Tutorial

Part 1

- A Precooled Linde – Hampson System has Nitrogen and R134a as primary and secondary fluids respectively. Determine the Liquid yield and FOM. The operating conditions and other useful data are as given below.

N_2	r	Point 2
I	0.05	101.3 bar
II	0.07	101.3 bar
III	0.05	202.6 bar
IV	0.1	202.6 bar

	a	b	c
p (bar)	1.013	8.104	8.104
T (K)	247	314	305
h (J/g)	380	420	240
R134a			

Tutorial

Part 2

- Also, calculate the $\mathbf{y_{max}}$ for each of the pressures mentioned and their corresponding **r** values. Plot the data graphically and comment on the nature of **y**, work requirement, FOM versus **r**.

N_2	r	Point 2
I	0.05	101.3 bar
II	0.07	101.3 bar
III	0.05	202.6 bar
IV	0.1	202.6 bar

	a	b	c
p (bar)	1.013	8.104	8.104
T (K)	247	314	305
h (J/g)	380	420	240
R134a			

Tutorial

Given : Part 1
Cycle : Precooled L – H Cycle with N_2.
Temperature : 300 K
Refrigerant : R134a, 1 atm → 8 atm

For this cycle, Calculate and comment	
1	Liquid Yield **y**
2	Work/unit mass of gas compressed
3	Work/unit mass of gas liquefied
4	FOM

N_2	r	Point 2	N_2	r	Point 2
I	0.05	101.3 bar	**III**	0.05	202.6 bar
II	0.07	101.3 bar	**IV**	0.1	202.6 bar

Tutorial

Given : Part 2
Cycle : Precooled L – H Cycle with N_2.
Temperature : 300 K
Refrigerant : R134a, 1 atm → 8 atm

For this cycle, Calculate and comment	
1	Liquid Yield $\mathbf{y_{max}}$
2	Work/unit mass of gas compressed
3	Work/unit mass of gas liquefied
4	FOM

N_2	Point 2
r @ $\mathbf{y_{max}}$	101.3 bar
r @ $\mathbf{y_{max}}$	202.6 bar

Methodology

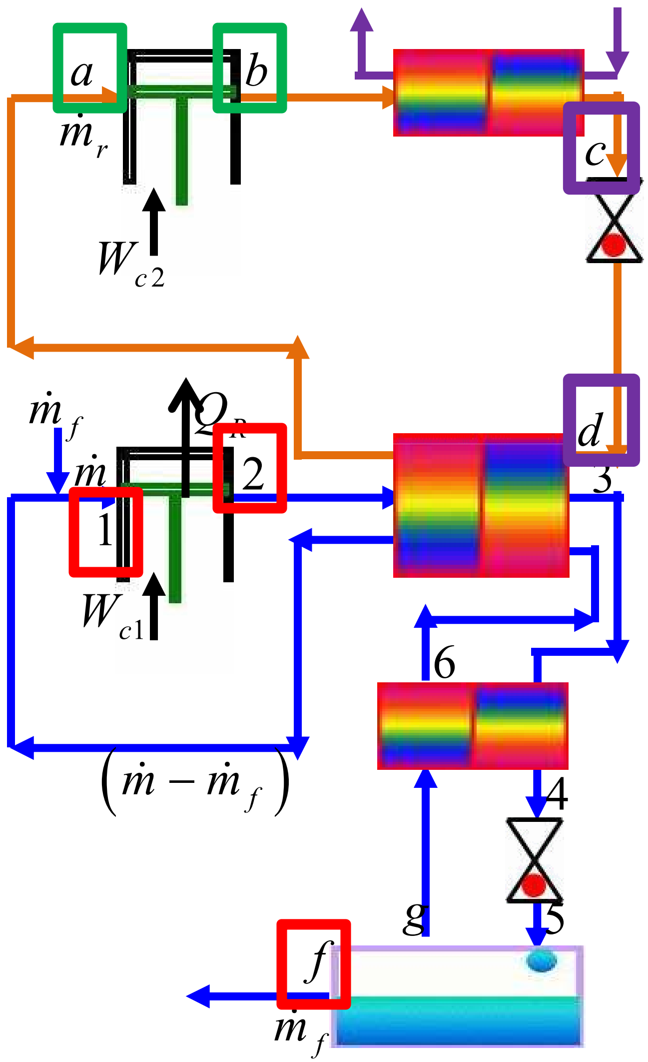
- The two pressures conditions under study are 101.3 bar and 202.6 bar.

- The Liquid yield and FOM are calculated only for 101.3 bar pressure condition.

- Also, the calculations for $\mathbf{y_{max}}$ and for an **r** value beyond $\mathbf{y_{max}}$ condition are calculated only for 101.3 bar pressure condition.

- Calculations pertaining to 202.6 bar condition are left as an exercise to students.

Tutorial



N_2	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42

	a	b	c
p (bar)	1.013	8.104	8.104
T (K)	247	314	305
h (J/g)	380	420	240
R134a			

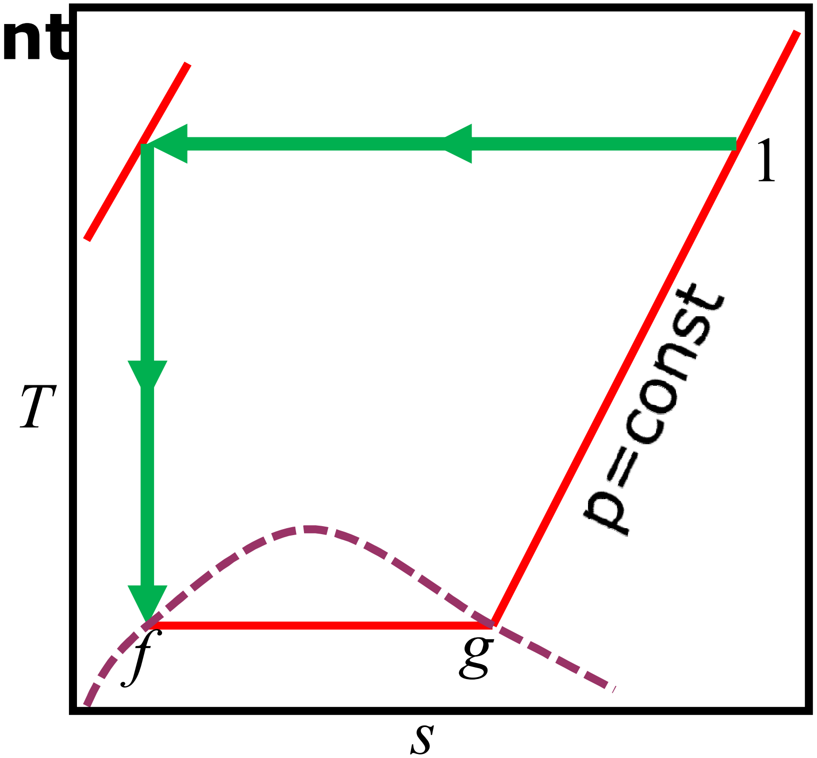
- $h_d = h_c$, since the expansion is isenthalpic.

Tutorial

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$

N_2	1	f
p (bar)	1.013	1.013
T (K)	300	77
h (J/g)	462	29
s (J/gK)	4.42	0.42

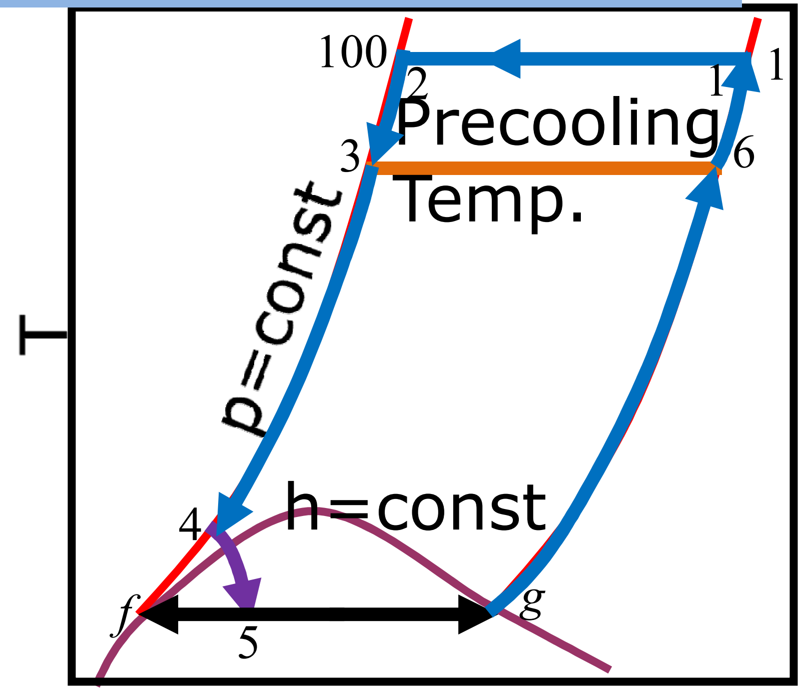


$$-\frac{W_c}{\dot{m}} = 300\left(4.42 - 0.42\right) - \left(462 - 29\right) = 767\ J/g$$

Tutorial : Part – 1

- The T – s diagram for a Precooled Linde – Hampson system is as shown.

- The state properties are as tabulated below.



N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

Tutorial : Part – 1

- **Liquid yield**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

N_2	r	Point 2
I	0.05	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$y\big|_1 = \frac{(462-445)}{(462-29)} + 0.05\frac{(380-240)}{(462-29)} = \frac{(17)}{(433)} + 0.05\frac{(140)}{(433)} = 0.055$$

Tutorial : Part – 1

- **Work/unit mass of N_2 compressed**

N_2	r	Point 2
I	0.05	101.3 bar

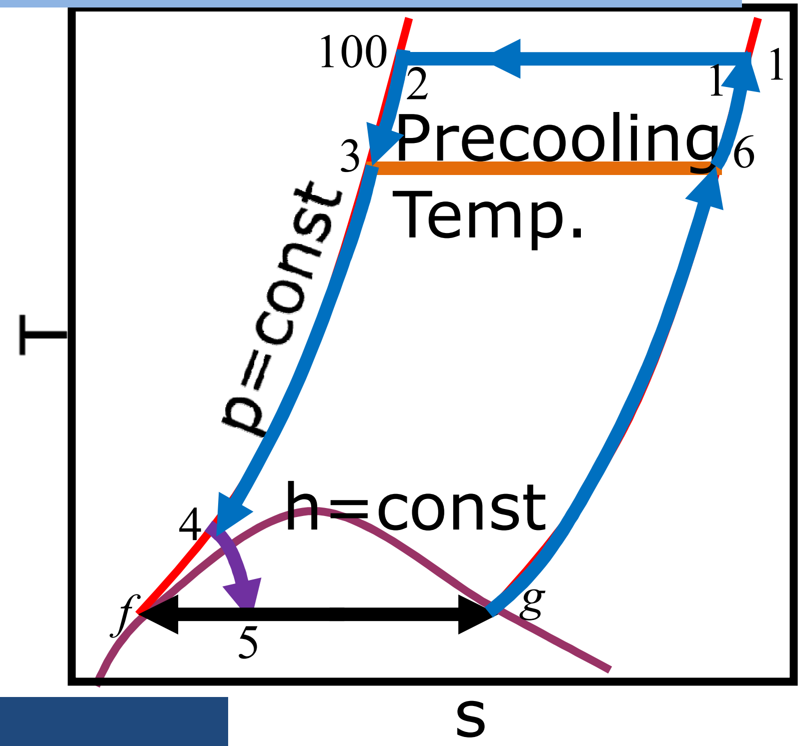
$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) + r\left(h_{b,r} - h_{a,r}\right)$$

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\frac{W_c}{\dot{m}}\bigg|_1 = 300\left(4.42 - 3.1\right) - \left(462 - 445\right) + 0.05\left(420 - 380\right) = 381 J/g$$

Tutorial : Part – 1

- **Work/unit mass of N_2 liquefied**



$$-\frac{W_c}{\dot{m}}\bigg|_1 = 381$$

$$y\big|_1 = 0.055$$

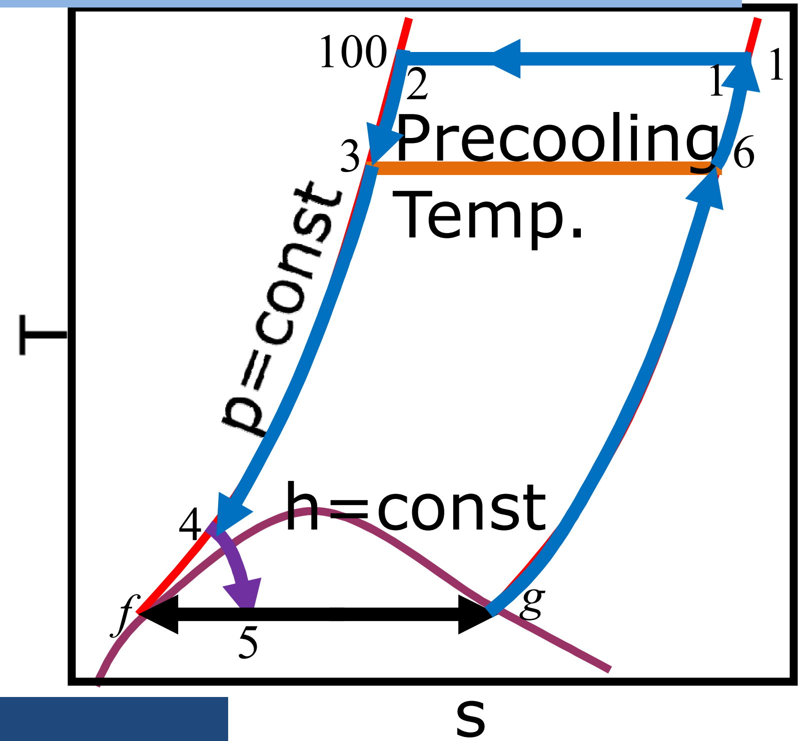
$$-\frac{W_c}{\dot{m}_f}\bigg|_1 = -\frac{W_c}{y\dot{m}} = \frac{381}{0.055} = 6927.2\ J/g$$

Tutorial : Part – 1

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f}\bigg|_1 = 6927.2$$

$$-\frac{W_i}{\dot{m}_f} = 767$$



$$FOM\Big|_1 = \frac{\dfrac{W_i}{\dot{m}_f}}{\dfrac{W_c}{\dot{m}_f}} = \frac{767}{6927.2} = 0.1107$$

Tutorial : Part – 1

- **Liquid yield**

$$y=\frac{\dot{m}_f}{\dot{m}}=\frac{h_1-h_2}{h_1-h_f}+r\left(\frac{h_{a,r}-h_{d,r}}{h_1-h_f}\right)$$

N_2	r	Point 2
II	0.07	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$y\big|_2=\frac{(462-445)}{(462-29)}+0.07\frac{(380-240)}{(462-29)}=\frac{(17)}{(433)}+0.07\frac{(140)}{(433)}=0.062$$

Tutorial : Part – 1

- **Work/unit mass of N_2 compressed**

N_2	r	Point 2
II	0.07	101.3 bar

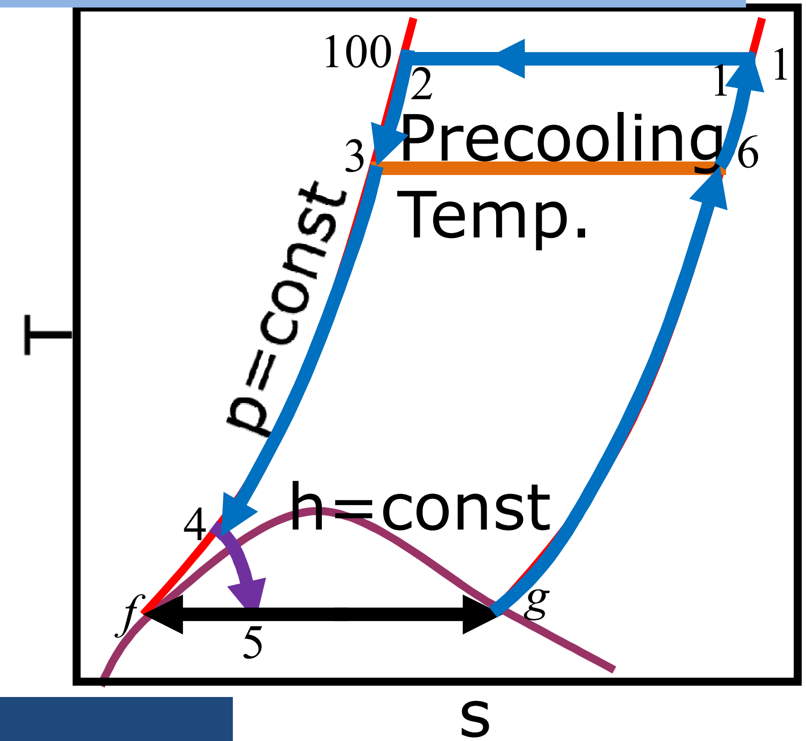
$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) + r\left(h_{b,r} - h_{a,r}\right)$$

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\left.\frac{W_c}{\dot{m}}\right|_2 = 300\left(4.42 - 3.1\right) - \left(462 - 445\right) + 0.07\left(420 - 380\right) = 381.8$$

Tutorial : Part – 1

- **Work/unit mass of N_2 liquefied**



$$-\left.\frac{W_c}{\dot{m}}\right|_2 = 381.8$$

$$y|_2 = 0.062$$

$$-\left.\frac{W_c}{\dot{m}_f}\right|_2 = -\frac{W_c}{y\dot{m}} = \frac{381.8}{0.062} = 6158.06\ J/g$$

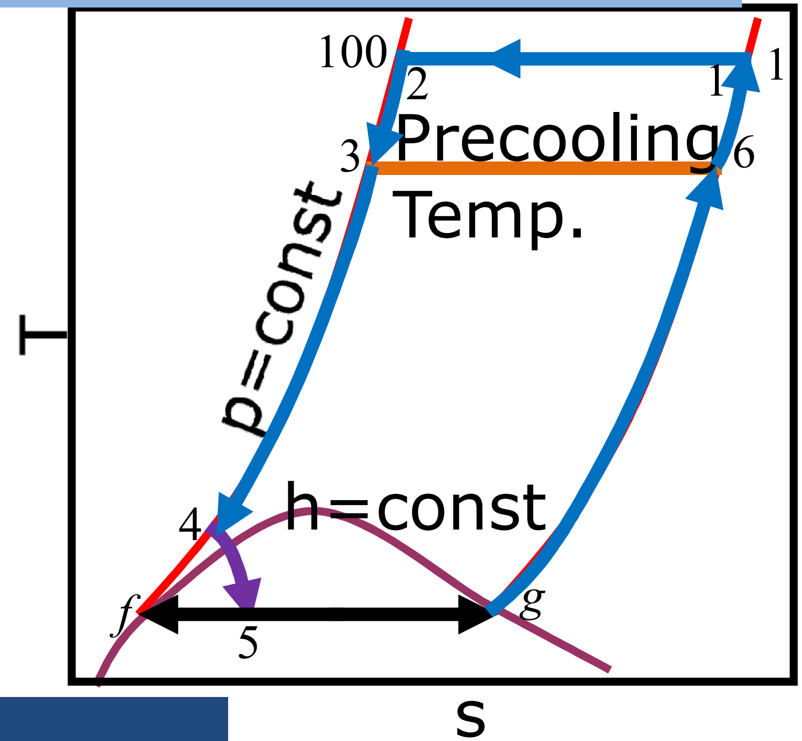
Tutorial : Part – 1

- **Figure of Merit (FOM)**

$$-\left.\frac{W_c}{\dot{m}_f}\right|_2 = 6158.06$$

$$-\frac{W_i}{\dot{m}_f} = 767$$

$$FOM\Big|_2 = \frac{W_i}{\dot{m}_f}\Big/\frac{W_c}{\dot{m}_f} = \frac{767}{6158.06} = 0.1245$$



Tutorial : Part – 2

- **Maximum Liquid yield**

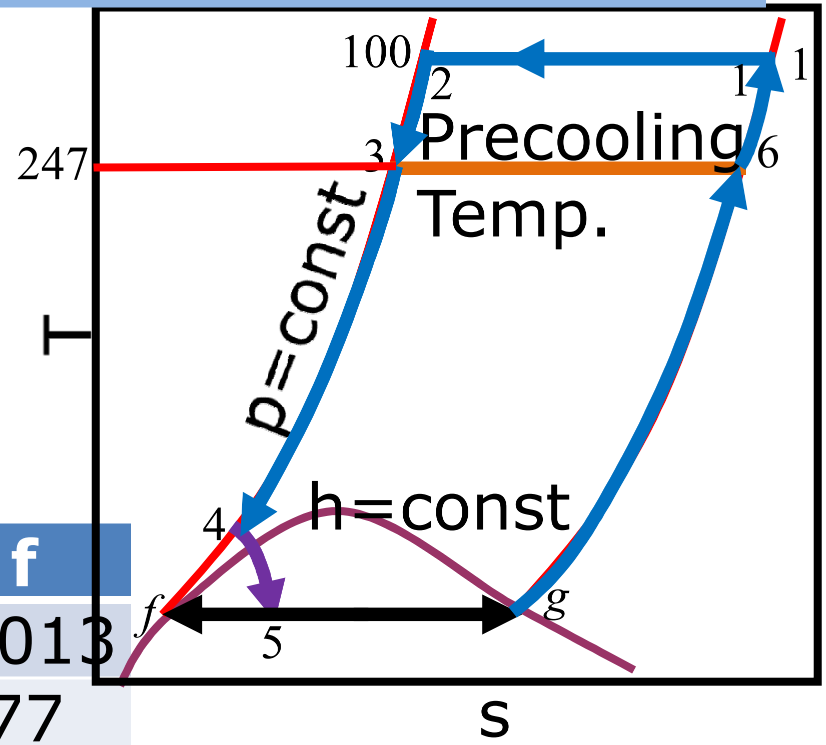
$$y = y_{max}$$

$$T_3 = T_6 = T_d = B\,P_{ref}$$

$$T_3 = T_6 = T_d = 247\,K$$

$$y_{max} = \frac{h_6 - h_3}{h_6 - h_f}$$

N_2	3	6	f
p (bar)	101.3	1.013	1.013
T (K)	247	247	77
h (J/g)	380	408	29



N_2	Point 2
@ y_{max}	101.3 bar

$$y_{max}\Big|_3 = \frac{(408-380)}{(408-29)} = \frac{(28)}{(379)} = 0.074$$

Tutorial : Part – 2

- **r corresponding to y_{max}**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

N_2	Point 2
@ y_{max}	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$\frac{(462-445)}{(462-29)} + r\frac{(380-240)}{(462-29)} = 0.074 \qquad \Rightarrow r = 0.11$$

Tutorial : Part – 2

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) + r\left(h_{b,r} - h_{a,r}\right)$$

@ y_{max}	Point 2
r=0.11	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$\left.-\frac{W_c}{\dot{m}}\right|_3 = 300\left(4.42 - 3.1\right) - \left(462 - 445\right) + 0.11\left(420 - 380\right) = 384 J/g$$

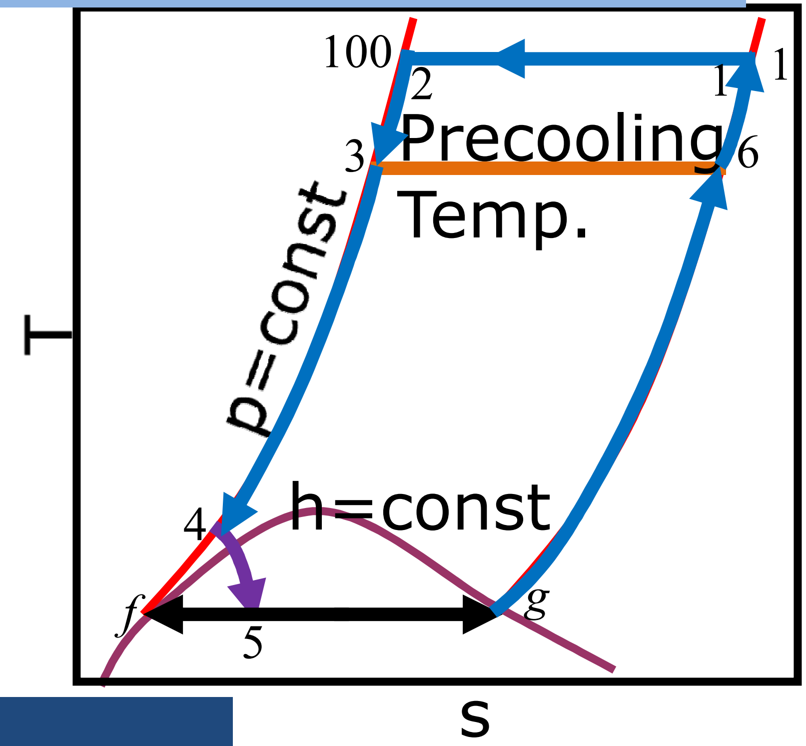
Tutorial : Part – 2

- **Work/unit mass of N_2 liquefied**

$$-\left.\frac{W_c}{\dot{m}}\right|_3 = 384$$

$$y_{max}\big|_3 = 0.074$$

$$-\left.\frac{W_c}{\dot{m}_f}\right|_3 = -\frac{W_c}{y\dot{m}} = \frac{384}{0.074} = 5189.2\ J/g$$

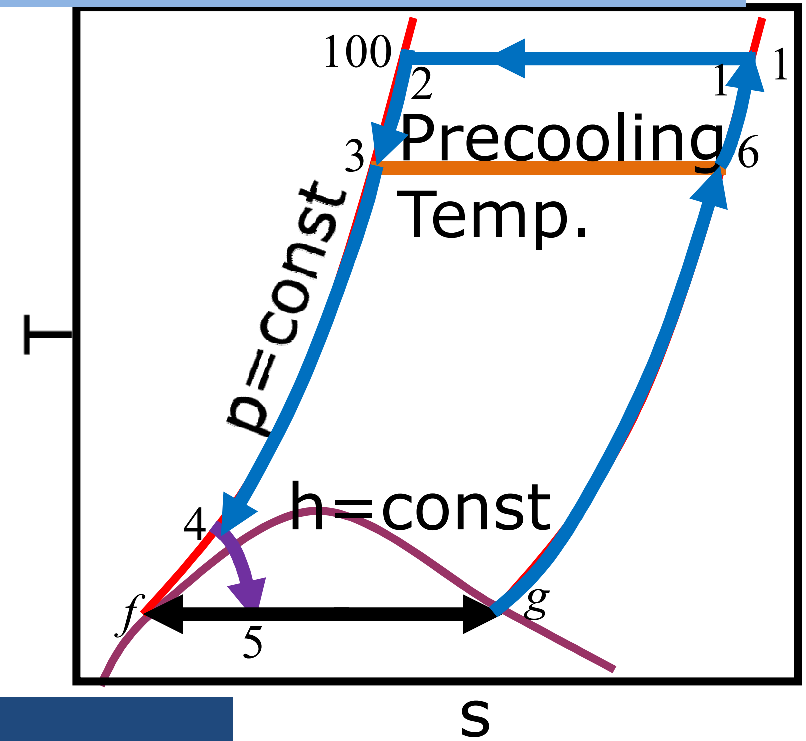


Tutorial : Part – 2

- **Figure of Merit (FOM)**

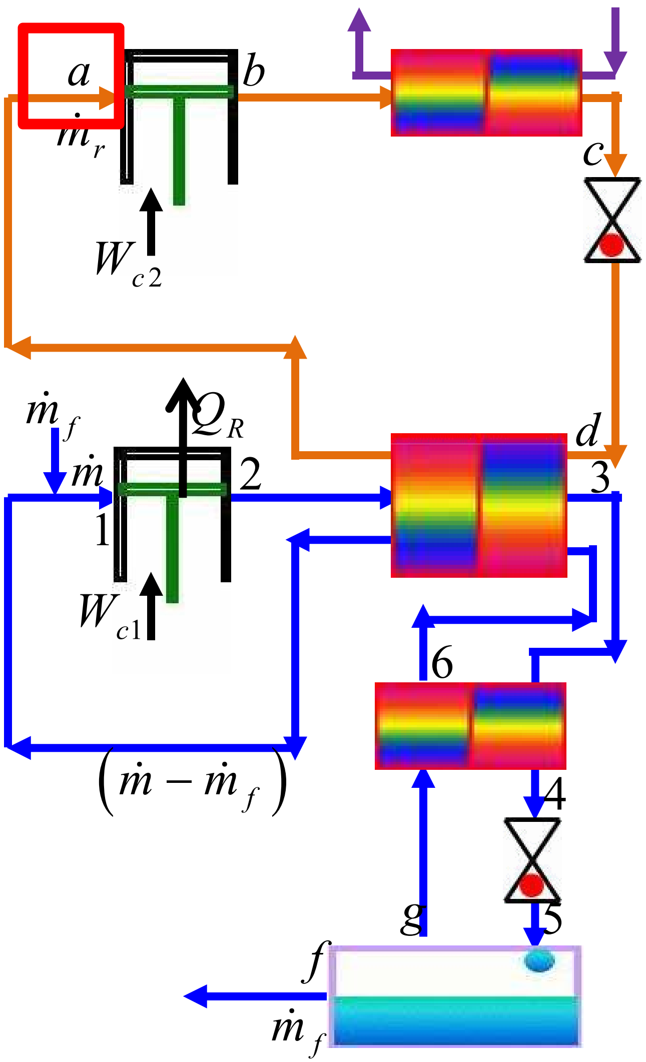
$$-\frac{W_c}{\dot{m}_f}\bigg|_3 = 5189.2$$

$$-\frac{W_i}{\dot{m}_f} = 767$$



$$FOM\Big|_3 = \frac{W_i}{\dot{m}_f}\Bigg/\frac{W_c}{\dot{m}_f} = \frac{767}{5189.2} = 0.1478$$

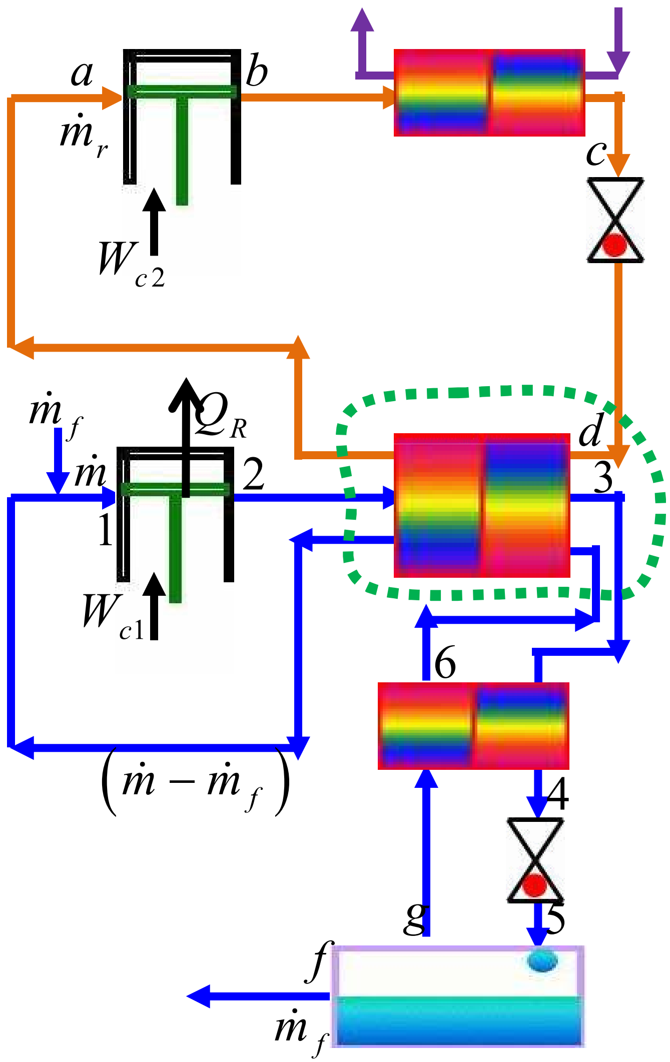
Tutorial : Part – 2



- From the above calculations, the value of **r** corresponding to $\mathbf{y_{max}}$ is 0.11 at the compression pressure of 101.3 bar.

- For r= 0.12, the enthalpy of the refrigerant at the state **a** is calculated by applying the energy balance across the 3 – fluid heat exchanger.

Tutorial : Part – 2



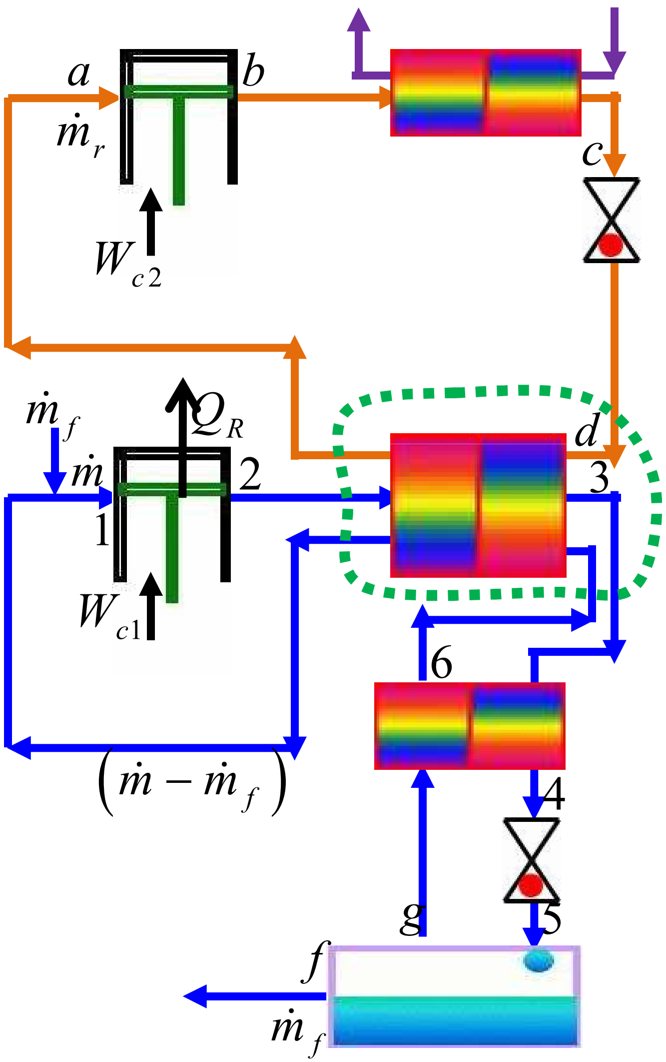
- Consider a control volume enclosing the 3 fluid heat exchanger.

IN	OUT
m_r @ d	m_r @ a
m @ 2	m @ 3
m – m_f @ 6	m – m_f @ 1

- Applying the heat balance, we have

$$\dot{m}_r h_{d,r} + \dot{m} h_2 + \left(\dot{m} - \dot{m}_f\right) h_6 = \dot{m}_r h_{a,r} + \dot{m}_3 h_3 + \left(\dot{m} - \dot{m}_f\right) h_1$$

Tutorial : Part – 2



- Rearranging the terms,

$$\dot{m}_r\left(h_{a,r} - h_{d,r}\right) + \dot{m}\left(h_3 - h_2 + h_1 - h_6\right) = \dot{m}_f\left(h_1 - h_6\right)$$

- Denoting the ratios

$$\frac{\dot{m}_r}{\dot{m}} = r \qquad y = \frac{\dot{m}_f}{\dot{m}}$$

$$r\left(h_{a,r} - h_{d,r}\right) + \left(h_3 - h_2 + h_1 - h_6\right) = y\left(h_1 - h_6\right)$$

Tutorial : Part – 2

$$r\left(h_{a,r} - h_{d,r}\right) + \left(h_3 - h_2 + h_1 - h_6\right) = y\left(h_1 - h_6\right)$$

- The equation of **y** at this refrigerant flow rate **r** is given by

$$y = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

- The only unknowns in these two equations are $\mathbf{h_{a,r}}$ and **y**.

- The values of $\mathbf{h_{a,r}}$ and **y** are obtained by solving these two simulations equations.

Tutorial : Part – 2

- Substituting the values, we have

N_2	1	2	f	6	a	c
p (bar)	1.013	101.3	1.013	1.013	1.013	8.104
T (K)	300	300	77	247	247	305
h (J/g)	462	445	29	408	$h_{a,r}$	240
					R134a	

$$y = r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_6}\right) + \left(\frac{h_3 - h_2 + h_1 - h_6}{h_1 - h_6}\right)$$

$$54y - 0.12h_{a,r} = -39.8$$

$$y = \frac{h_1 - h_2 + r\left(h_{a,r} - h_{d,r}\right)}{h_1 - h_f}$$

$$h_{a,r} - 3610.1y = 98.55$$

Tutorial : Part – 2

- Solving the simultaneous equations we have values as $y_4 = 0.074$ $h_{a,r} = 364.9$
- It is important to note that the value of **y** is same as $\mathbf{y_{max}}$ = 0.074.
- Also, the value of enthalpy at point **a** after the heat exchanger for r=0.12 is 364.9 J/g.
- This value is less than the value at the saturated vapor (380 J/g) indicating that the fluid is now a two – phase mixture.

Tutorial : Part – 2

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) + r\left(h_{b,r} - h_{a,r}\right)$$

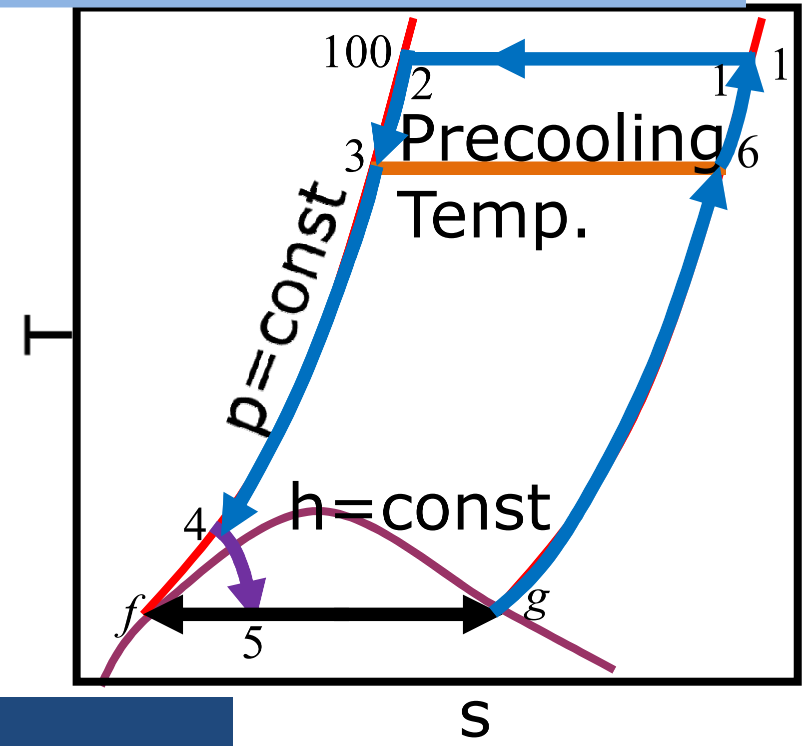
above y_{max}	Point 2
r=0.12	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	364.9	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\left.\frac{W_c}{\dot{m}}\right|_4 = 300\left(4.42 - 3.1\right) - \left(462 - 445\right) + 0.12\left(420 - 364.8\right) = 385.6 J/g$$

Tutorial : Part – 2

- **Work/unit mass of N_2 liquefied**



$$-\frac{W_c}{\dot{m}}\bigg|_4 = 385.6$$

$$y|_4 = 0.074$$

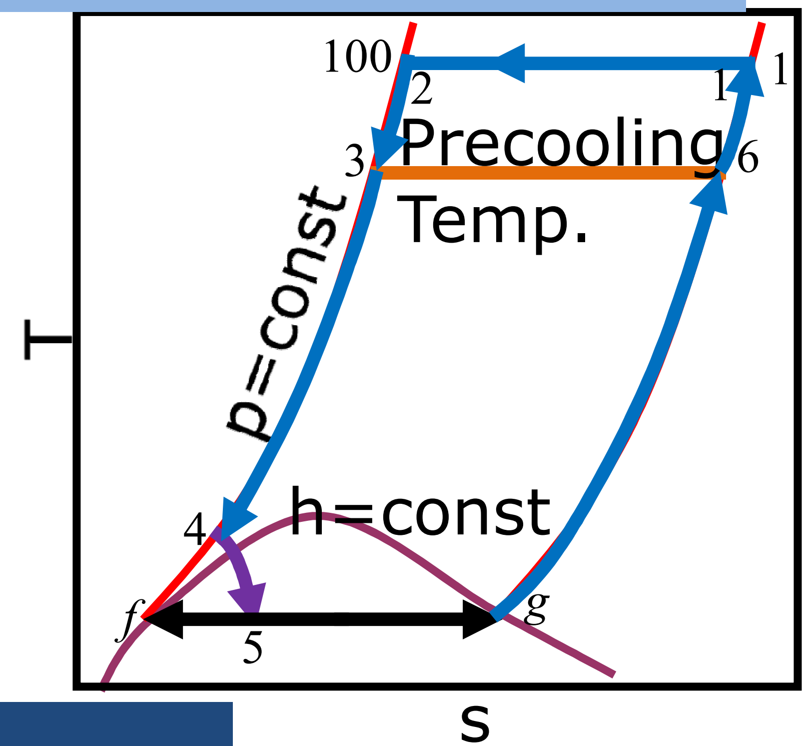
$$-\frac{W_c}{\dot{m}_f}\bigg|_4 = -\frac{W_c}{y\dot{m}} = \frac{385.6}{0.074} = 5239.13\ J/g$$

Tutorial : Part – 2

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f}\bigg|_4 = 5239.13$$

$$-\frac{W_i}{\dot{m}_f} = 767$$



$$FOM\big|_4 = \frac{W_i / \dot{m}_f}{W_c / \dot{m}_f} = \frac{767}{5239.13} = 0.1464$$

Tutorial : Part – 1

- Tabulating the results for 101.3 bar pressure condition, we have the following comparison for the various values of

  - Refrigerant flow rate (m_r).

	r	y	$-\frac{W}{\dot{m}}$	$-\frac{W}{\dot{m}_f}$	FOM
I	0.05	0.055	381.0	6927.2	0.111
II	0.07	0.062	381.8	6158.1	0.125
III (y_{max})	0.11	0.074	384.0	5189.2	0.148
IV	0.12	0.074	385.6	5239.1	0.146

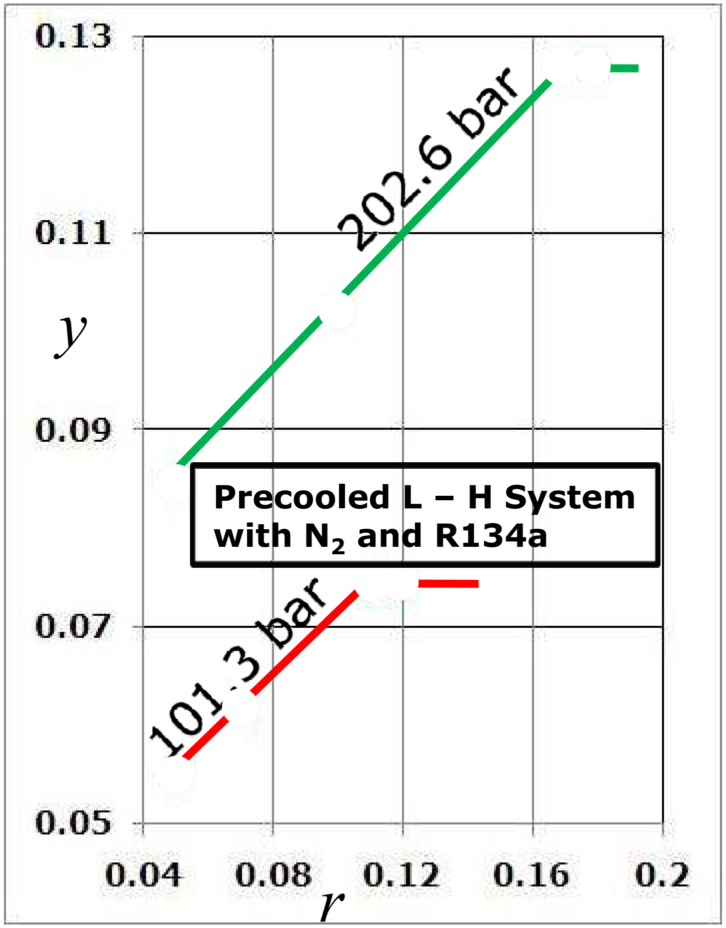
Tutorial : Part – 1

- Similarly, calculating the results for 202.6 bar pressure condition, we have the following comparison for the various values of
  - Refrigerant flow rate (m_r).

	r	y	$-\frac{W}{\dot{m}}$	$-\frac{W}{\dot{m}_f}$	FOM
I	0.05	0.085	476.0	5600.0	0.137
II	0.1	0.102	478.0	4704.7	0.163
III (y_{max})	0.17	0.127	479.0	3783.5	0.203
IV	0.18	0.127	483.5	3819.6	0.201

Tutorial : Part – 2

- **Liquid yield v/s. r**

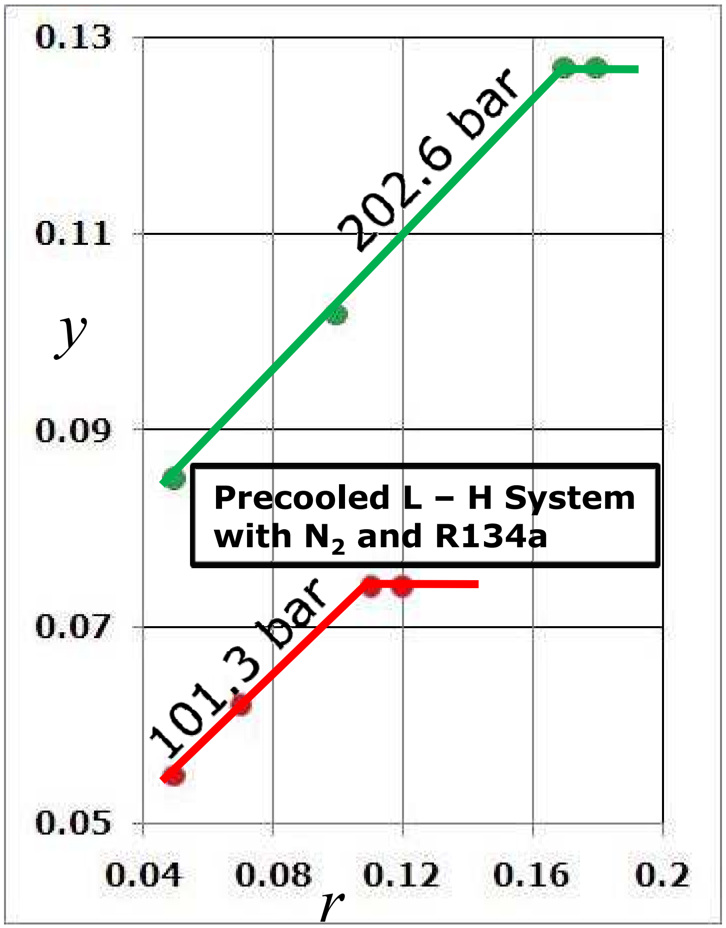


101.3	r	y
I	0.05	0.055
II	0.07	0.062
III (y_{max})	0.11	0.074
IV	0.12	0.074

202.6	r	y
I	0.05	0.085
II	0.1	0.102
III (y_{max})	0.17	0.127
IV	0.18	0.127

Tutorial : Part – 2

- **Liquid yield v/s. r**

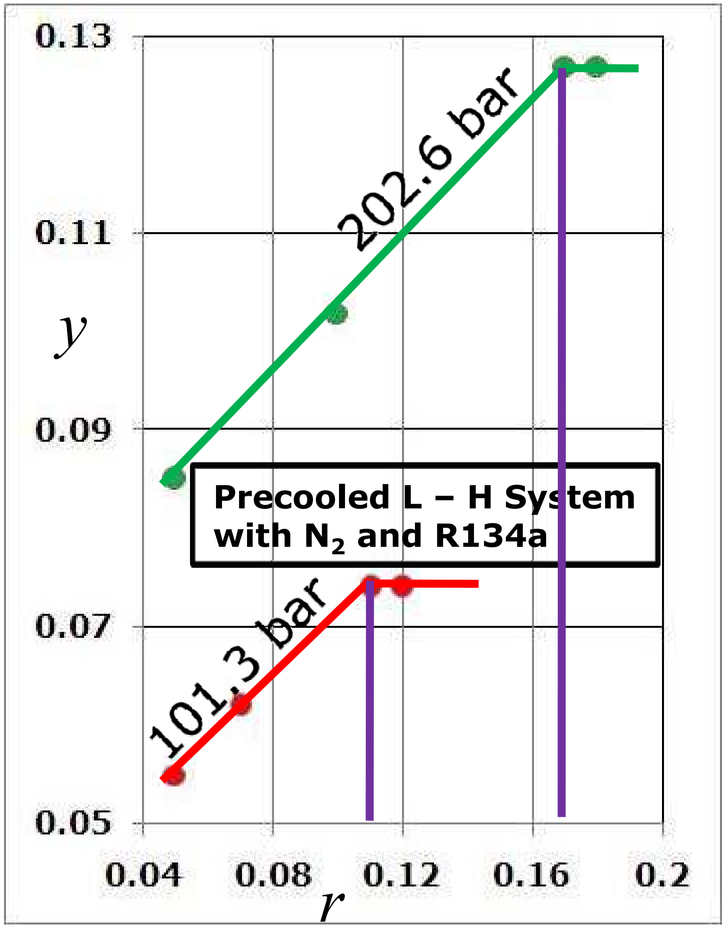


- It is clear that the yield of the system increases with the increase in the refrigerant flow rate for a given compression pressure.

- As the compression pressure increases, the yield increases for a given amount of the refrigerant flow rate.

Tutorial : Part – 2

- **Liquid yield v/s. r**

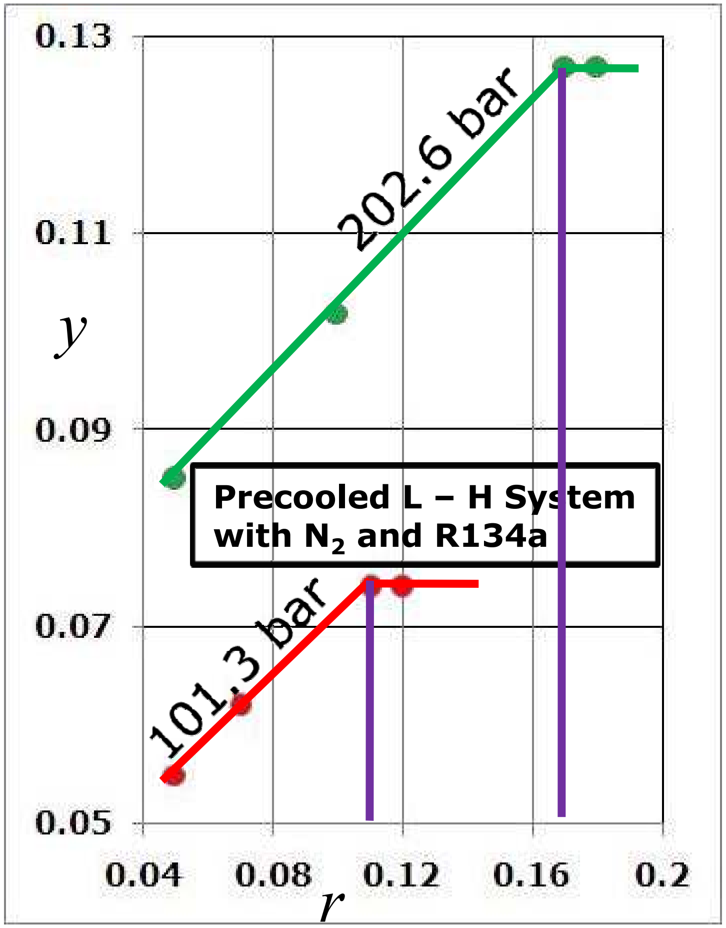


- For each compression pressure, the yield reaches to a maximum values and thereafter, it remains constant.

- This value of **r** is the limiting value.

Tutorial : Part – 2

- **Liquid yield v/s. r**

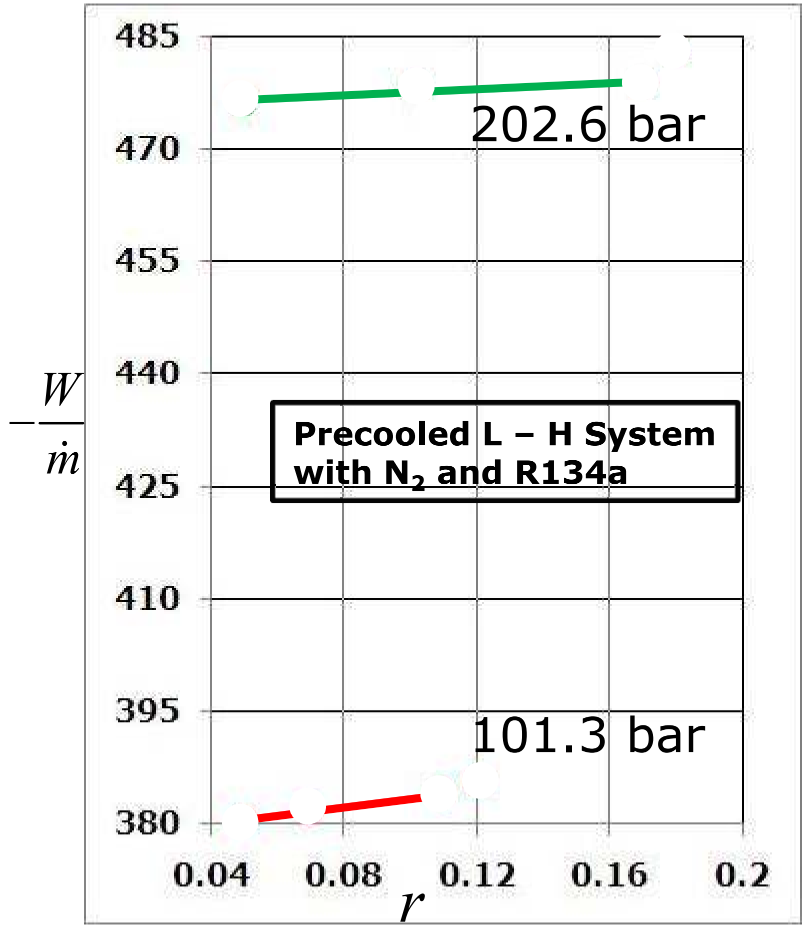


- Any additional increase in **r** leads to the liquid flow into the refrigerant compressor, which is not a desirable condition.

- Also, the limiting value increases with the increase in the compression pressure.

Tutorial : Part – 2

- **Work/unit mass compressed v/s. r**

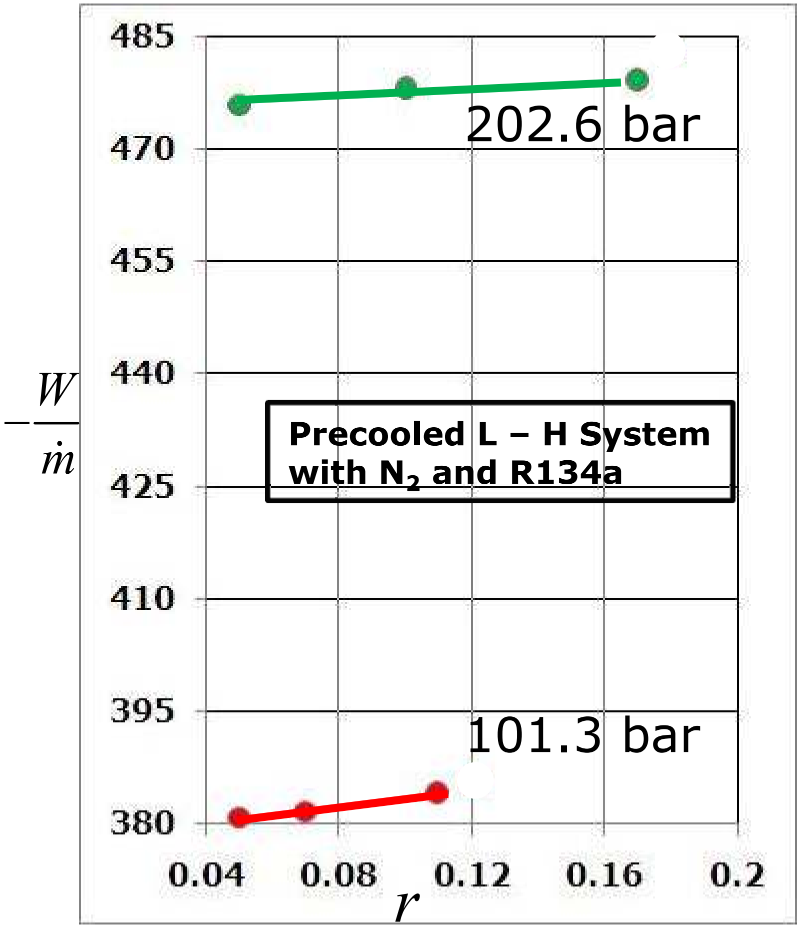


101.3	r	$\frac{W}{\dot{m}}$
I	0.05	381.0
II	0.07	381.8
III (y_{max})	0.11	384.0
IV	0.12	385.6

202.6	r	$\frac{W}{\dot{m}}$
I	0.05	476.0
II	0.1	478.0
III (y_{max})	0.17	479.0
IV	0.18	483.5

Tutorial : Part – 2

- **Work/unit mass compressed v/s. r**

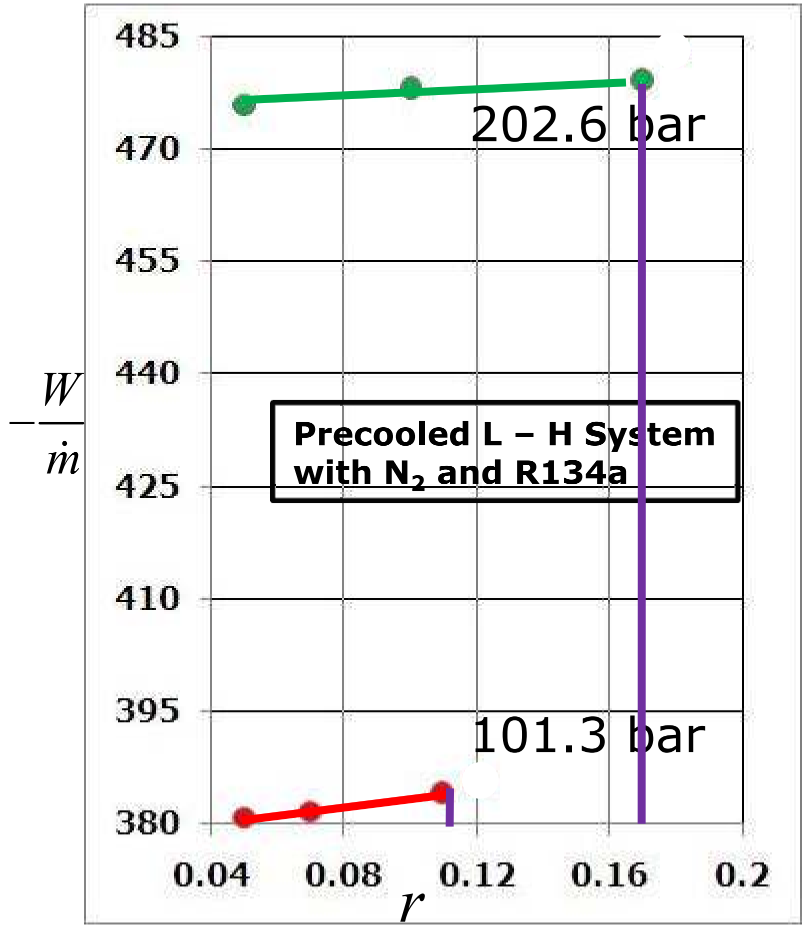


- We see that the work/unit mass of gas compressed increases with the increase in the refrigerant flow rate for a given compression pressure.

- As the compression pressure increases, work requirement also increases.

Tutorial : Part – 2

- **Work/unit mass compressed v/s. r**



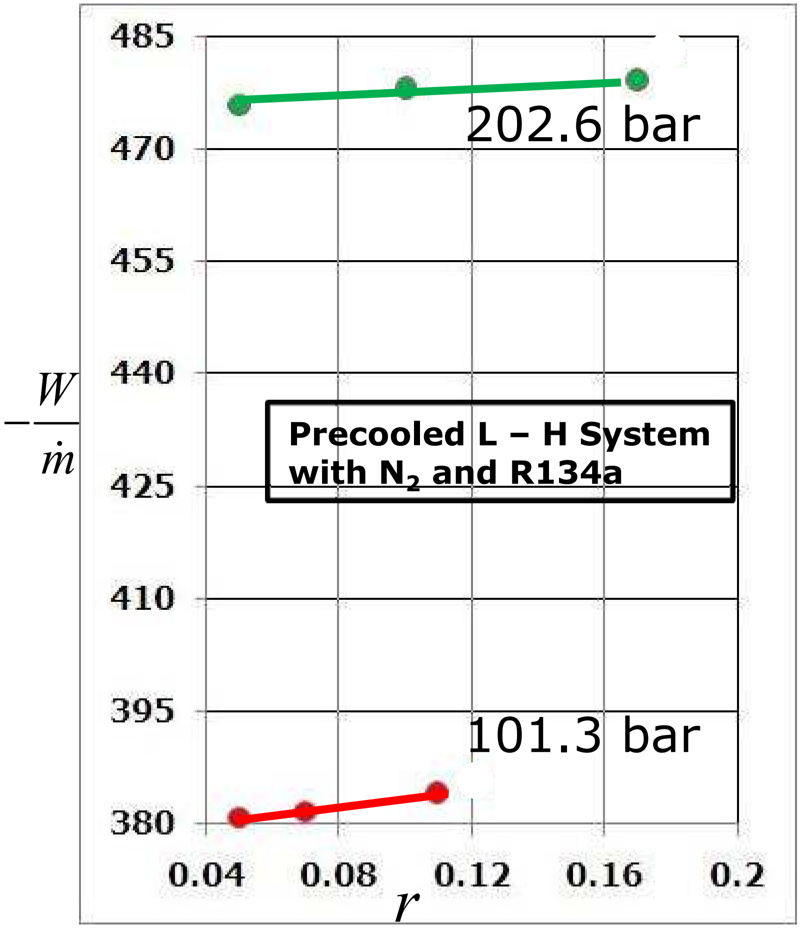
101.3	r	$-\frac{W}{\dot{m}}$
IV	0.12	385.6

202.6	r	$-\frac{W}{\dot{m}}$
IV	0.18	483.5

- It is clear that the work requirement is increased when the **r** value is increased beyond the limiting value.

Tutorial : Part – 2

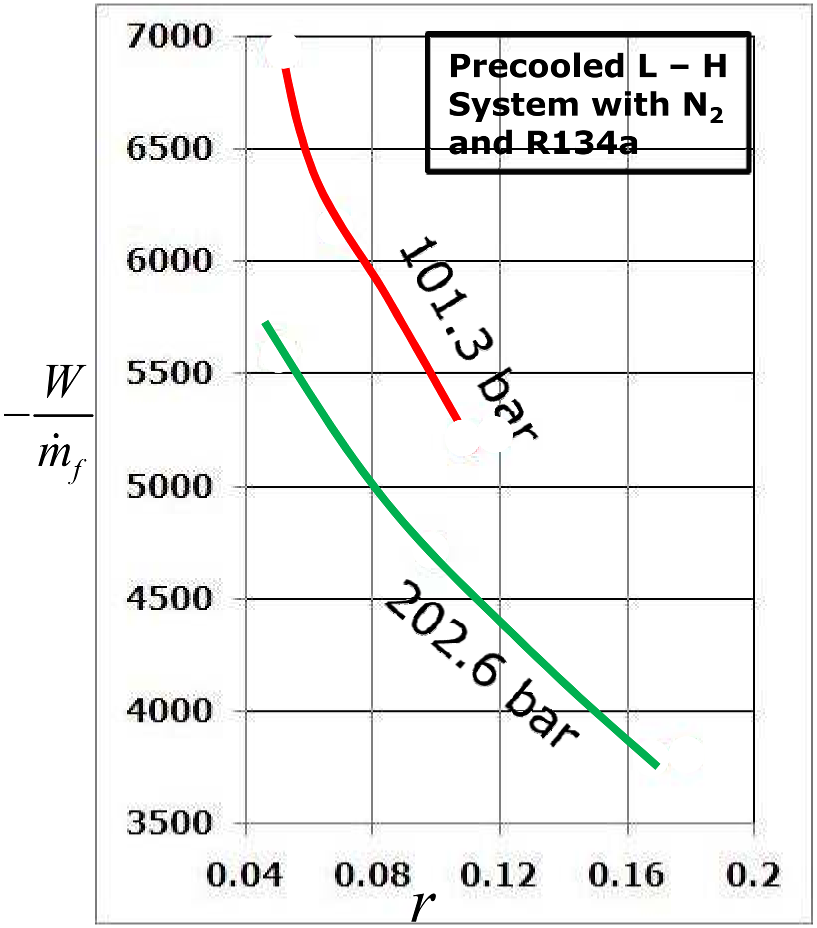
- **Work/unit mass compressed v/s. r**



- For each compression pressure, the increase in the work requirement is very small.
- Hence, the work requirement for the precooling compressor is negligible as compared to that of liquefaction compressor.

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**

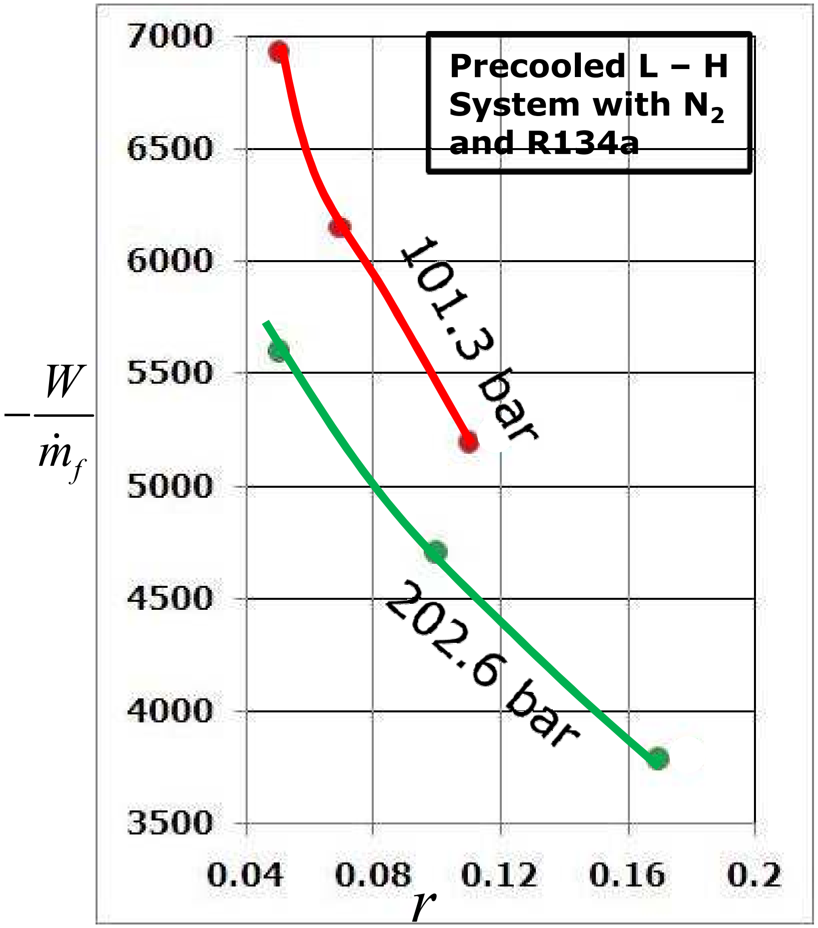


101.3	r	$-\frac{W}{\dot{m}_f}$
I	0.05	6927.2
II	0.07	6158.1
III (y_{max})	0.11	5189.2
IV	0.12	5239.1

202.6	r	$-\frac{W}{\dot{m}_f}$
I	0.05	5600.0
II	0.1	4704.4
III (y_{max})	0.17	3783.5
IV	0.18	3819.0

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**

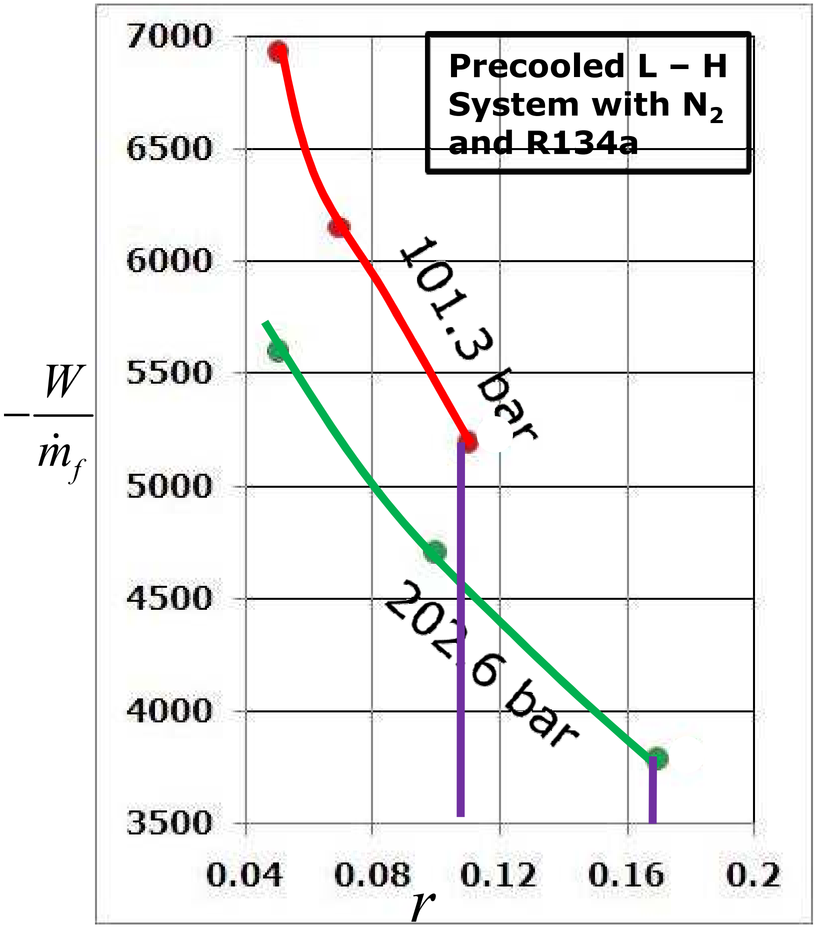


- For each compression pressure, the work requirement decreases with the increase in the refrigerant flow rate.

- As the compression pressure increases, the work requirement decreases for a given amount of the refrigerant flow rate **r**.

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**



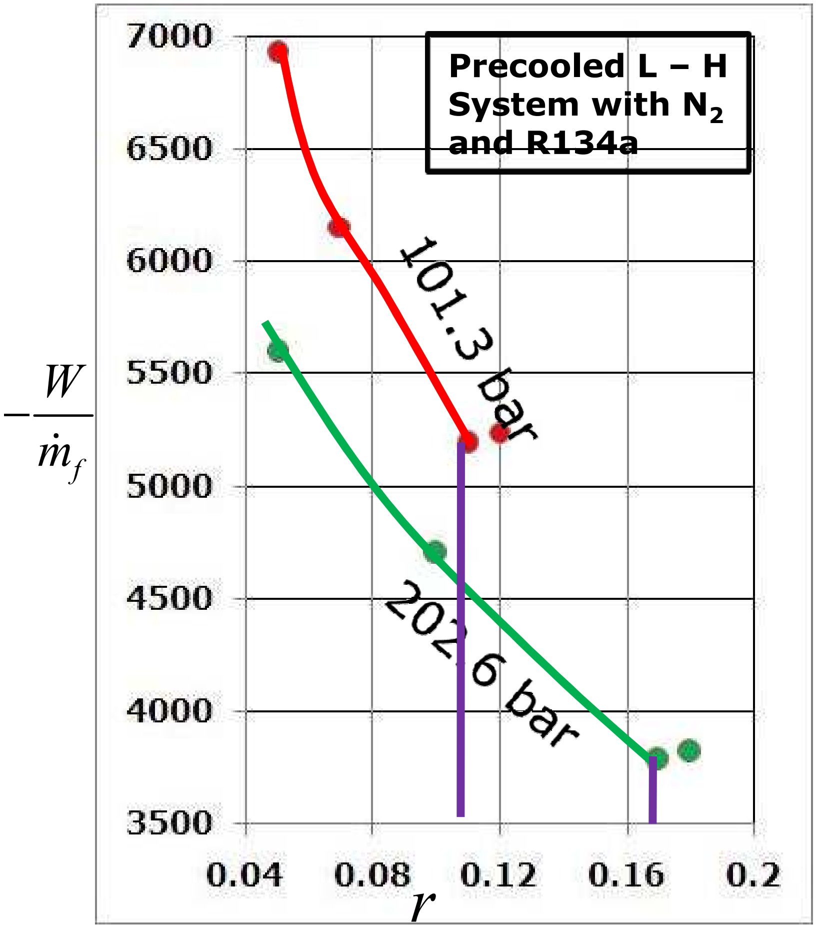
- The limiting values of **r** are as shown.
- Plotting the values of **r** above the limiting values we have as shown

101.3	r	$-\frac{W}{\dot{m}_f}$
IV	0.12	5239.1

202.6	r	$-\frac{W}{\dot{m}_f}$
IV	0.18	3819.0

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**

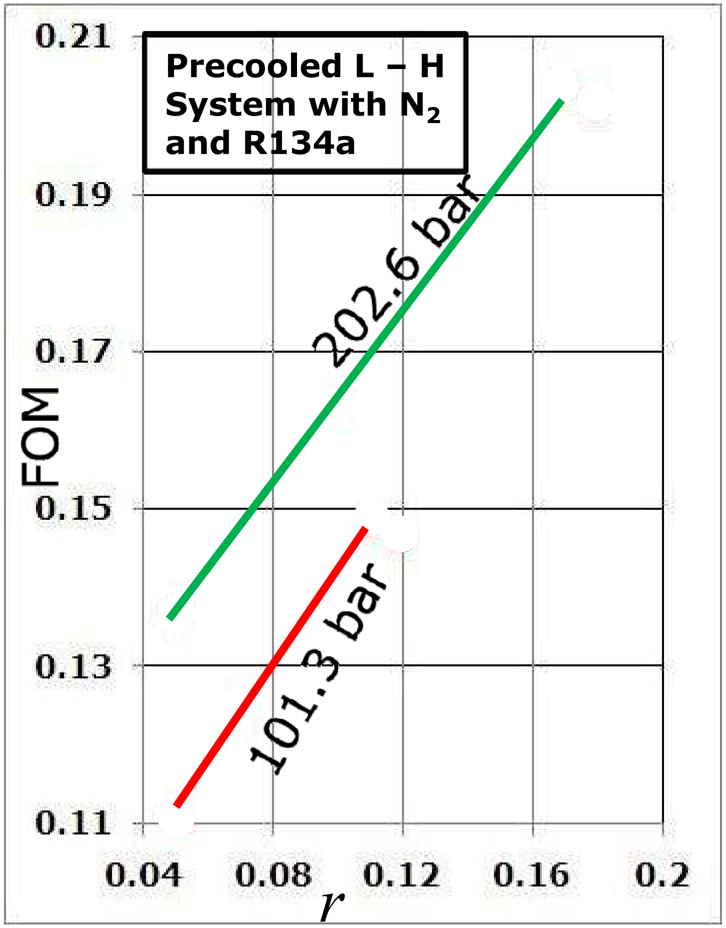


- Any further increase in the **r**, increases the work input. But under such conditions, the liquid refrigerant would enter the precooling compressor.

- This is undesirable for compressor operation.

Tutorial : Part – 2

- **FOM v/s. r**

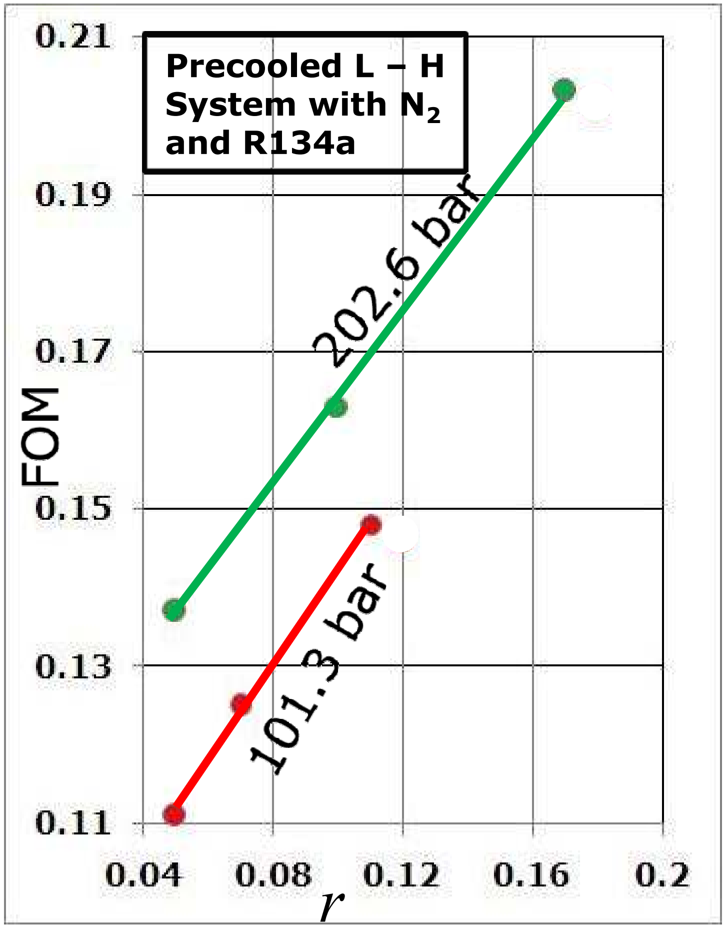


101.3	r	FOM
I	0.05	0.111
II	0.07	0.125
III (y_{max})	0.11	0.148
IV	0.12	0.146

202.6	r	FOM
I	0.05	0.137
II	0.1	0.163
III (y_{max})	0.17	0.203
IV	0.18	0.201

Tutorial : Part – 2

- **FOM v/s. r**

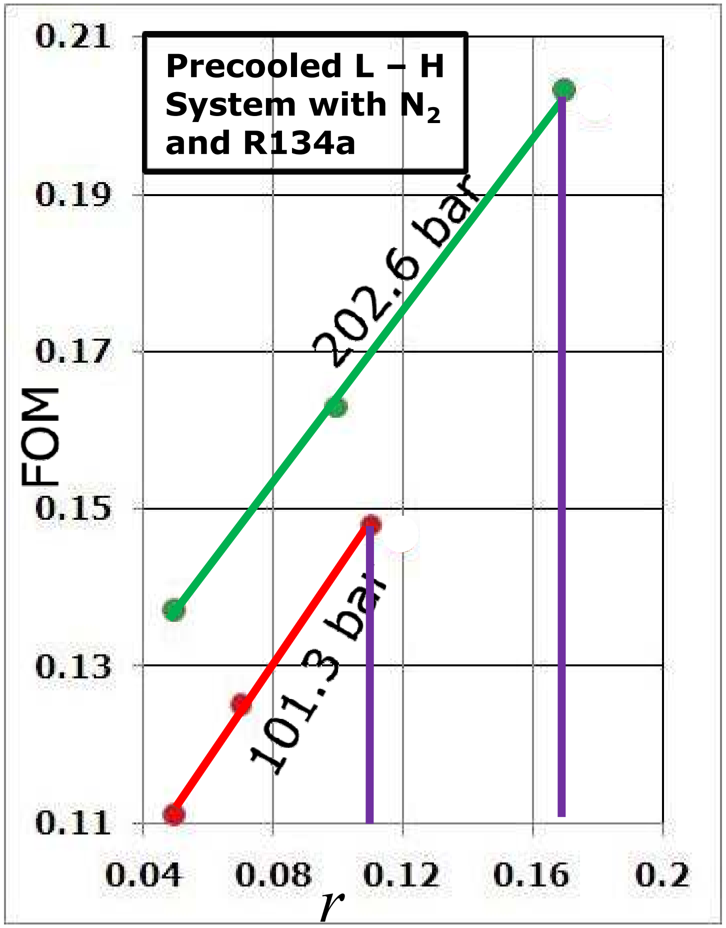


- For each compression pressure, the FOM increases with the increase in the refrigerant flow rate.

- As the compression pressure increases, the FOM increase for a given amount of the refrigerant flow rate **r**.

Tutorial : Part – 2

- **FOM v/s. r**



- The limiting values of **r** are as shown.

101.3	r	FOM
IV	0.12	0.146

202.6	r	FOM
IV	0.18	0.201

- Any further increase in the **r**, decreases the FOM. But, the liquid refrigerant would enter the precooling compressor.