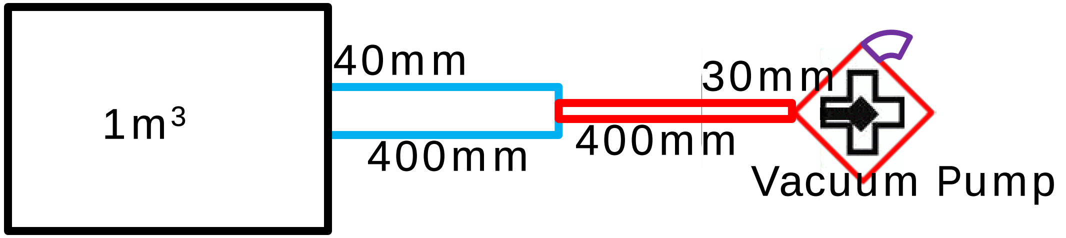
Calculation of S_s

- $V=1m^3$, $p_1 = 1.013x10^5$ Pa, $p_2 = 1000$ Pa, $p_u = 0.1x10^{-3}$ Pa, $t_p = 3600$ s.

$$S_s = \frac{V}{t_p}\ln\left(\frac{p_1 - p_u}{p_2 - p_u}\right)$$

$$S_s = \frac{1}{3600}\ln\left(\frac{101300 - 0.1\left(10^{-3}\right)}{1000 - 0.1\left(10^{-3}\right)}\right) = 0.0012 m^3/s$$

Tutorial – 2



From the earlier tutorial, we have

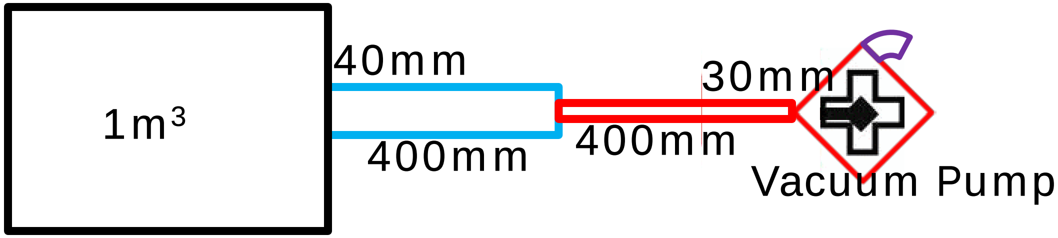
- $\mathbf{N_{Kn}}$ for pipe 1: 1.132, $\mathbf{N_{Kn}}$ for pipe 2: 22.65.

- $\mathbf{N_{Kn}}$>0.3, the flow is free molecular flow.

- The conductance of these Short Pipes
- Pipe 1 ($\mathbf{C_1}$) : 0.0173, Pipe 2 ($\mathbf{C_2}$) : 0.0079.

- The Overall Conductance ($\mathbf{C_o}$) is 0.00542 m^3/s.

Tutorial – 2



Calculation of S_p

- C_o = 0.00542, S_s = 0.0012.

$$\frac{1}{S_s} = \frac{1}{S_p} + \frac{1}{C_o}$$

$$\frac{1}{S_p} = \frac{1}{S_s} - \frac{1}{C_o}$$

$$\frac{1}{S_p} = \frac{1}{0.0012} - \frac{1}{0.00542}$$

$$S_p = 0.00154 m^3 / s$$

$$S_p = 92.4 Lit / \min$$

Summary

- Correlations for conductance for some commonly used pipes and pipe joints are given.
- Pump Speed : $S_p = \frac{Q}{p_i}$ System Speed : $S_s = \frac{Q}{p}$

$$\frac{1}{S_s} = \frac{1}{S_p} + \frac{1}{C_o}$$

- $\mathbf{S_p}$ depends on vacuum pump and therefore, in order to maximize $\mathbf{S_s}$, $\mathbf{C_o}$ should be maximum.
- For a constant $\mathbf{S_s}$, we have $t_p = \frac{V}{S_s} \ln\left(\frac{p_1 - p_u}{p_2 - p_u}\right)$

Thank You!