# Tutorial – 1

- Determine the liquid yield for a Linde – Hampson cycle with air as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%, 95%, 90% and 85%. Comment on the results.

- The T – s diagram of Linde – Hampson system if assumed the heat exchanger to be 100% effective is as shown.

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$$\varepsilon_1 = 1$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_1 = \frac{(28.47 + 8.37) - \cancel{(1-1)(28.47 + 199)}}{(28.47 + 406) - \cancel{(1-1)(28.47 + 199)}} = 0.085$$

$$y_1 = 0.085$$

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$$\varepsilon_2 = 0.95$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_2 = \frac{(28.47 + 8.37) - (1 - 0.95)(28.47 + 199)}{(28.47 + 406) - (1 - 0.95)(28.47 + 199)} = \frac{25.466}{423.1} = 0.060$$

$$y_2 = 0.060$$

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$$\varepsilon_3 = 0.90$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_3 = \frac{(28.47 + 8.37) - (1 - 0.90)(28.47 + 199)}{(28.47 + 406) - (1 - 0.90)(28.47 + 199)} = \frac{14.093}{411.7} = 0.034$$

$$y_3 = 0.034$$

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

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Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$
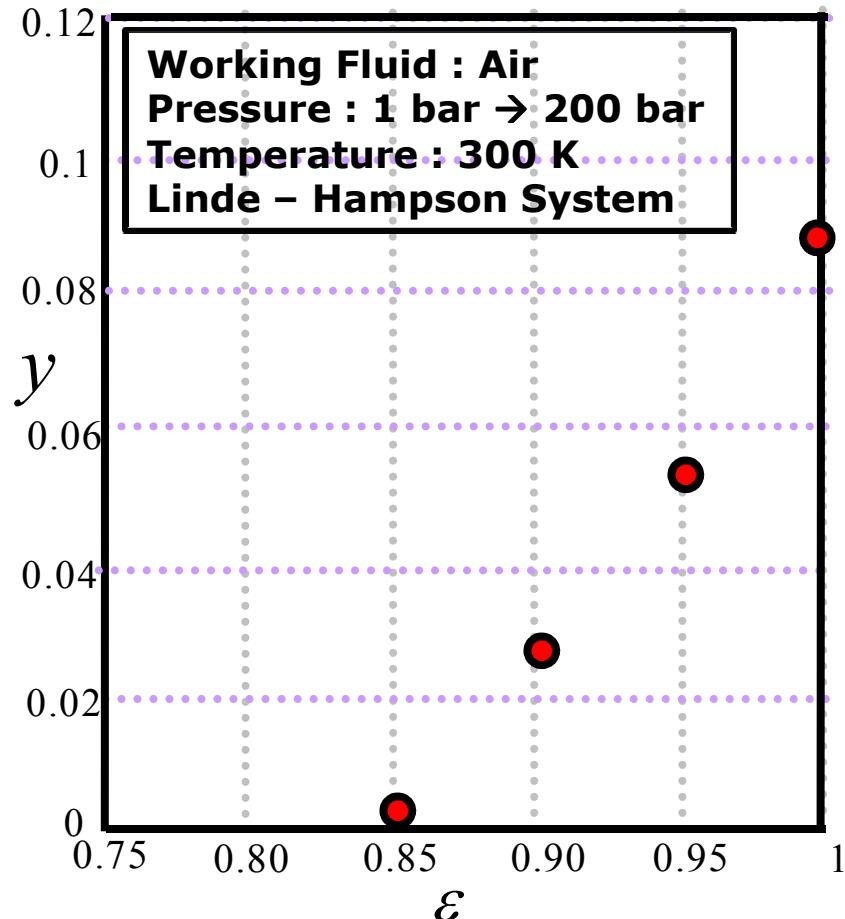
$$\varepsilon_4 = 0.85$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_4 = \frac{(28.47 + 8.37) - (1 - 0.85)(28.47 + 199)}{(28.47 + 406) - (1 - 0.85)(28.47 + 199)} = \frac{2.7195}{400.3} = 0.006$$

$$y_4 = 0.006$$

Tutorial – 1

Working Fluid : Air
Pressure : 1 bar → 200 bar
Temperature : 300 K
Linde – Hampson System
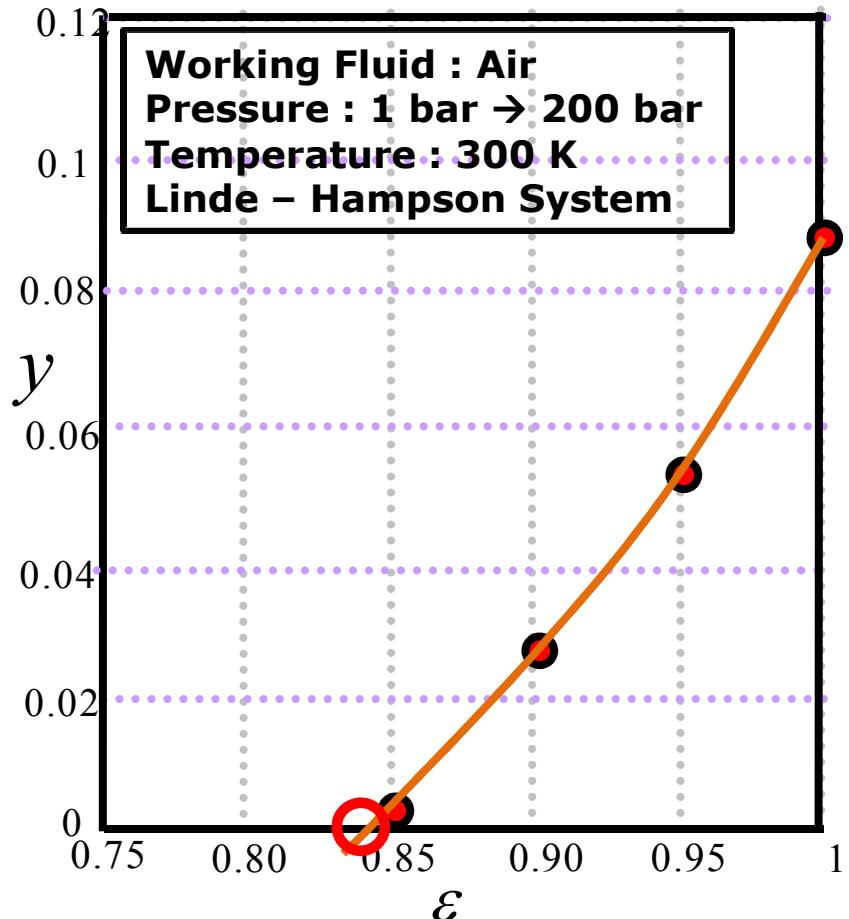
	ε	y
1	1	0.085
2	0.95	0.060
3	0.90	0.034
4	0.85	0.006

- Plotting the values for the above conditions, we have the trend as shown.

Tutorial – 1

Working Fluid : Air
Pressure : 1 bar → 200 bar
Temperature : 300 K
Linde – Hampson System

- Joining the points, we have the trend as shown in the figure.

- It is clear that as the effectiveness decreases, the yield **y** decreases drastically.

- Furthermore, the effectiveness should be more than 85% in order to have a liquid yield.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

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Figure of Merit (FOM)

- In the earlier lecture, we have seen that the ideal cycle is used as a benchmark to compare various liquefaction systems.

- In effect, a parameter called Figure of Merit (FOM) is defined.

- It is the ratio of the ideal work input to the actual work input of the system per unit mass of gas liquefied.

- Mathematically, $\dfrac{W_i}{W}$

Tutorial – 2
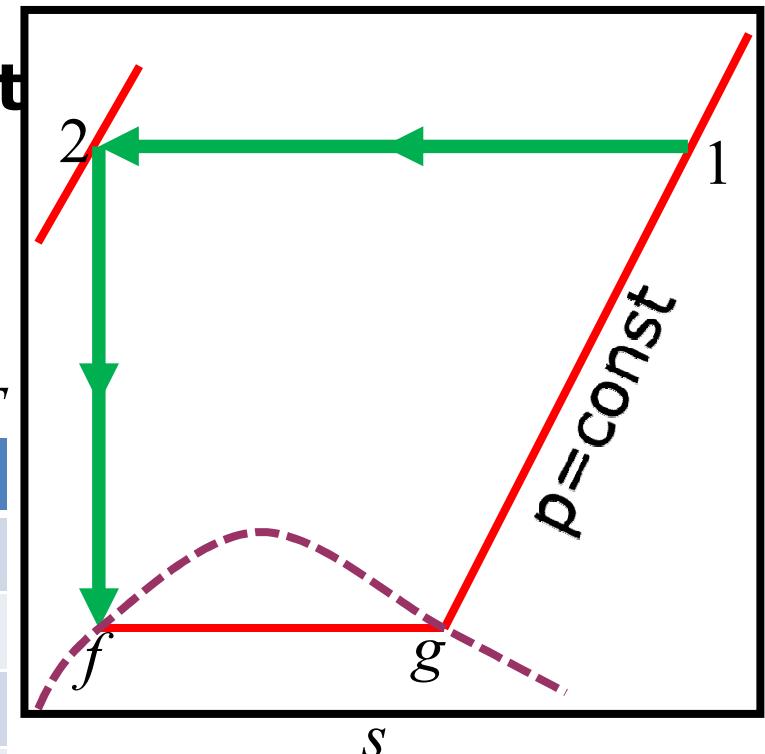
- Determine the following for a Linde – Hampson system with Nitrogen as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Tutorial – 2

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42
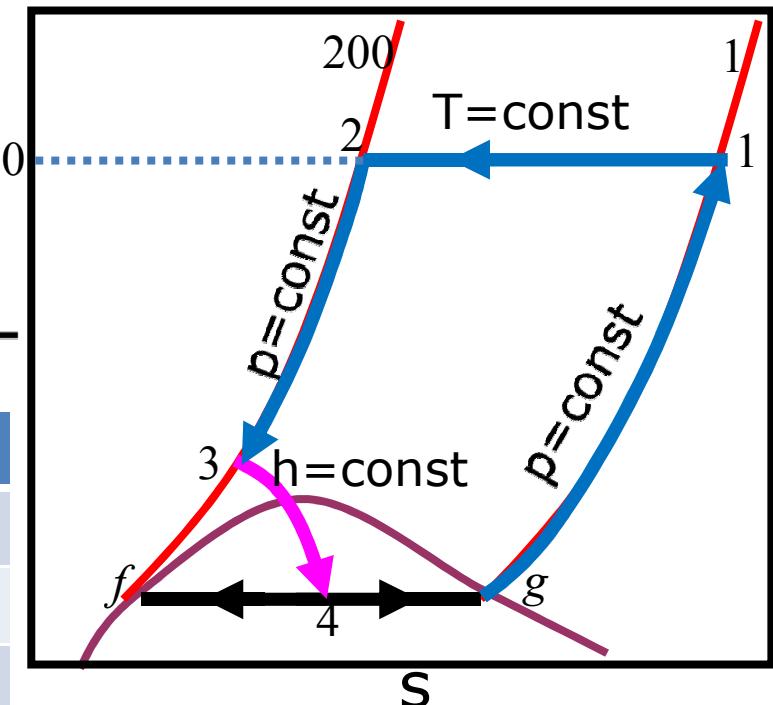
$$-\frac{W_c}{\dot{m}} = 300\left(4.4 - 0.42\right) - \left(462 - 29\right) \ = 761 \ J/g$$

Tutorial – 2

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

|	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42
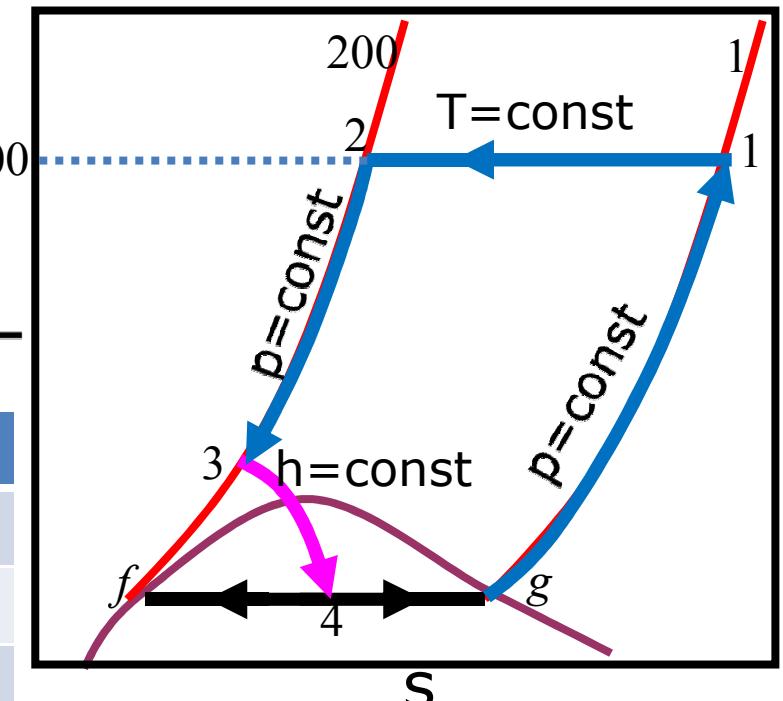
$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{462 - 430}{462 - 29} \right) = \left(\frac{32}{433} \right) = 0.074$$

Tutorial – 2

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42
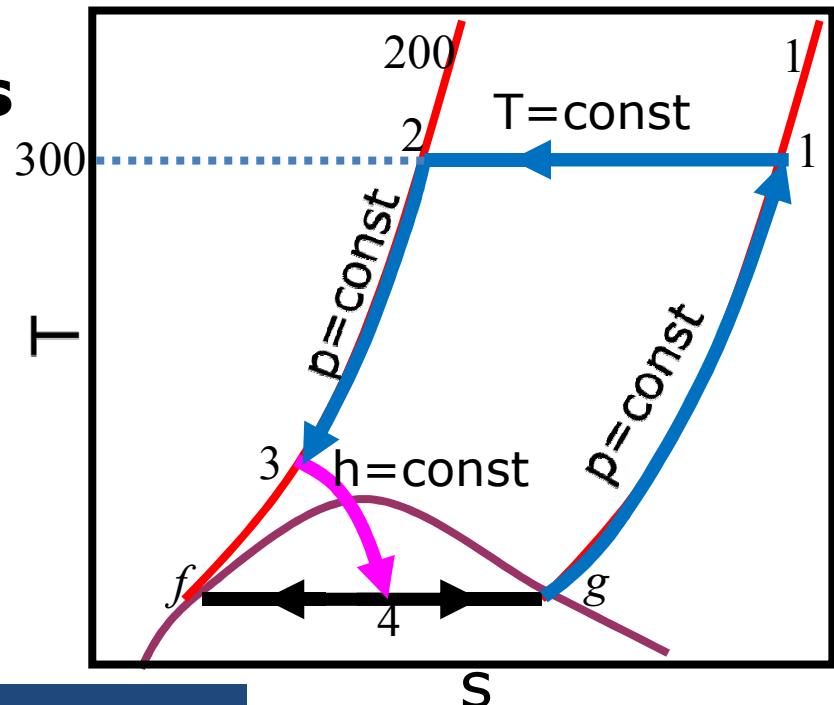
$$-\frac{W_c}{\dot{m}} = 300\left(4.4 - 2.75\right) - \left(462 - 430\right) = 463 \; J/g$$

Tutorial – 2

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 463$$

$$y = 0.074$$
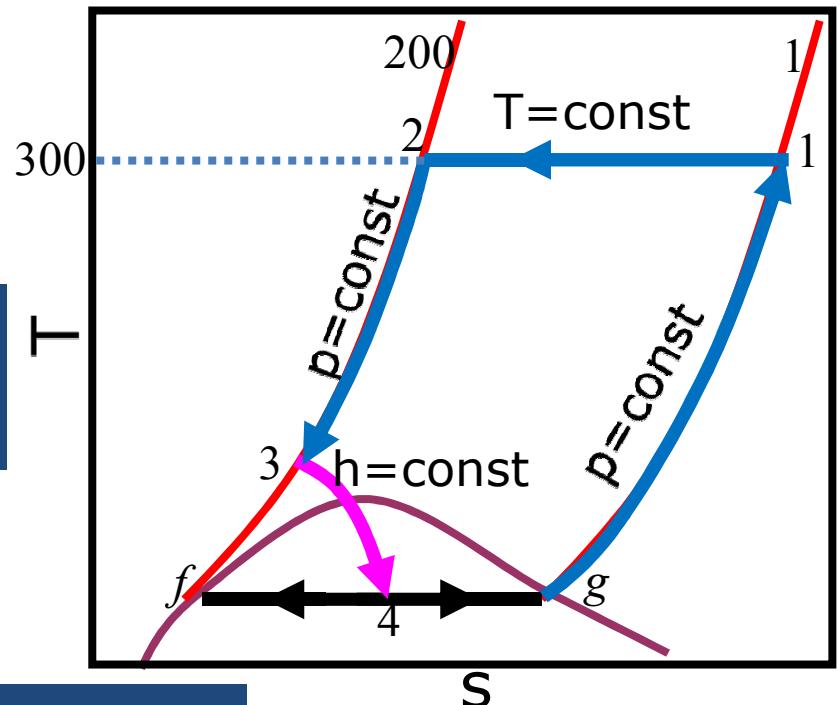
$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{463}{0.074} = 6265.22 \ J/g$$

Tutorial – 2

- ## **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f} = 6265.22$$

$$-\frac{W_i}{\dot{m}_f} = 767$$

$$FOM = \frac{\dfrac{W_i}{\dot{m}_f}}{\dfrac{W_c}{\dot{m}_f}} = \frac{767}{6265.22} = 0.1225$$

Tutorial – 3
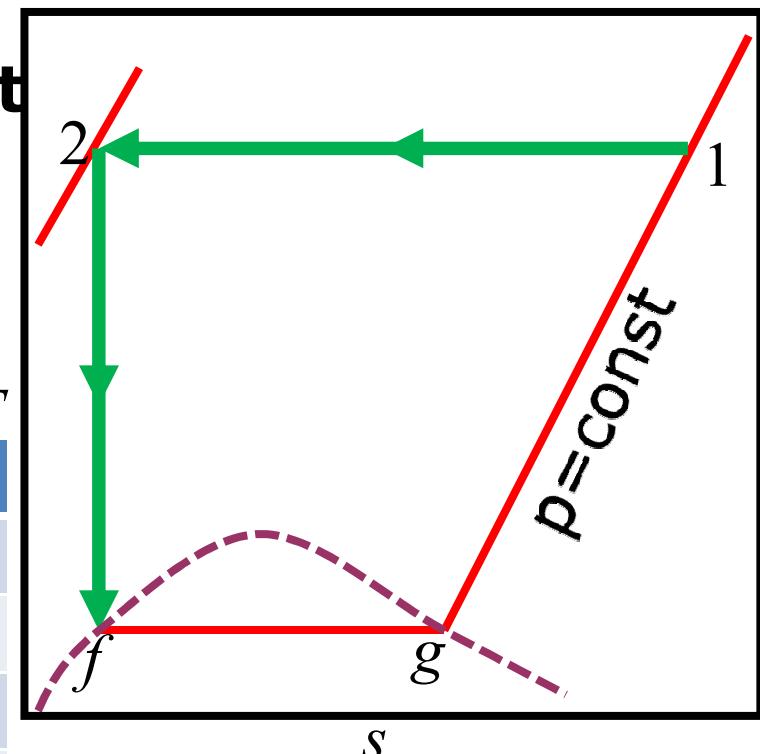
- Determine the following for a Linde – Hampson system with Argon as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Tutorial – 3

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$

	1	**2**	**f**
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35
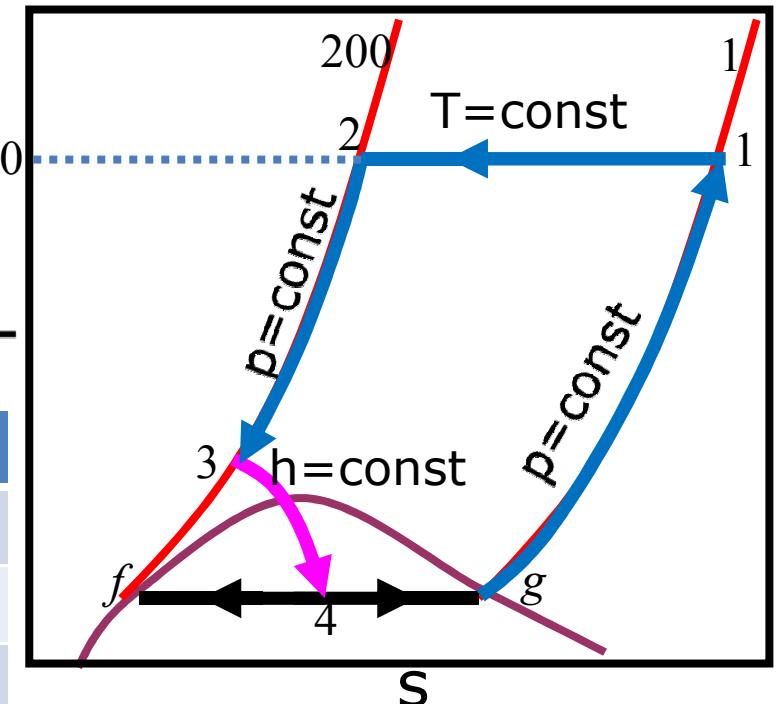
$$-\frac{\dot{W}_c}{\dot{m}} = 300\left(3.9 - 1.35\right) - \left(349 - 60\right) \quad = 476 \; J/g$$

Tutorial – 3

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

|	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35
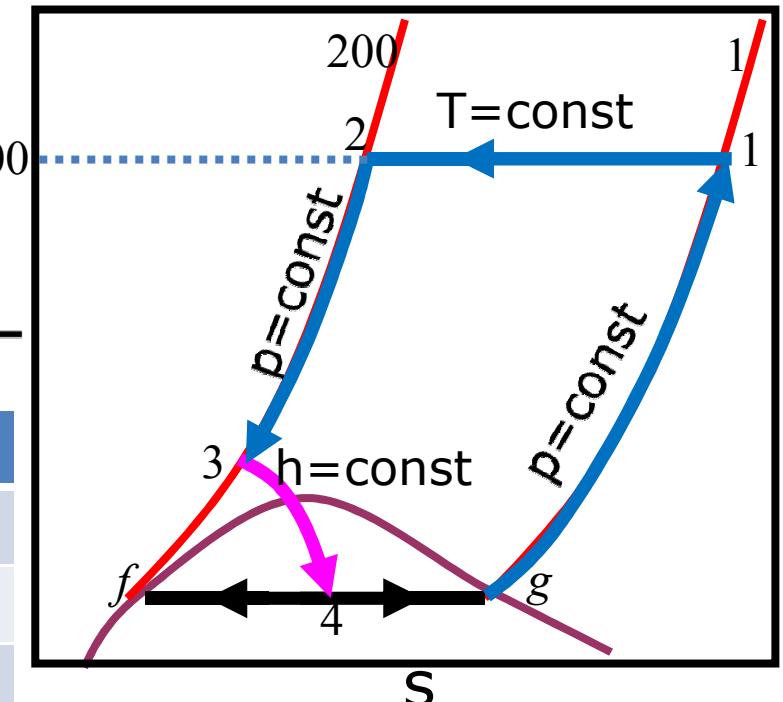
$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{349 - 315}{349 - 60} \right) = \left(\frac{34}{289} \right) = 0.1176$$

Tutorial – 3

- ## Work/unit mass of gas compressed

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35
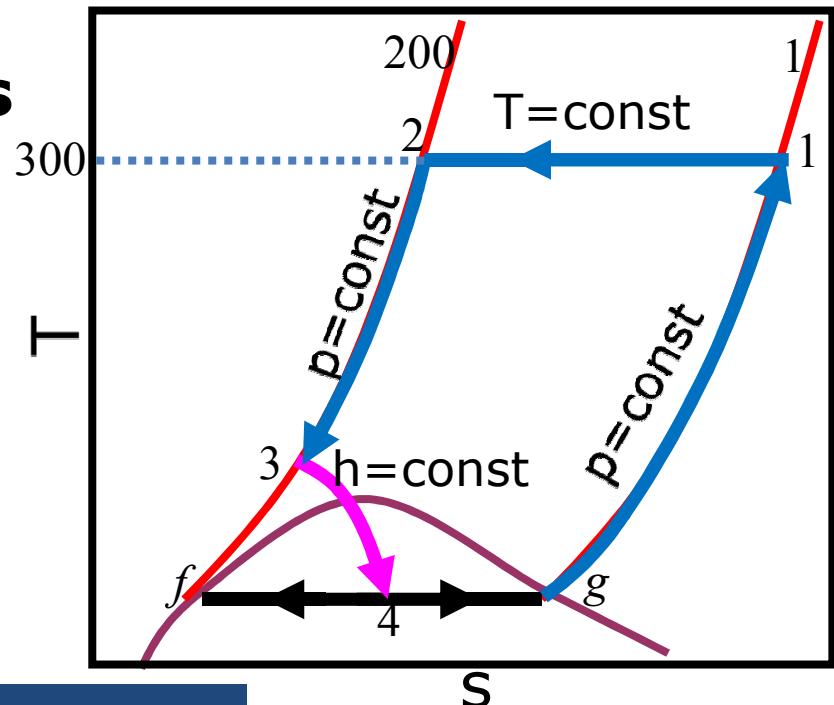
$$-\frac{W_c}{\dot{m}} = 300\left(3.9 - 2.7\right) - \left(349 - 315\right) \quad = 326 \, J/g$$

Tutorial – 3

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 326$$

$$y = 0.1176$$
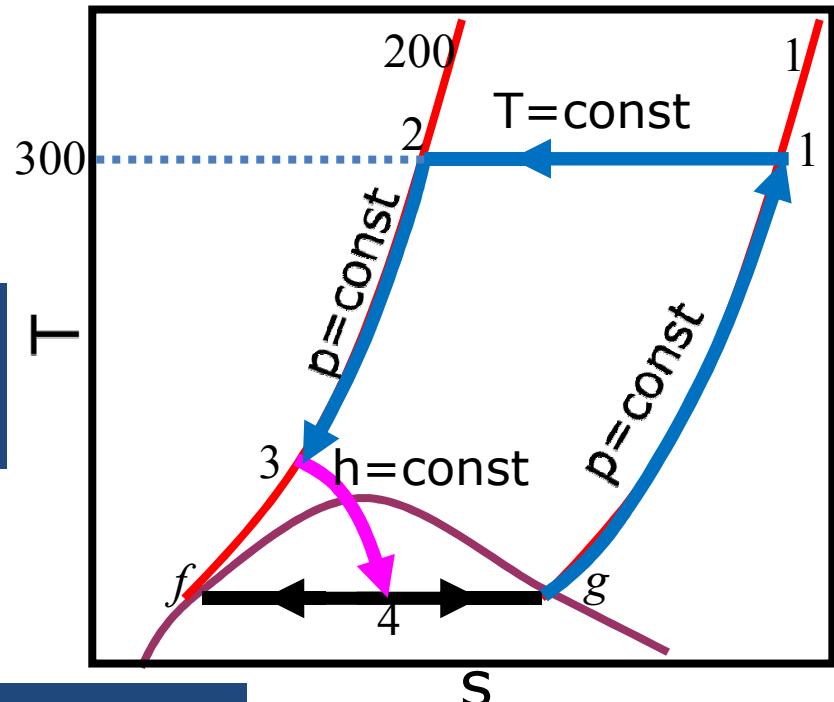

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{326}{0.1176} = 2772.1 \; J/g$$

Tutorial – 3

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f} = 2772.1$$

$$-\frac{W_i}{\dot{m}_f} = 476.0$$

$$FOM = \frac{\dfrac{W_i}{\dot{m}_f}}{\dfrac{W_c}{\dot{m}_f}} = \frac{476.0}{2772.1} = 0.1717$$

Performance of L – H System

Fluid	Boil. Pt	y	$-\dfrac{W_c}{\dot{m}}$	$-\dfrac{W_c}{\dot{m}_f}$	FOM
N$_2$	77.3	0.074	463	6265.2	0.122
Ar	87.2	0.117	326	2772.1	0.171
Air	78.8	0.081	454	5621.0	0.131
O$_2$	90.1	0.106	405	3804.0	0.167

- The above table is for a Linde – Hampson system when the pressures are from 1 bar to 200 bar at 300K.

- The heat exchanger effectiveness is 100%.

Tutorial – 4

- For the conditions specified in Tutorial 2 for the Linde – Hampson system with Nitrogen as working fluid, calculate the following when the effectiveness of HX is 90%.

- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

- Comment on the results

Tutorial – 4

- **Liquid yield**

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$$\varepsilon = 0.90$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$y = \frac{(462 - 430) - (1 - 0.90)(462 - 230)}{(462 - 29) - (1 - 0.90)(462 - 230)} = 0.021$$

Tutorial – 4

- **Location of $h_{1'}$ and Additional Work**

$$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

$$h_{1'} = \varepsilon\left(h_1 - h_g\right) + h_g$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$h_{1'} = 0.9\left(462 - 230\right) + 230 \quad = 438.8 \; J/g$$

$$\left(-\frac{W_c}{\dot{m}}\right)_{add} = h_1 - h_{1'} \quad = 462 - 438.8 \quad = 23.2 \; J/g$$

Tutorial – 4

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$-\frac{W_c}{\dot{m}} = 300\left(4.4 - 2.75\right) - \left(462 - 430\right) = 463 \; J/g$$

$$\left(-\frac{W_c}{\dot{m}}\right)_{total} = \left(-\frac{W_c}{\dot{m}}\right)_{h_1 \to h_1} + \left(-\frac{W_c}{\dot{m}}\right)_{add} = 463 + 23.2 = 486.2 \; J/g$$

Tutorial – 4

- ## Work/unit mass of gas liquefied

$$-\frac{W_c}{\dot{m}} = 486.2$$

$$y = 0.0215$$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} \quad = \frac{486.2}{0.0215} = 22613.95 \; J/g$$

- ## Figure of Merit (FOM)

$$-\frac{W_c}{\dot{m}_f} = 22613.95$$

$$-\frac{W_i}{\dot{m}_f} = 761$$

$$FOM = \frac{W_i}{\dot{m}_f} / \frac{W_c}{\dot{m}_f} \quad = \frac{761}{22613.95} = 0.0336$$

Tutorial – 4

	$\varepsilon = 1$	$\varepsilon = 0.9$	% change
y	0.074	0.021	71.62
$-\dfrac{W_c}{\dot{m}}$	463	486.2	-5.01
$-\dfrac{W_c}{\dot{m}_f}$	6265.2	22614	-261
FOM	0.1225	0.0336	72.57

- The above table highlights the significance of heat exchanger effectiveness for a Linde – Hampson system