CRYOGENIC ENGINEERING

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Lecture No - 6

Properties of Materials

Sr. No.	Property
1	Mechanical
2	Thermal
3	Electrical
4	Magnetic

From Mech Engg. perspective

From SC perspective

Earlier Lecture

- Introduction to material properties
- Structure of Metals and Plastics
- Stress – strain relationship
- Mechanical properties of Metals at low temperature
- Mechanical properties of Plastics at low temperature

Outline of the Lecture

Title : Material Properties at Low Temperature (contd)

- Introduction
- Thermal properties
- Electric and Magnetic properties

Introduction

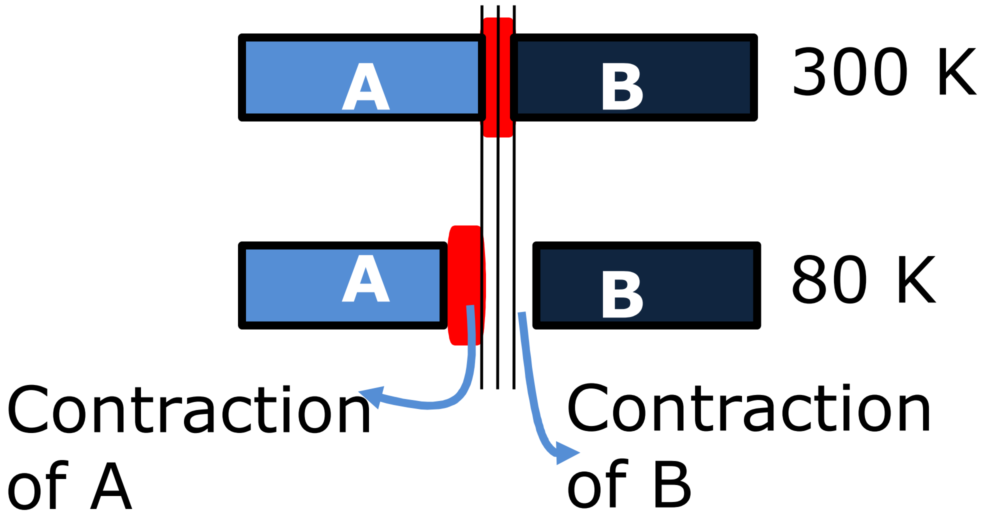
- The properties of materials change, when cooled to cryogenic temperatures (demo video).
- The electrical resistance of a conductor decreases as the temperature decreases.
- Shrinkage of metals occur when cooled to lower temperatures.
- Systems cool down faster at low temperatures due to decrease in the specific heat.
- Hence, a study of properties of materials at low temperatures is necessary for the proper design.

Material Properties

Sr. No.	Thermal Properties
1	Thermal Expansion/Contraction
2	Specific Heat of Solids
3	Thermal Conductivity

Thermal Expansion

- Reduction (contraction) in the dimensions of a material occur when cooled to low temperatures.



Thermal Expansion

- The linear coefficient of thermal expansion (λ_t)

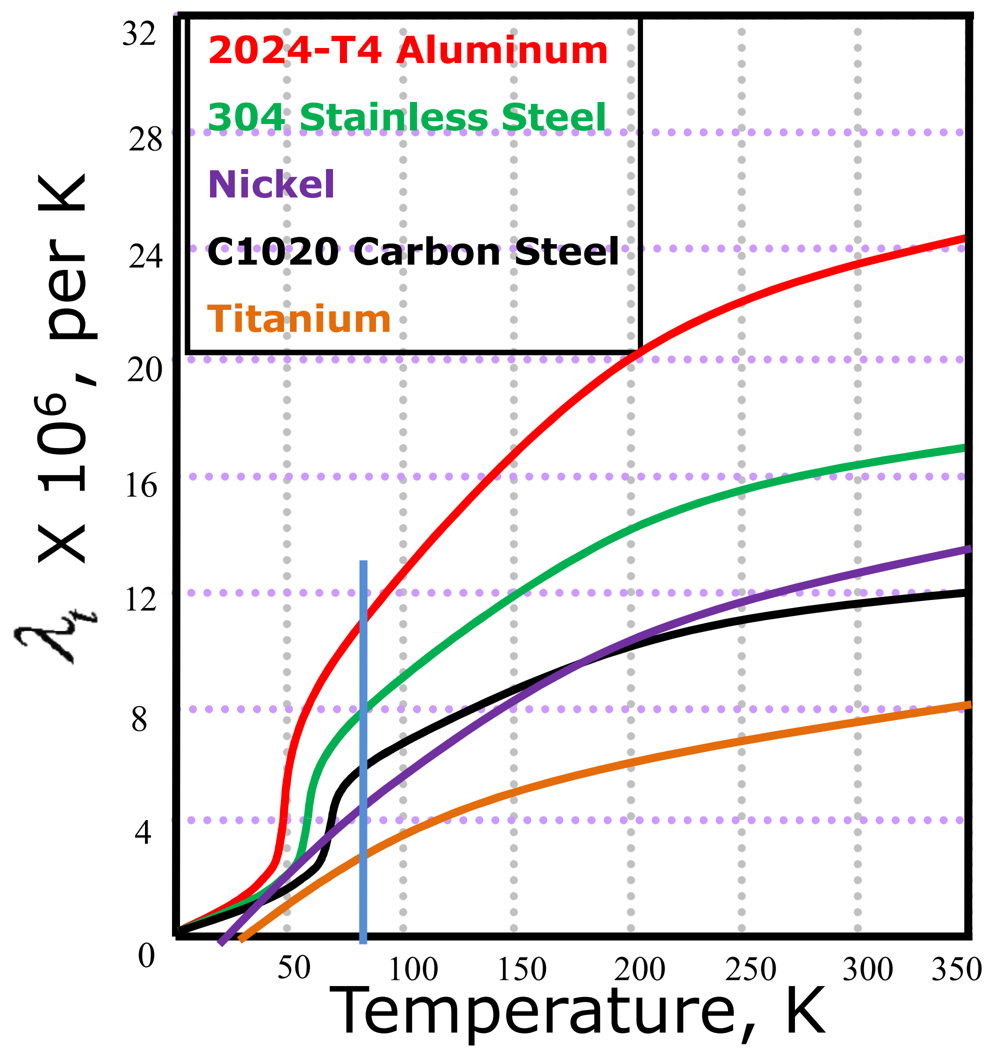
$$\lambda_t = \frac{(\delta L / L)}{\delta T} \qquad K^{-1}$$

- is the fractional change in length per unit change in temperature while the stress is constant.

- Similarly, the volumetric coefficient of thermal expansion (β) is the fractional change in volume per unit change in temperature while the pressure is constant.

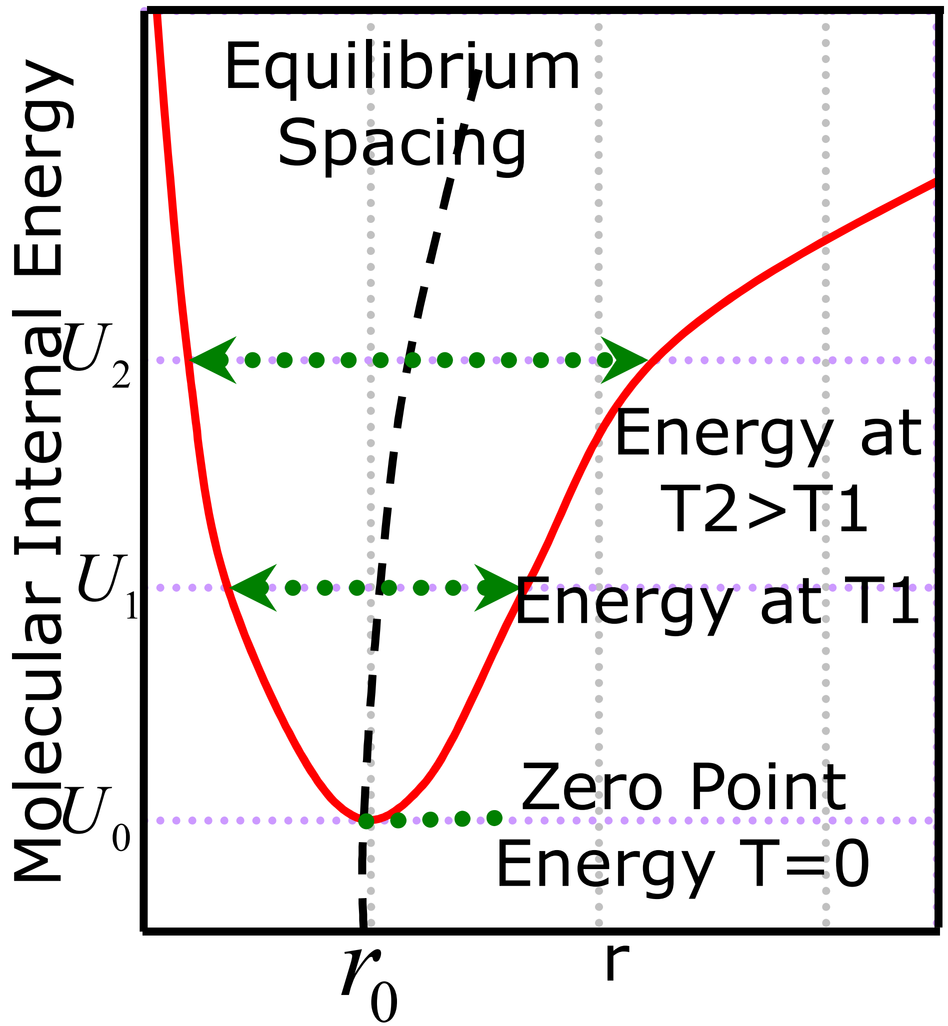
- For isotropic materials $\beta = 3\lambda_t$

Thermal Expansion



- The variation of λ_t for few of the commonly used materials is as shown.
- In general, the coefficient of thermal expansion decreases with the decrease in temperature.
- Most contraction occurs till 80 K.

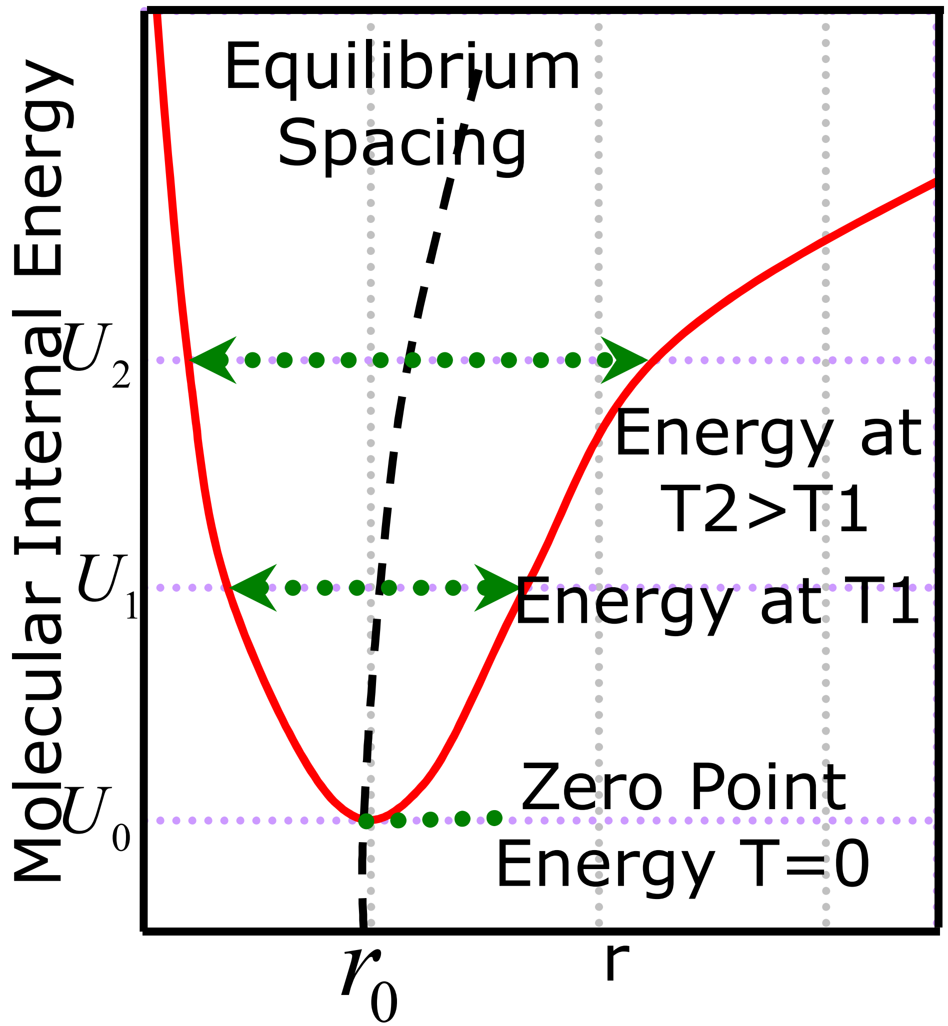
Thermal Expansion



- The variation of Molecular Internal Energy (U) with the intermolecular distance (r) is as shown. Here, r_o is the intermolecular distance at 0 K.

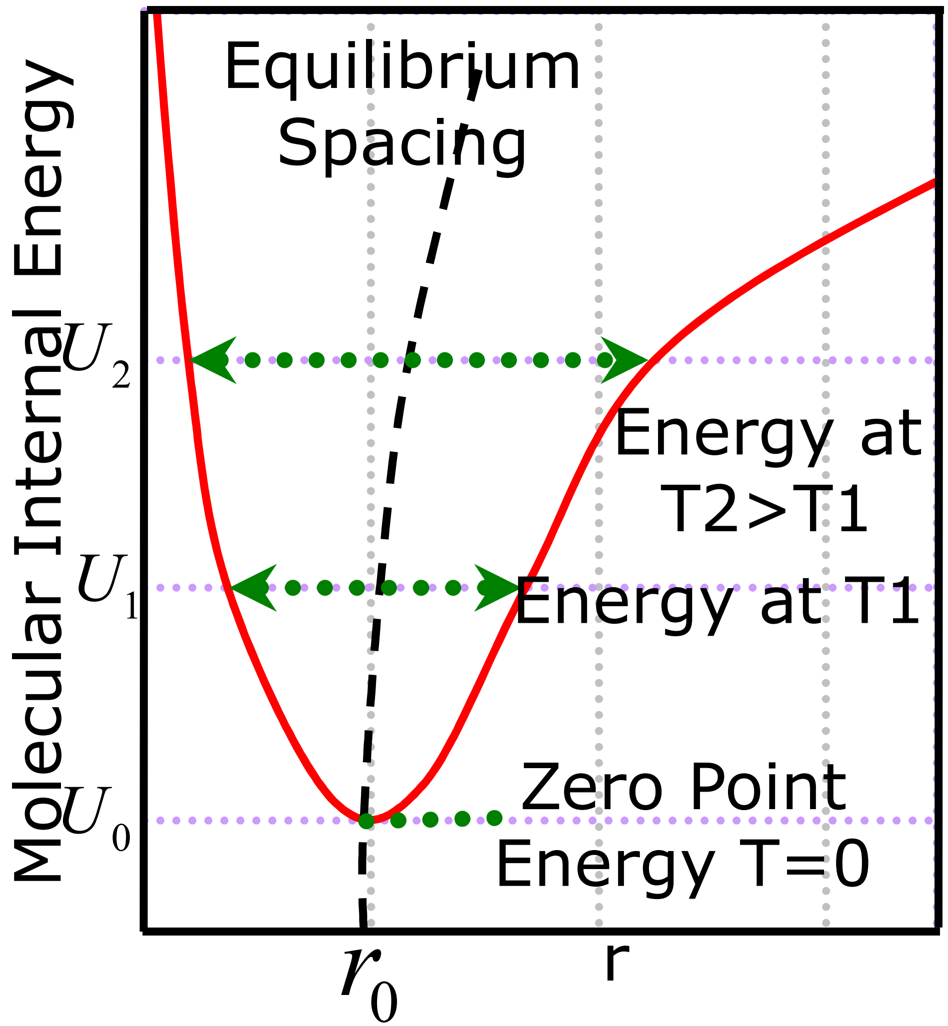
- The equilibrium spacing depicts the mean position of the atoms about which it oscillates.

Thermal Expansion



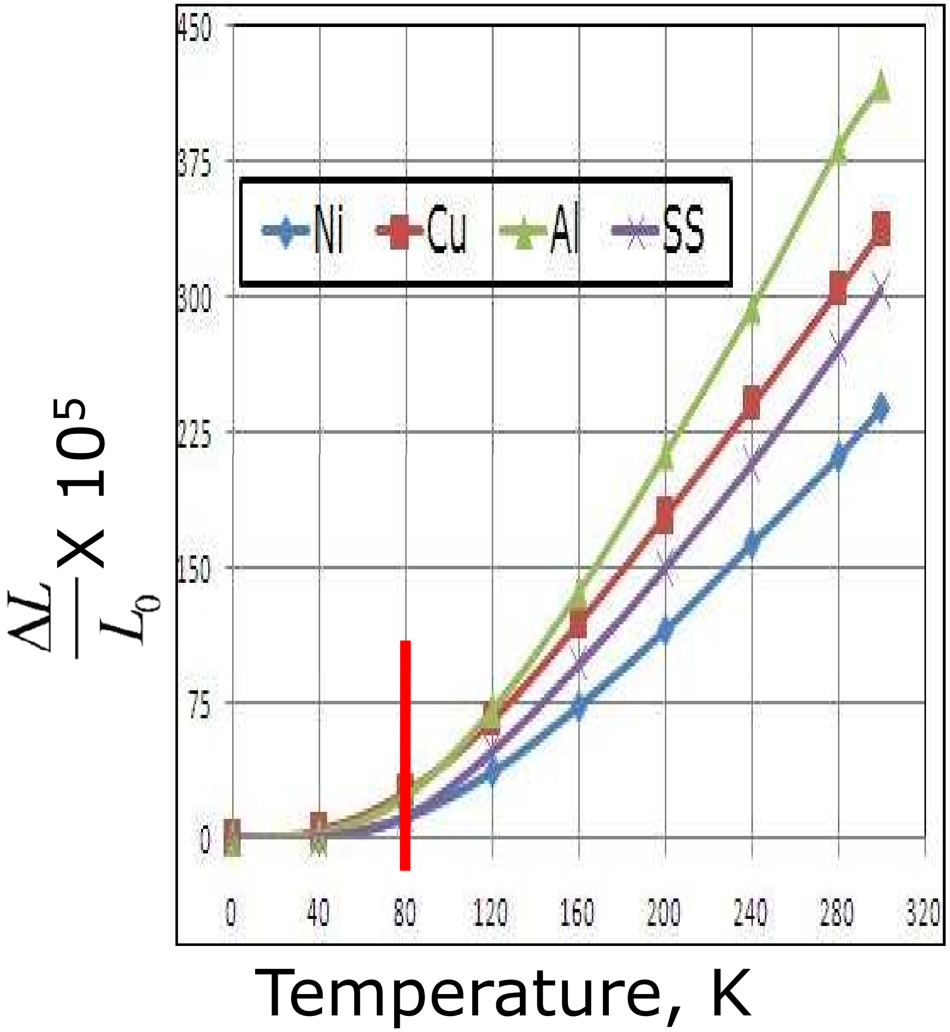
- With the rise in the temperature, the increased thermal agitation leads to increased inter molecular distance.
- The energy curve is asymmetric about the point r_o, stating that the atomic vibration is asymmetric.

Thermal Expansion



- The rate of increase of intermolecular distance increases with the increase in the temperature.
- Hence, the coefficient of thermal expansion (λ_t) increases with the increase in temperature.

Mean Linear Thermal Exp.



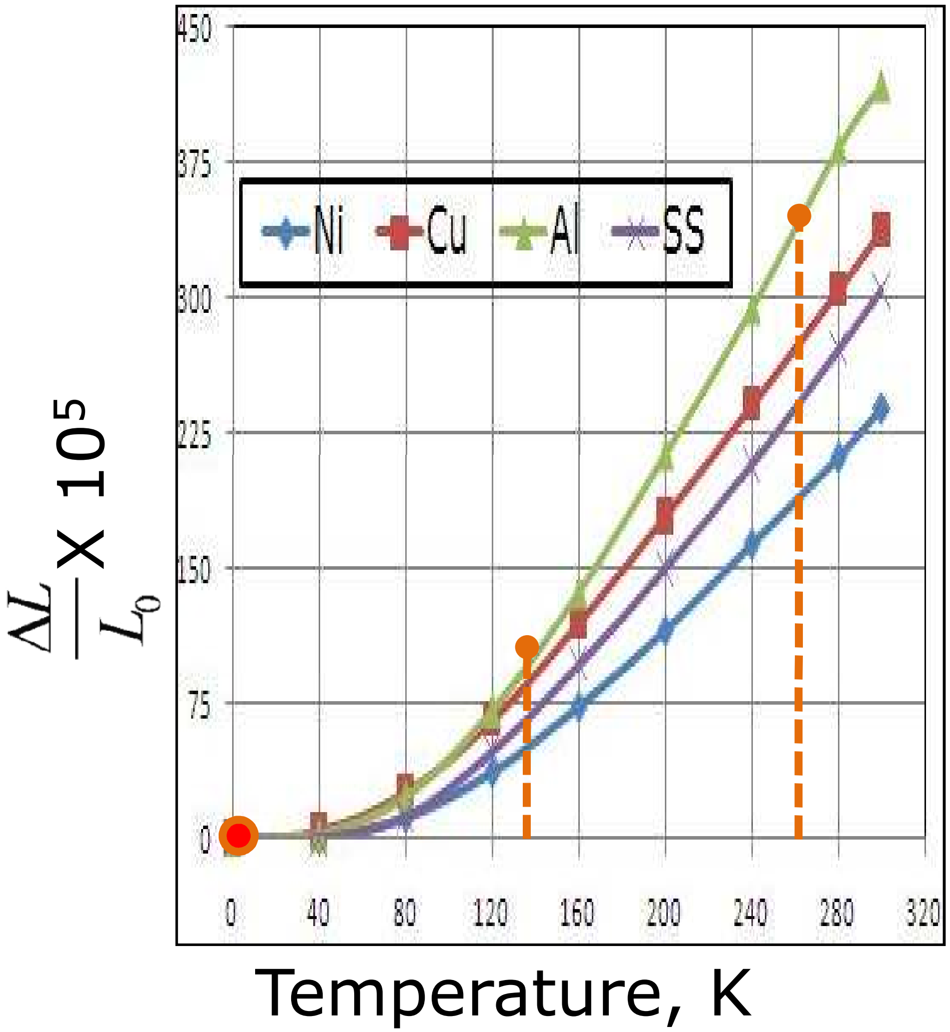
- Mean linear thermal expansion is defined as

$$\frac{\Delta L}{L_0} = \frac{L_T - L_0}{L_0}$$

- Here L_0 is the length at 0 K and L_T is length at any temperature T.

- Slope of curves is very high upto 80 K, and thereafter flattens.

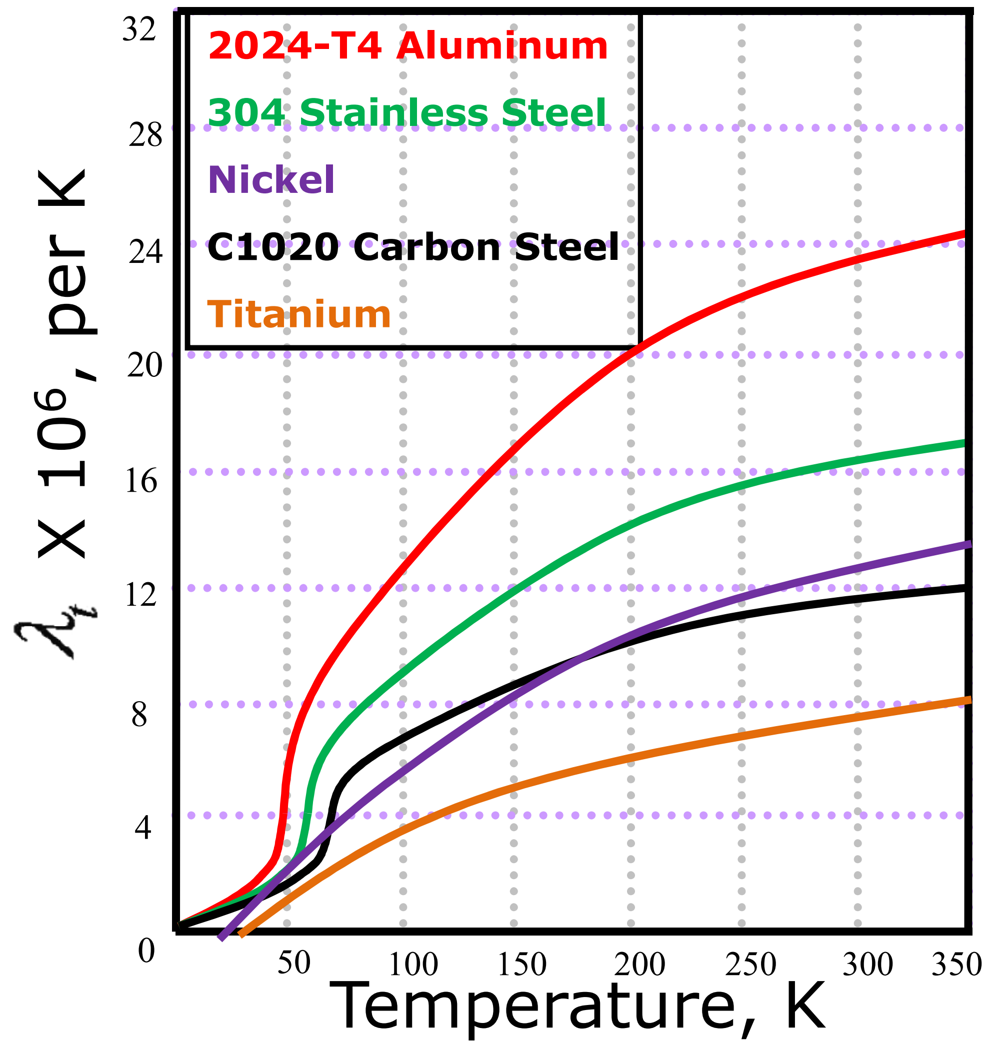
Mean Linear Thermal Exp.



- Mean linear thermal expansion can also be evaluated between two different temperatures.
- If L_{T1} & L_{T2} be the lengths of the specimen at T_1 and T_2 respectively. Then change in length is given by

$$\frac{\Delta L}{L_0} = \frac{L_{T1} - L_{T2}}{L_0}$$

Mean Linear Thermal Exp.



$\frac{L_{280}-L_{80}}{L_0}.10^5$	$\frac{L_{80}-L_{20}}{L_0}.10^5$
360	24
259	13
201	12
177	10

- $SS_{(280-80)}$ → dL=2.59 mm.
- $SS_{(80-20)}$ → dL=0.13 mm.
- when L_0 = 1m.

Specific Heat of Solids

- It is the energy required to change the temperature of a unit mass of substance by 1° C, holding the volume or pressure as constant.

$$C_v = \frac{dU}{dT} \qquad C_p = \frac{dH}{dT}$$

- In 1911, Dulong and Petit observed that the heat capacities of the solids are independent of temperature. Each lattice point absorbs same energy as the every other lattice point. Therefore, by the principle of equipartition of energy.

$$U = 3RT \therefore C_v = 3R$$

Einstein & Debye Theory

- Einstein treated the solid as a system of simple harmonic oscillators. It was assumed that, all the oscillators are of same frequency.
- However, Debye treated solid as an infinite elastic continuum and considered all the possible standing waves in the material.
- A parabolic frequency distribution was derived for the atoms vibrating in lattice.
- He presented a model to compute lattice heat capacity per mole, which accounts for all the vibration frequencies of all the lattice points.

Debye Theory

- The Debye model gives the following expression for the lattice heat capacity per mole.

$$c_v = 9R\left(\frac{T}{\theta_D}\right)^3 \int_0^{\frac{\theta_D}{T}} \frac{x^4 e^x dx}{\left(e^x - 1\right)^2} \Rightarrow = 3R\left(\frac{T}{\theta_D}\right)^3 D\left(\frac{T}{\theta_D}\right)$$

Debye function

- x is a dimensionless variable. $x = \frac{h\upsilon}{kT}$

- In the equation, only the value of θ_D changes from material to material.

- θ_D is called as Debye Characteristic Temperature.

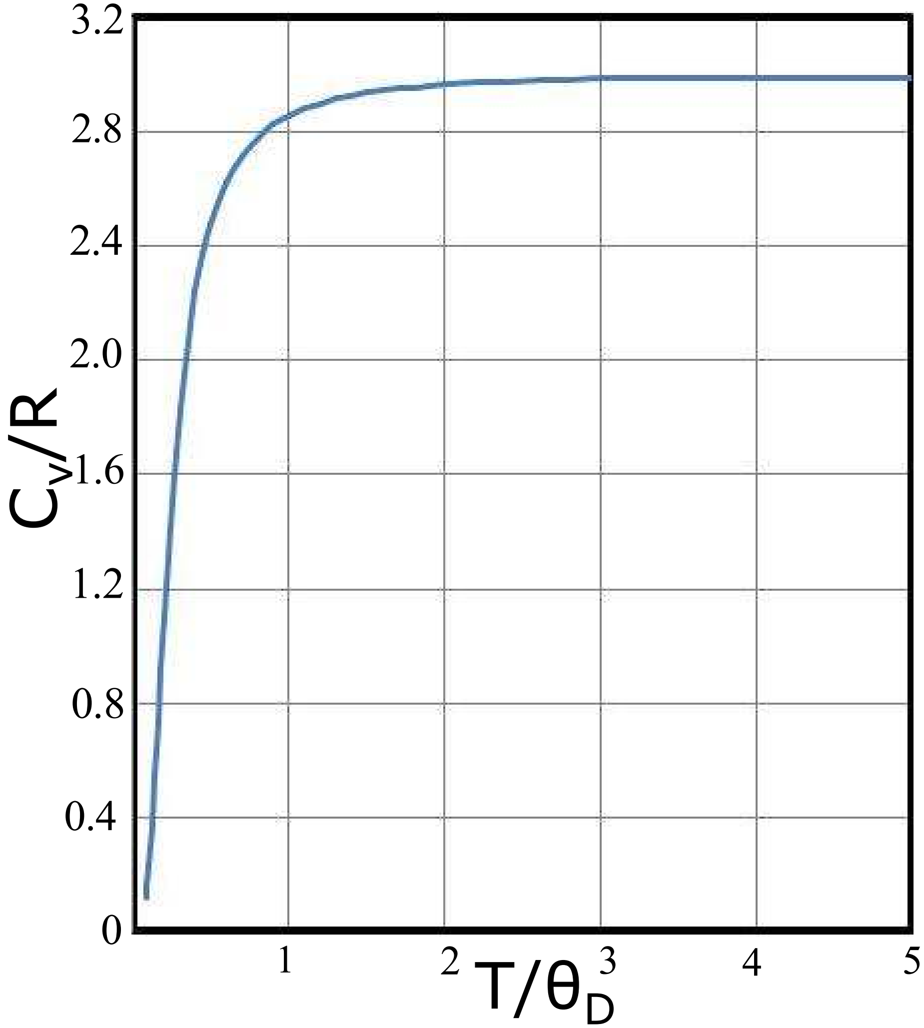
Debye Theory

- At high temperatures ($T > 2\theta_D$), specific heat obtained from the above equation approaches 3R. This is called as Dulong and Petit Value.
- At low temperatures ($T < \theta_D/12$), the Debye function approaches a constant value of $D(0) = 4\pi^4/5$.

$$c_v = \frac{12\pi^4 R}{5}\left(\frac{T}{\theta_D}\right)^3$$

- The variation is a cubic equation in absolute temperature at very low temperatures.

Specific Heat Curve



- The variation of C_v/R with T/θ_D is as shown.
- In general, the specific heat decreases with the decrease in temperature.

Debye Characteristic Temp.

Material	θ_D	Material	θ_D
Aluminum	390	Mercury	95
Argon	85	Molybdenum	375
Beryllium	980	Neon	63
Calcium	230	Nickel	375
Copper	310	Platinum	225
Diamond	1850	Silver	220
α-Iron	430	Titanium	350
γ-Iron	320	Tungsten	315
Lead	86	Vanadium	280
Lithium	430	Zirconium	280

Calculation of C_v

- The calculation of C_v for a particular material at a particular temperature, T, involves the following procedure.
- Refer the table and find the θ_D.
- Calculate T/θ_D and interpolate the value on the graph to obtain C_v/R.
- C_v can be known by multiplying it with R.
- If the value of T/θ_D is less than 1/12, correlation can be used to evaluate the C_v value directly.

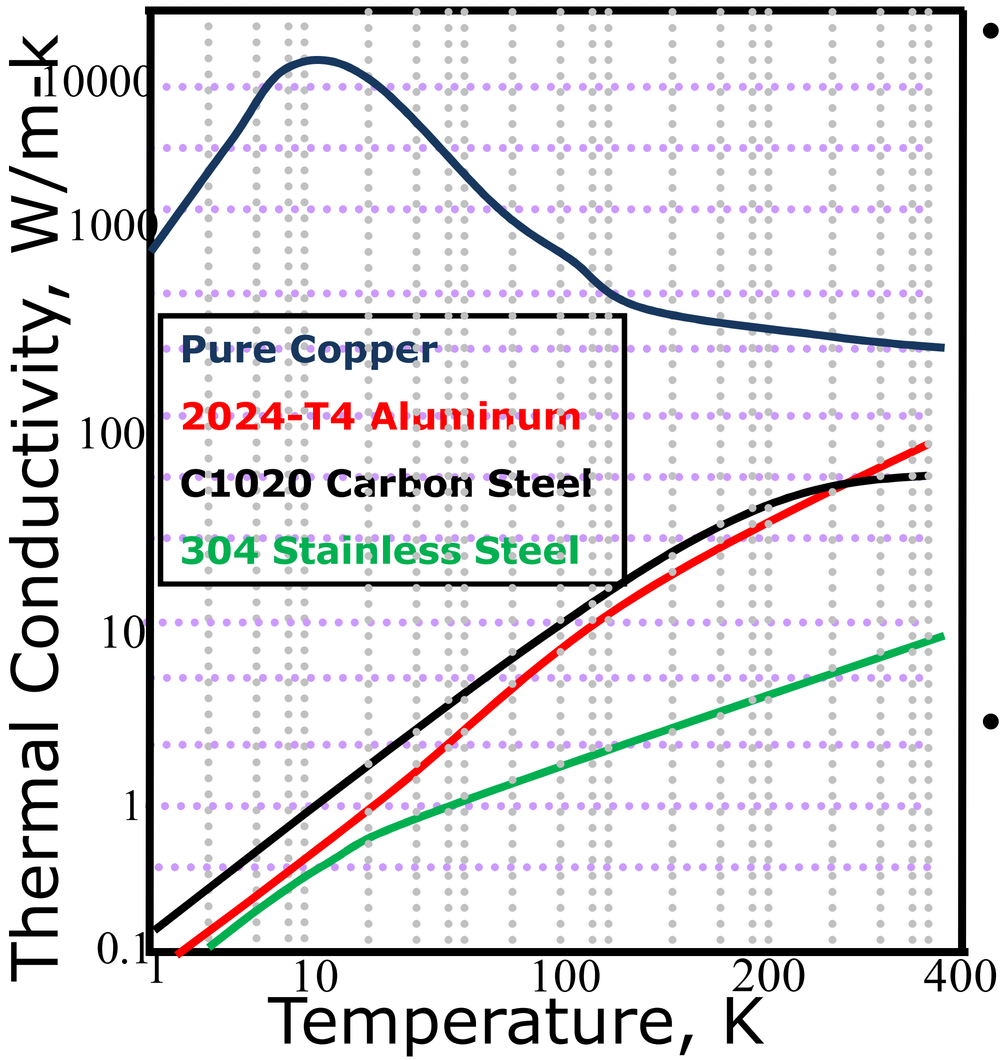
Thermal Conductivity in Solids

- In a cryostat, the solid members made of a metal or a non metal conduct heat from high temperature to low temperature.

- For these members, the thermal conductivity, k_T, should be as low as possible to minimize the heat loss.

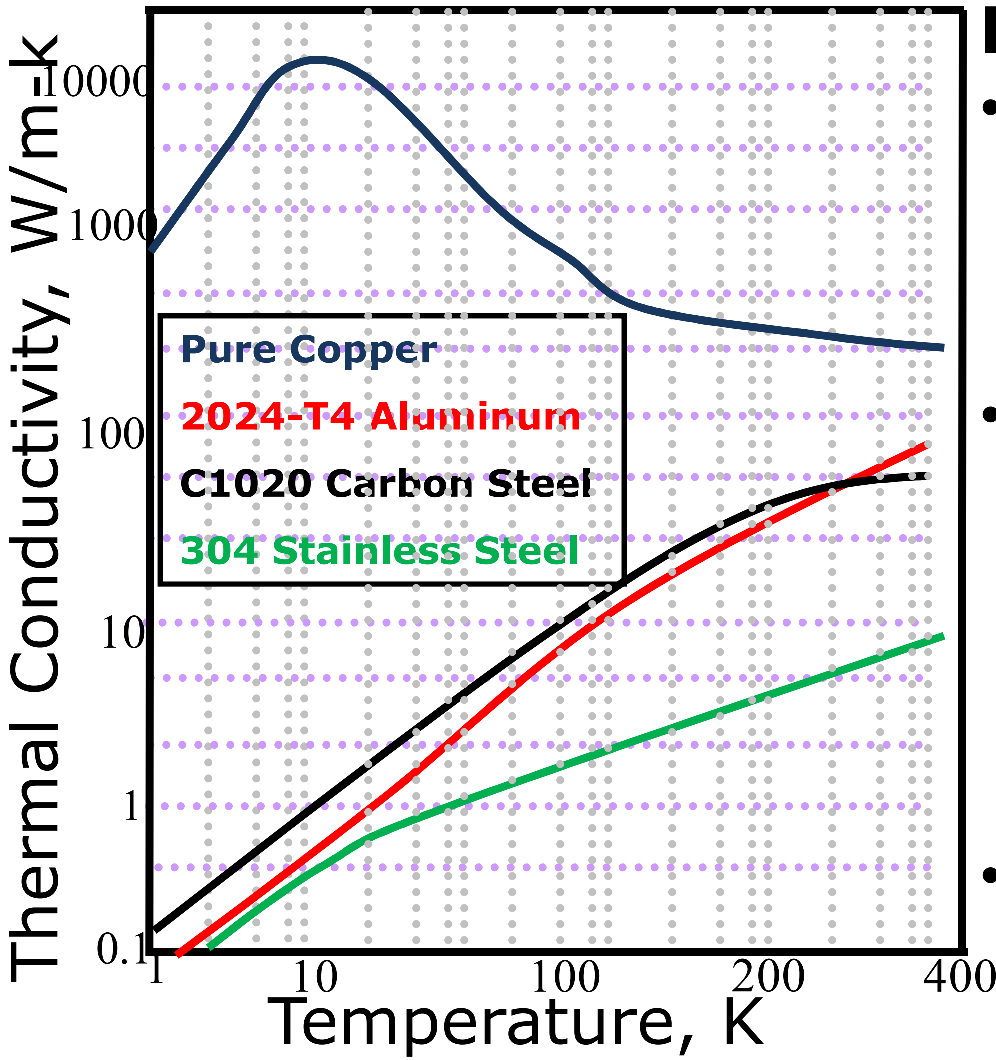
- On the other hand, for achieving best heat transfer of cold generated, copper can be used as a medium due to its very high thermal conductivity.

Thermal Conductivity in Solids



- Thermal conductivity, k_T, is the property of a material which indicates its ability to conduct heat. In general, k_T decreases with the decrease in the temperature.
- However, for pure metals the variation is slightly different from that of impure metals and alloys.

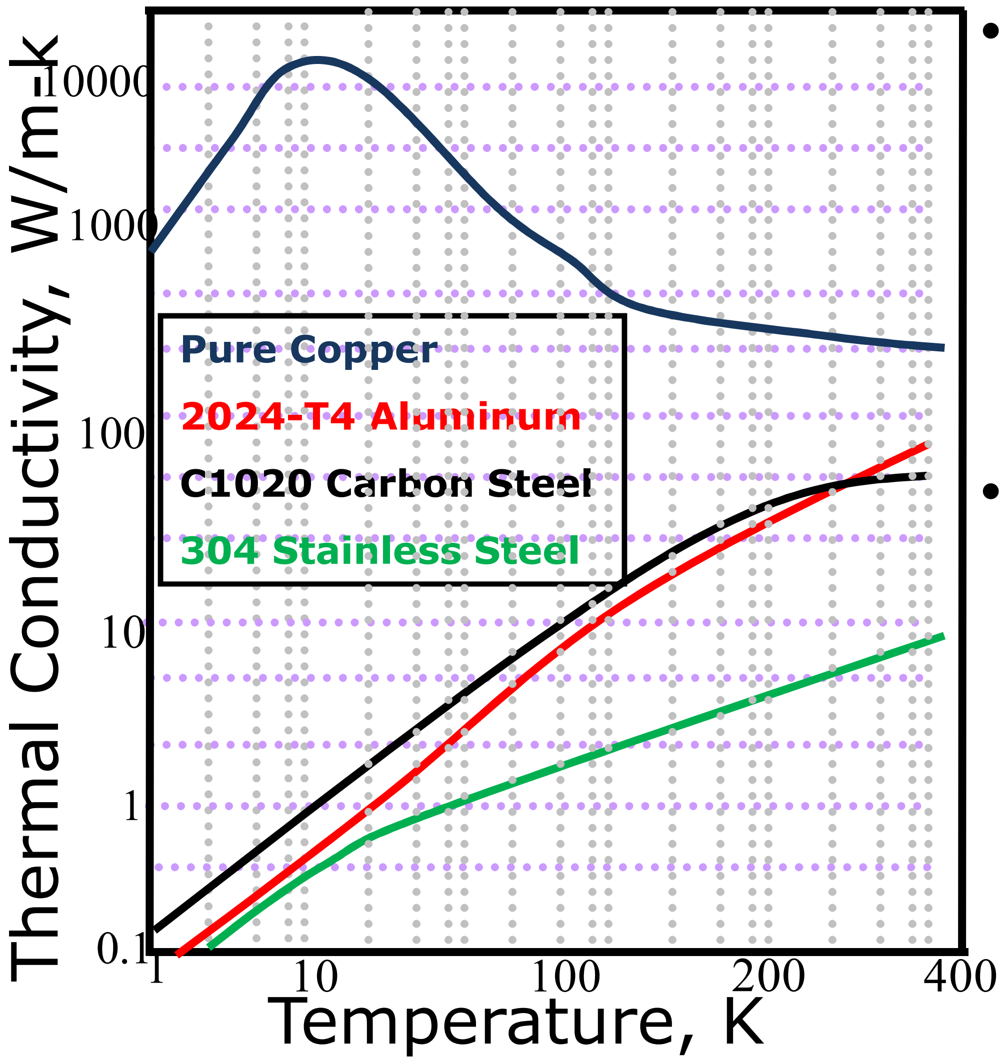
Conduction in Pure Metals



Pure Metals

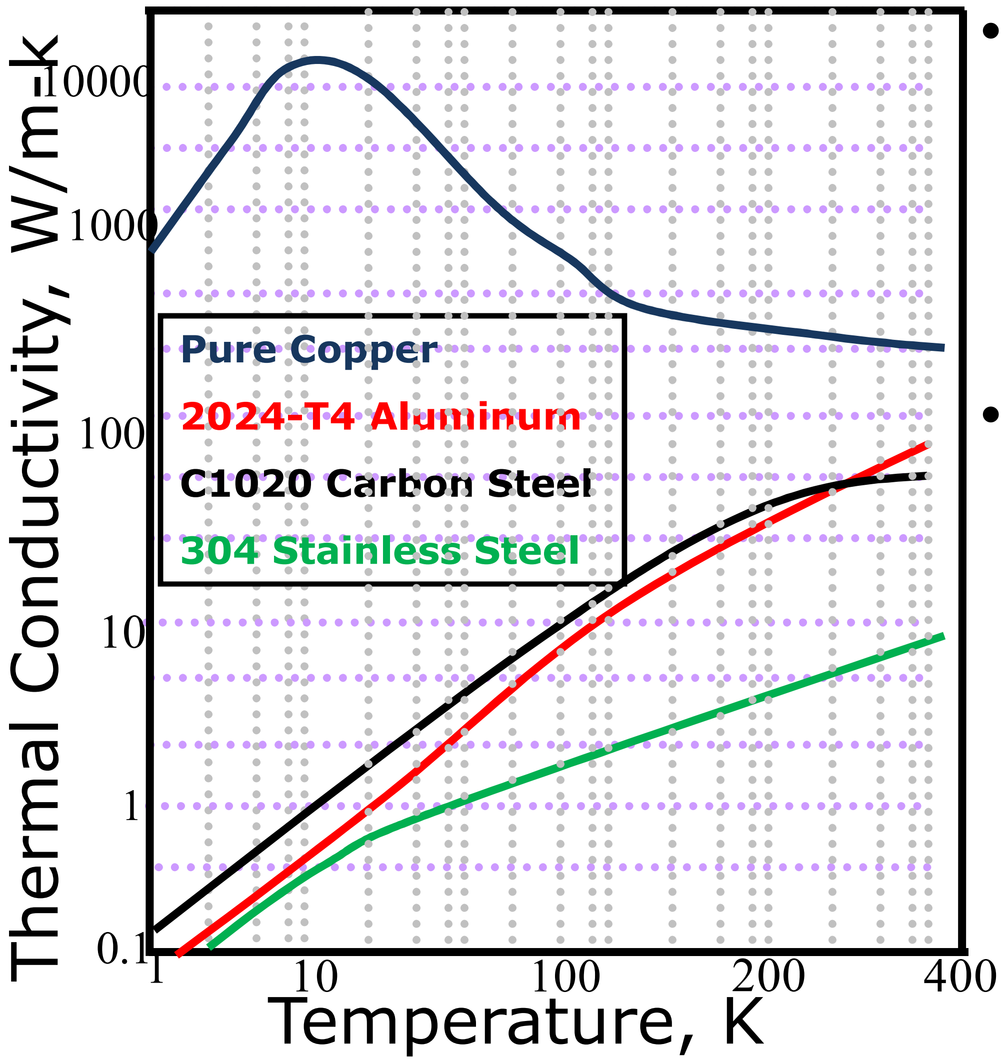
- The Electron and Phonon motion cause heat conduction.
- The contribution of electron motion to heat conduction is predominant above LN_2 temperature.
- At temperature below LN_2, phonon motion is predominant.

Conduction in Pure Metals



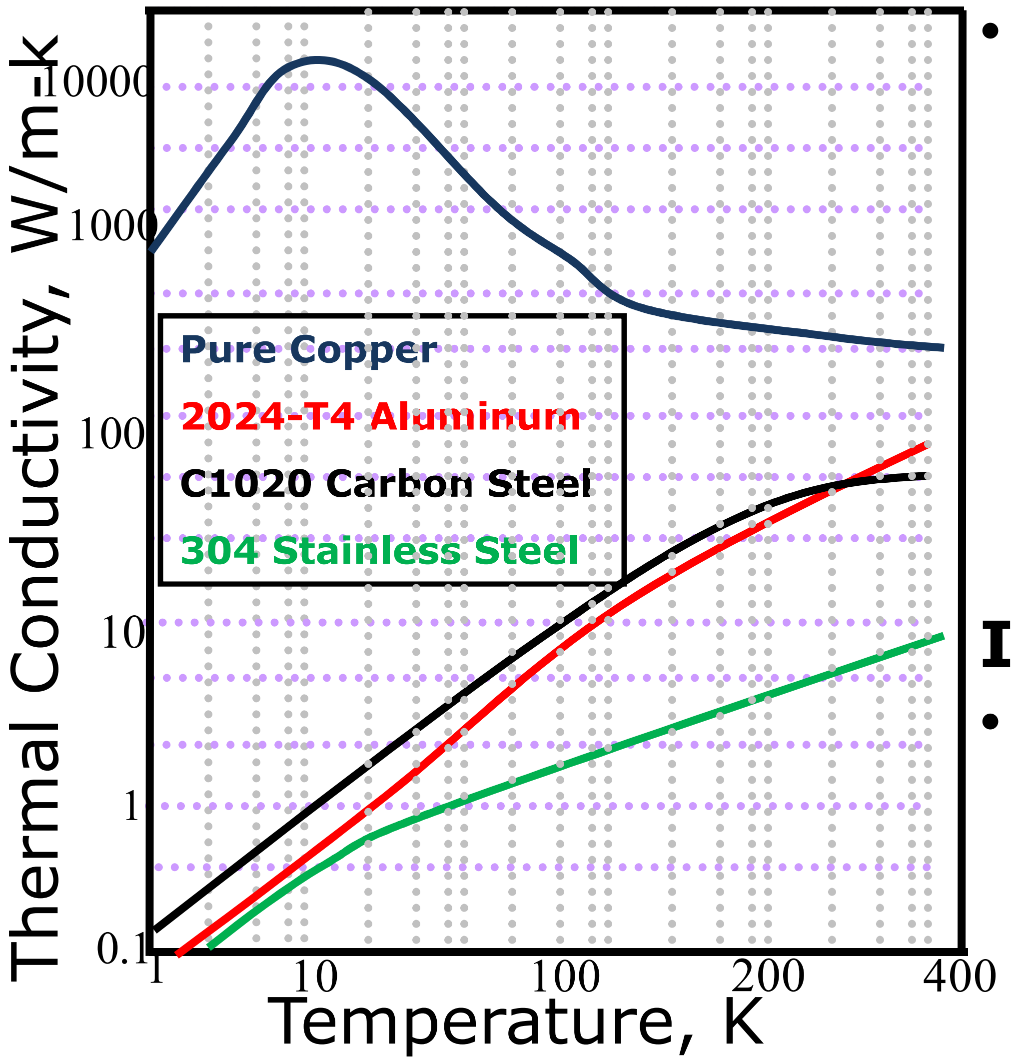
- Conduction depends on the product of electronic specific heat and mean free path.
- This product being a constant above LN_2, the k_T remains constant above LN_2.

Conduction in Pure Metals



- As the temperature is lowered, phonon contribution increases and k_T varies as $1/T^2$.
- It reaches a high value until the mean free path of the electrons equals to the dimensions of test specimen.

Conduction in Pure Metals

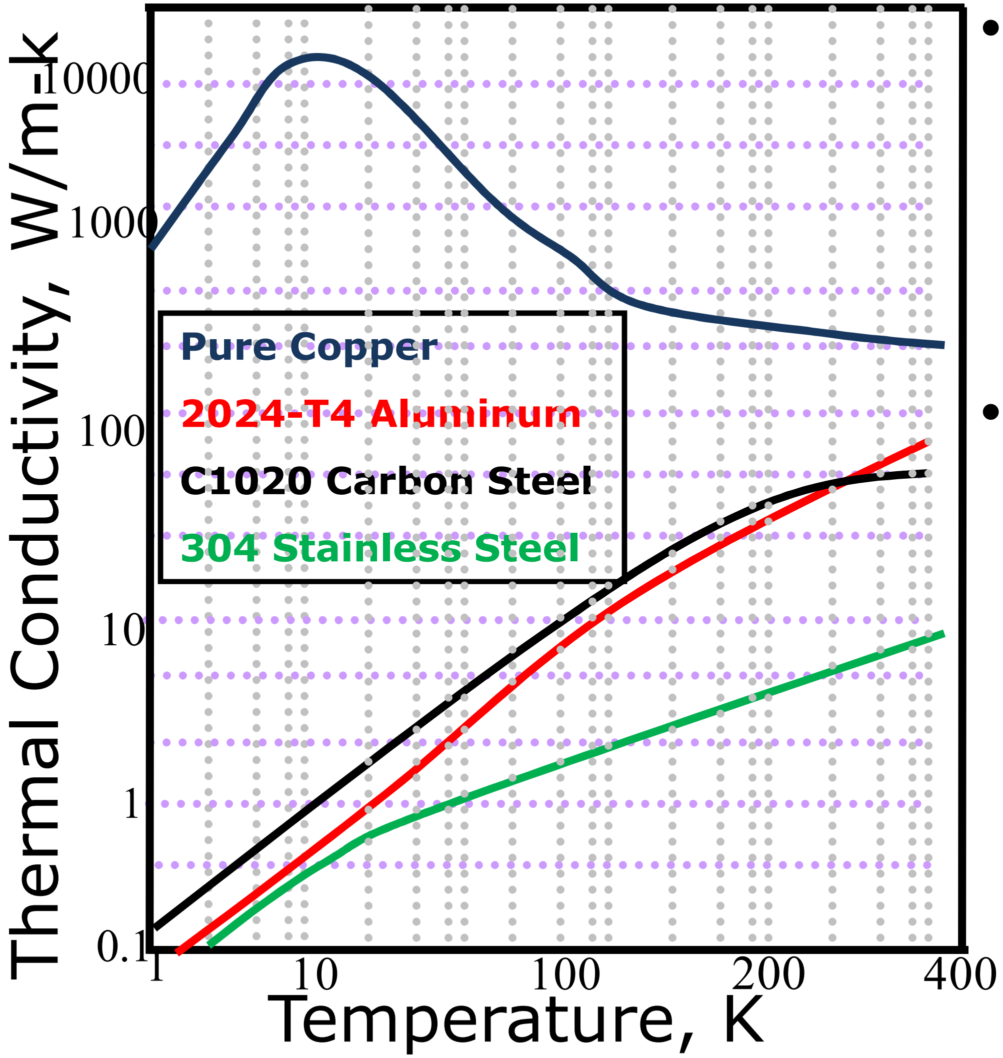


- When this condition is reached, the surface exhibits a resistance causing the k_T to decrease with the further drop in the temperature.

Impure & Alloy Metals

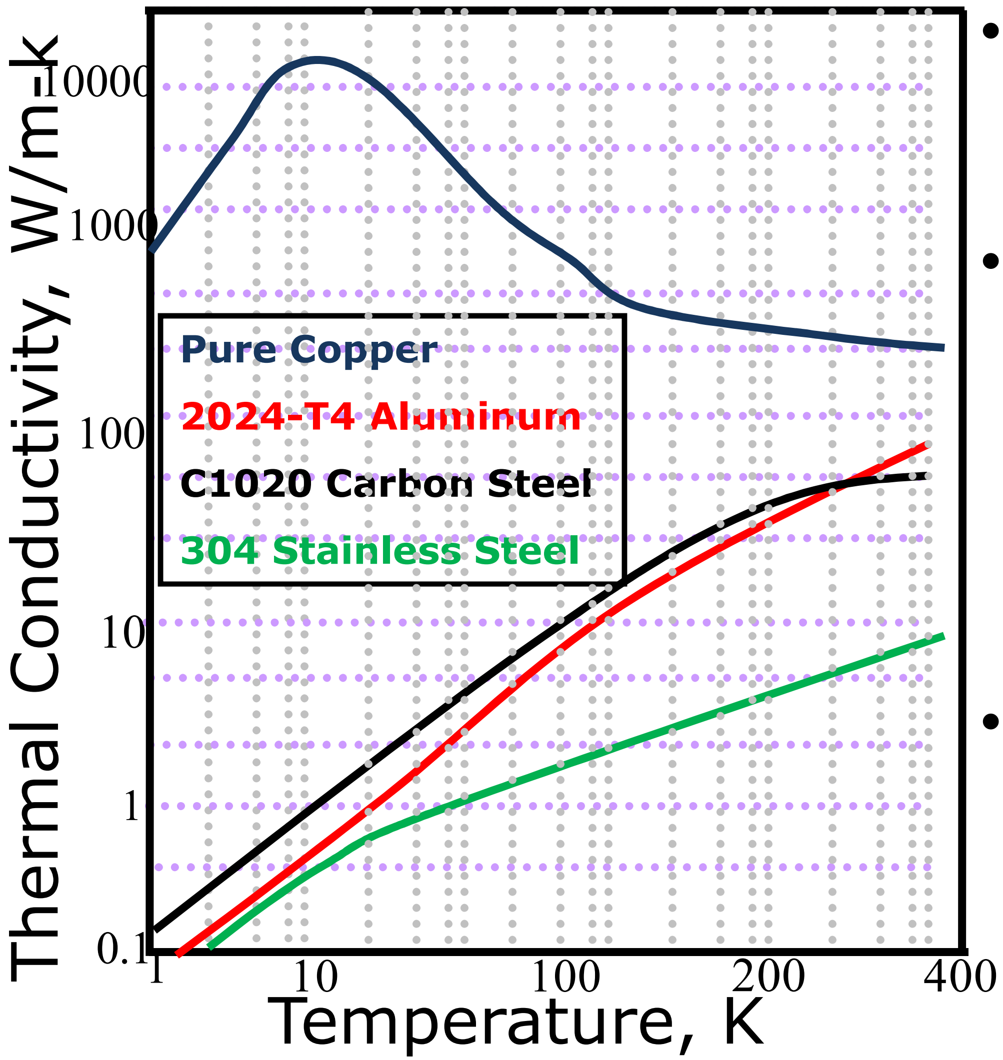
- Electron and phonon motion are of same magnitude in impure and alloy metals.

Conduction in Impure Metals



- The impure metals have imperfections like grain boundaries and dislocations.
- An additional scattering of electrons occur due to grain boundaries and dislocations which is proportional to T^3 and T^2 respectively, at temperatures lower than θ_D.

Conduction in Impure Metals



- At low temperatures, scattering decreases.
- As a result, k_T decreases with decrease in temperature in impure metals and alloys.
- These materials do not exhibit any high maxima like that of pure materials.

Thermal Conductivity Integrals

- As we found that, thermal conductivity, k_T, is a strong function of temperature, the term $k_T dT$ is very important.

- Due to this variation in k_T, the calculation of heat transfer (Q) can be very complicated.

- Therefore, a simple method is proposed in order to simplify calculation of Q.

- This method use $\int kdT$ or Thermal Conductivity Integral, which basically sums up the effect of variation of k_T with change in temperature, dT.

Thermal Conductivity Integrals

- The Fourier's Law of heat conduction is given by the following mathematical expression.

$$Q = -k(T)A(x)\frac{dT}{dx} \Rightarrow Q = -\left\{\frac{A(x)}{dx}\right\}\left\{k(T)dT\right\}$$

- In this method Q is expressed as given below.

$$Q = -G(\theta_2 - \theta_1)$$

- Here, θ_1 and θ_2 are expressed as Thermal Conductivity Integrals.

Thermal Conductivity Integrals

- kdT is taken as an integral called as Thermal Conductivity Integral evaluated w.r.t a datum temperature.

$$\theta_1 = \int_{T_d}^{T_1} k(T)dT$$

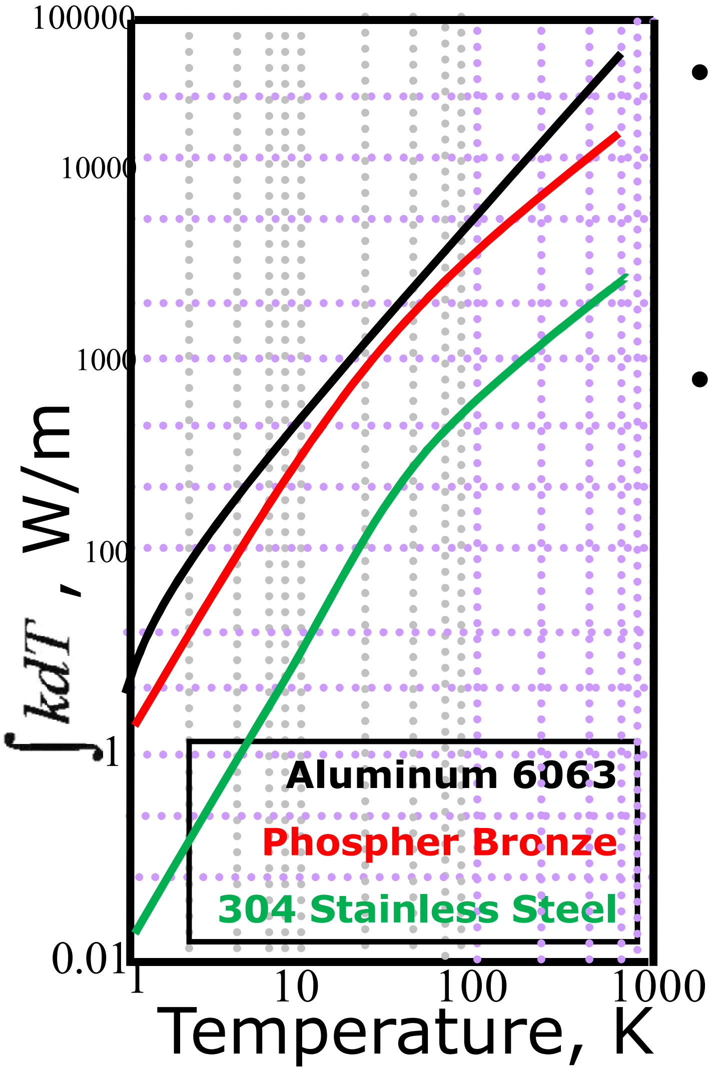
For Example

$$T_d = 0 \, or \, 4.2$$

- If A_{cs} is constant, G is defined as

$$G = A_{cs} / L$$

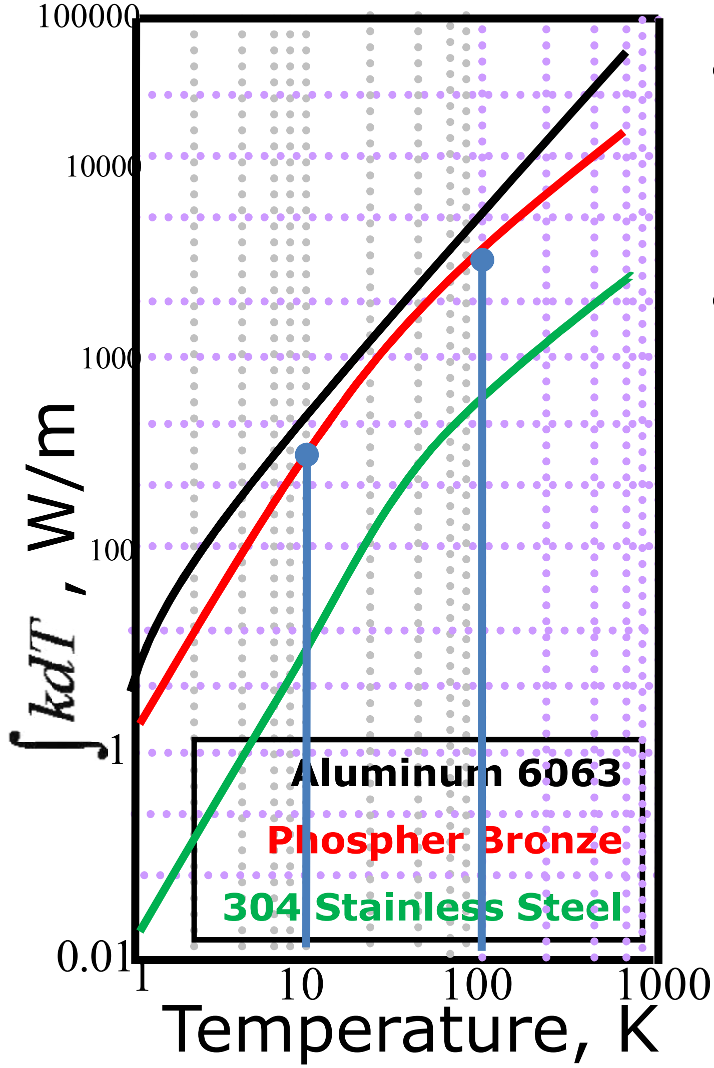
Thermal Conductivity Integrals



- The variation of kdT for few of the commonly used materials is as shown.

- In the calculations, the actual temperature distribution is not required, but only the end point temperatures.

Thermal Conductivity Integrals



- This technique is widely used in the analysis of heat leaks.
- If the datum temperature is taken as 0 K and the two ends of a specimen are maintained at 100 K and 10 K respectively, then the kdT integral is given by

$$\int_{10}^{100} kdT = \int_{0}^{100} kdT - \int_{0}^{10} kdT$$

Electric & Magnetic Properties

Electrical Conductivity

- It is defined as the electric current per unit cross sectional area divided by the voltage gradient in the direction of the current flow.

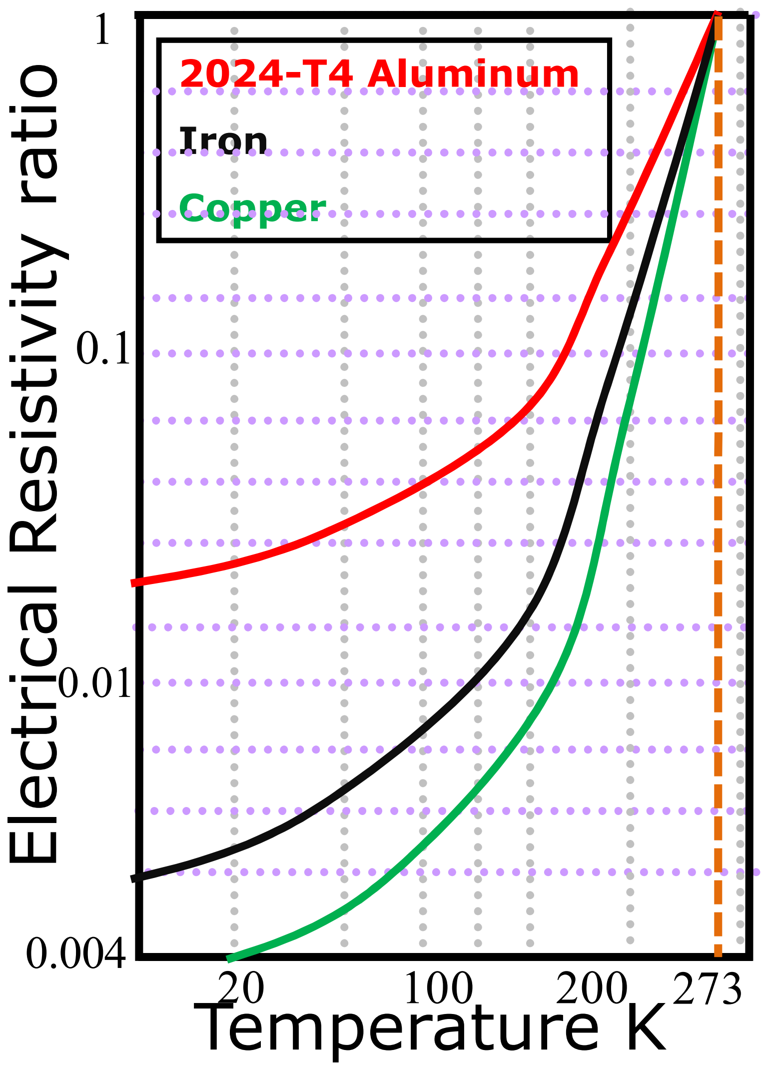
Electrical Resistivity

- It is the reciprocal of electrical conductivity.

- Decreasing the temperature decreases the vibration energy of the ions. This results in smaller interference with electron motion.

- Therefore, electrical conductivity of the metallic conductors increases at low temperature.

Electrical Resistivity



- Electrical resistivity ratio is defined as $\frac{R_T}{R_{273}}$
- The variation of electrical resistivity ratio for some commonly used materials is as shown.
- This ratio for a material decreases with the decrease in the temperature.

Electrical Conductivity

- Electrical and thermal conductivities are related by Wiedemann – Franz expression.

$$\frac{k_T}{k_e T} = \frac{1}{3}\left(\frac{\pi k}{e}\right)^2$$

- It means that the ratio of k_T and k_e is a product of constant and absolute temperature.

$$\frac{k_T}{k_e} = AT$$

Summary

- The coefficient of thermal expansion decreases with the decrease in temperature.
- For pure metals, k_T remains constant above LN_2 temperature. Below LN_2, it reaches a maxima and then after decreases steadily.
- For impure metals, k_T decreases with decrease in temperature. Integral kdT is used to calculate Q.
- Electrical conductivity of the metallic conductors increases at low temperature.
- k_e and k_t are correlated by Wiedemann–Franz Law.

- A self assessment exercise is given after this slide.

- Kindly asses yourself for this lecture.

Self Assessment

1. Coefficient of thermal expansion is the change in length to original length per ___________.

2. Coefficient of thermal expansion _______ with the decrease in temperature.

3. Metals undergo most of the contraction upto ___.

4. Mathematically, mean linear thermal expansion is defined as ________.

5. Dulong and Petit value for Specific heat is _____.

Self Assessment

6. Debye characteristic temperature is denoted by ____.

7. At low temperatures ($T < \theta_D/12$), the Debye function approaches a constant value of _____.

8. Expression for Q in Thermal conductivity integral form is ___________.

9. k_T decreases with the _________ in the temperature for impure metals.

Self Assessment

10. Specific heat of the material ___________ with decrease in temperature.

11. Electrical conductivity of the metallic conductors ___________ at low temperature.

12. k_e and k_t are correlated by __________ Law.

Answers

1. Unit rise in temperature.
2. Decreases
3. 80 K
4. $$\frac{\Delta L}{L_0} = \frac{L_T - L_0}{L_0}$$
5. 3R
6. θ_D
7. $4\pi^4/5$
8. $Q = -G(\theta_2 - \theta_1)$

Answers

9. Decrease

10. Decrease

11. Increase

12. Wiedemann–Franz

Thank You!