Tutorials

- There are four tutorial problems in the forthcoming slides.

1. Thermal expansion/contraction – 1 tutorial.

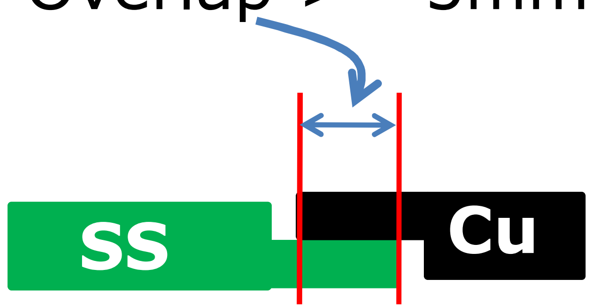
2. Estimation of C_v using the Debye Theory – 2 tutorials.

3. Thermal conductivity of materials – 1 tutorial.

Tutorial – 1

Calculate the overlap length of a brazed butt joint formed by SS 304 (L_0=1m) and Copper (L_0=0.5m). It is desired that the minimum overlap should be greater than 5mm. The joint is subjected to a low temperature of 80 K. Use the following data for the calculations.

Overlap >= 5mm



	SS $\frac{\Delta L}{L_0}.10^5$	Copper $\frac{\Delta L}{L_0}.10^5$
300 K	304	337
80 K	13	26

- This condition should be verified at 80 K.

Tutorial – 1

SS 304

- Mean linear expansion in SS 304 butt

$$\frac{\Delta L_{SS}}{L_0} = \left(\frac{L_{T1}}{L_0} - \frac{L_{T2}}{L_0}\right).10^{-5}$$

$$\frac{\Delta L_{SS}}{L_0} = (304 - 13).10^{-5}$$

L_0=1m, $\Delta L_{SS} = 2.91mm$

Cu

- Mean linear expansion in Cu butt

$$\frac{\Delta L_{Cu}}{L_0} = \left(\frac{L_{T1}}{L_0} - \frac{L_{T2}}{L_0}\right).10^{-5}$$

$$\frac{\Delta L_{Cu}}{L_0} = (337 - 26).10^{-5}$$

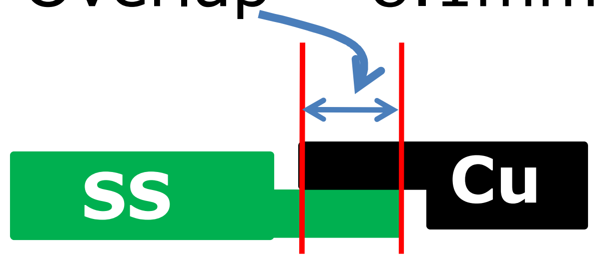
L_0=1m, $\Delta L_{Cu} = 3.11mm$

L_0=0.5m, $\Delta L_{Cu} = 1.55mm$

Tutorial – 1

- The greater of the two expansions is dL_{SS}
- The safe Butt joint should be more than dL_{SS} + 5 = 7.91mm.

Overlap = 8.1mm (say)



- When this joint is cooled to 80 K, the butt width in Cu after shrinkage is 6.55mm. Similarly, the butt width in SS after shrinkage is 5.19mm.

- Hence, the overlap being more than 5mm is a good design.

Debye Theory

- The expression for C_v, given by Debye theory is

$$C_v = 3R\left(\frac{T}{\theta_D}\right)^3 D\left(\frac{T}{\theta_D}\right)$$

- θ_D is called as Debye Characteristic Temperature.

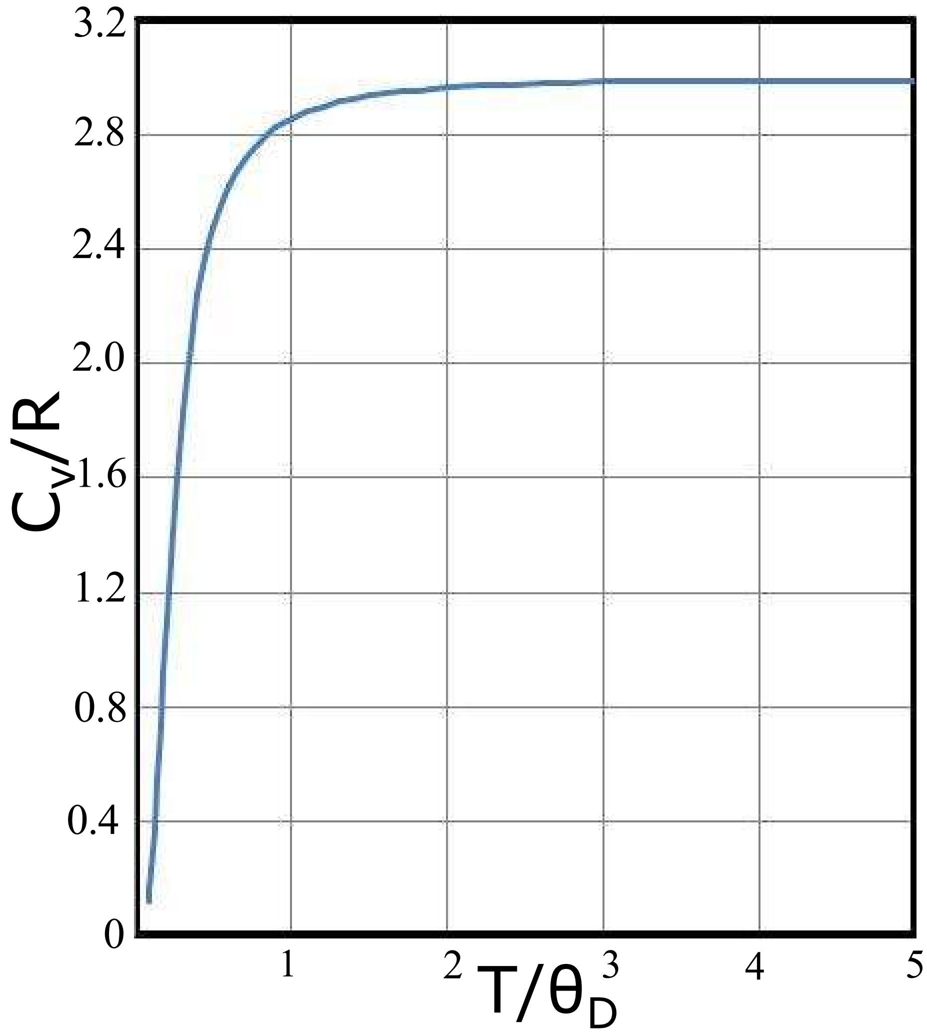
- At ($T > 2\theta_D$), C_v approaches 3R. This is called as Dulong and Petit Value.

- At ($T < \theta_D/12$), C_v is given by following equation.

$$c_v = \frac{12\pi^4 R}{5}\left(\frac{T}{\theta_D}\right)^3$$

- Also, D(0) is given a constant value of $4\pi^4/5$.

Specific Heat Curve



- The variation of C_v/R with T/θ_D is as shown.
- θ_D for few materials.

Material	θ_D
Aluminum	390
Lead	86
Nickel	375
Copper	310
Silver	220
α-Iron	430
Titanium	350

Tutorial – 2

Determine the lattice specific heat of copper at 100 K. Given that the molecular weight is 63.54 g/mol.

- Step 1:
- Calculation of T/θ_D ratio.

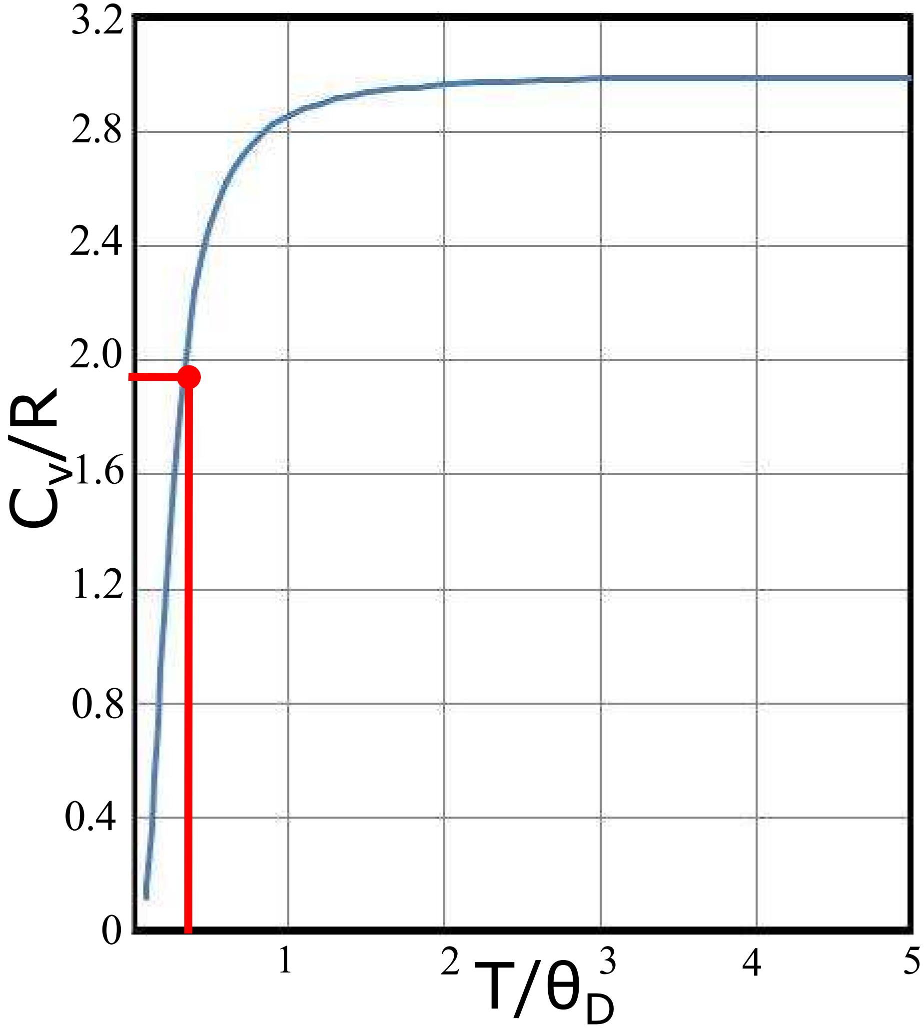
Material	θ_D
Aluminum	390
Lead	86
Nickel	375
Copper	**310**
Silver	220
Titanium	350

T = 100 K
θ_D = 310 K

$$\frac{T}{\theta_D} = \frac{100}{310} = 0.3225$$

- The value of T/θ_D is greater than 1/12 (0.0833).

Tutorial – 2



- The value of T/θ_D = 0.3225.
- From the graph, C_v/R = 1.93.

$$R = \frac{8.314}{0.06354} = 130.85$$

$$C_v = 130.85 X 1.93$$

$$= 252.534$$ **J/kg-K**

Tutorial – 3

Determine the lattice specific heat of Aluminum at 25 K. Given that the molecular weight is 27 g/mol.

- Step 1:
- Calculation of T/θ_D ratio.

Material	θ_D
Aluminum	**390**
Lead	86
Nickel	375
Copper	310
Silver	220
Titanium	350

T = 25 K
θ_D = 390 K

$$\frac{T}{\theta_D} = \frac{25}{390} = 0.0641$$

- The value of T/θ_D is less than 1/12 (0.0833).

Tutorial – 3

- Since, the T/θ_D ratio is less than 1/12, the equation to calculate the specific heat is as given below.

$$c_v = \frac{12\pi^4 R}{5}\left(\frac{T}{\theta_D}\right)^3$$

$$R = \frac{8.31434}{0.0270} = 307.9$$

$$c_v = \frac{233.78RT^3}{\theta_D^3}$$

$= 18.958$ **J/kg-K**

Thermal Conductivity Integrals

- The Fourier's Law of heat conduction is

$$Q = -k(T)A(x)\frac{dT}{dx}$$

- To make calculations less difficult and to account for the variation of k_T with temperature, Q is expressed as $Q = -G(\theta_2 - \theta_1)$

- kdT is taken as an integral called as Thermal Conductivity Integral evaluated w.r.t a datum.

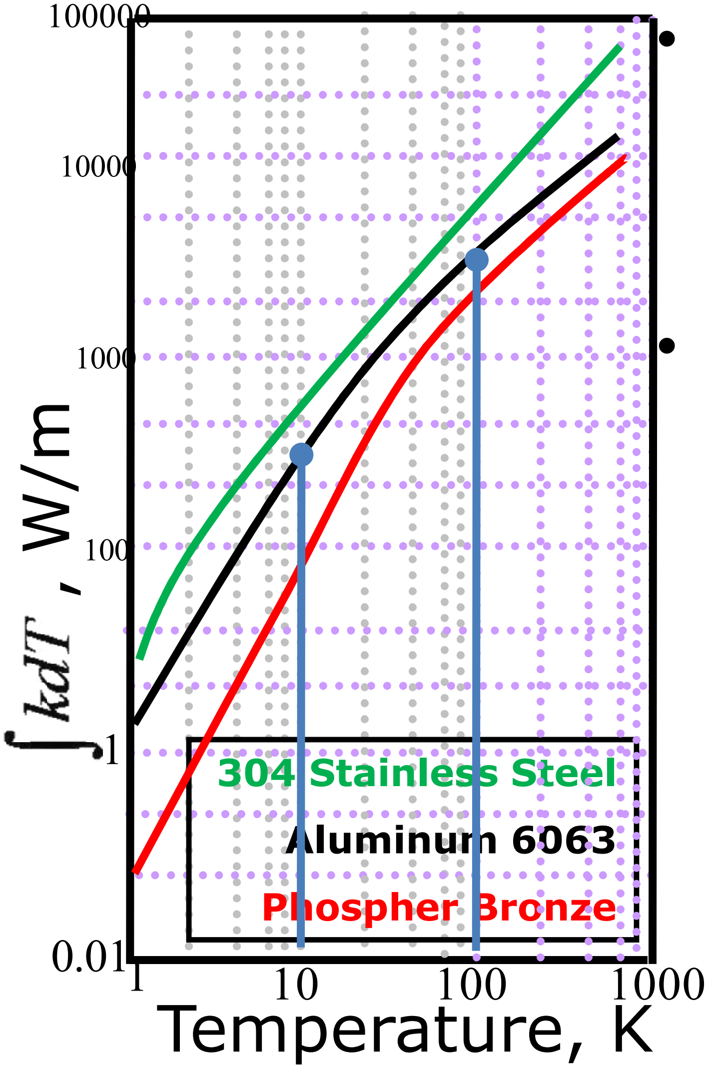
$$\theta_1 = \int_{T_d}^{T_1} k(T)dT$$

For Example

$$T_d = 0 \, or \, 4.2$$

- If A_{cs} is constant, G is defined as $G = A_{cs} / L$

Thermal Conductivity Integrals

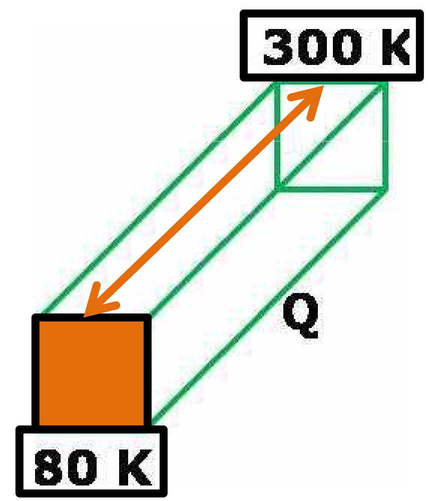


- The variation of kdT for few of the commonly used materials is as shown.
- In the calculations, the actual temperature distribution is not required, but only the end point temperatures.

$$\int_{10}^{100} kdT = \int_{0}^{100} kdT - \int_{0}^{10} kdT$$

Tutorial – 4

Determine the heat transferred in an copper slab of uniform cross section area 1cm^2 and length of 0.1m, when the end faces are maintained at 300 K and 80 K respectively. Compare the heat transferred by k_{avg} and kdT methods.

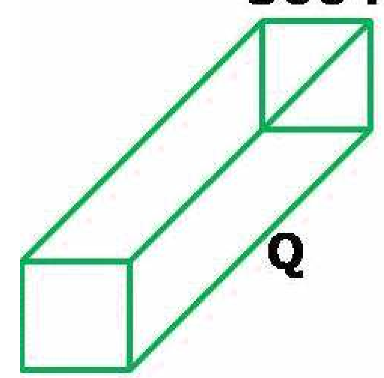


Given

- Area of cross section : 10^{-4} m^2
- Length of specimen: 0.1 m
- T_1 = 300 K
- T_2 = 80 K

Tutorial – 4

k_{avg} Method



$$Q = -k_{avg} A \frac{dT}{dx}$$

$$Q = k_{avg} A \left(\frac{T_1 - T_2}{L} \right)$$

$$Q = 57.75 X 10^{-4} \left(\frac{300 - 80}{0.1} \right)$$

$Q = 18.958$ **W**

k_{300} = 78.5 W/m K
k_{80} = 37.0 W/m K

k_{avg} = 57.75 W/m K

Tutorial – 4

kdT Method

$$Q = -G(\theta_2 - \theta_1)$$

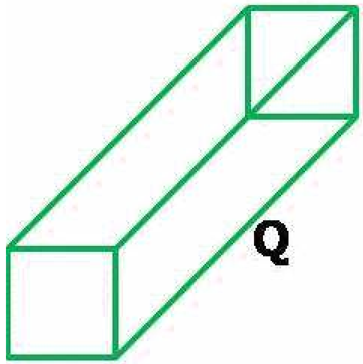
$$\theta_1 = \int_{4.2}^{300} k(T)dT = 15000$$

$$\theta_2 = \int_{4.2}^{80} k(T)dT = 1600$$

$$G = \frac{A_{cs}}{L} = \frac{10^{-4}}{0.1}$$

$$Q = -\frac{10^{-4}}{0.1}(1600 - 15000)$$

$$Q = 13.4 \text{ W}$$



Comparison

k_{avg}	kdT
18.98	13.4

- K_{avg} is more than the kdT method.