Introduction to Finite volume methods: Diffusion problems

Q1.

Finite volume discretization equation for scalar variable ϕ is obtained as:

$-5\phi_P = -3\phi_E - 2\phi_W + 5$

Is the above discretization expected to yield a physically unrealistic solution. Justify with reasoning.

Solution

$a_P\phi_P = a_E\phi_E + a_W\phi_W + b$

According to the basic rules, all coefficients in the discretized equation must be of the same sign.

Here, $a_P = -5$, $a_E = -3$, $a_W = -2$

The above discretization yields a physically realistic solution.

Q2.

For flow through a porous medium, the distributed resistance to flow is expressed by the source term $S = -C|u|u$, for the x – direction momentum equation. Here 'C' is a positive constant and 'u' is the velocity component in the x- direction. Write the best linearization for the source term by giving expressions for S_c and S_p.

Solution

Case (i) $u > 0$

$$S = -Cu^2$$

$$S - S^* = \left.\frac{dS}{du}\right|_* \left(u - u^*\right)$$

$$S - Cu^{*2} = -2Cu^*\left(u - u^*\right)$$

$$S = Cu^{*2} - 2Cu^*u$$

Since u^* is positive,

$$S_p = -2Cu^*$$

It gives negative S_p.

Case (ii) $u < 0$

$$S = Cu^2$$

$$S - S^* = \left.\frac{dS}{du}\right|_* \left(u - u^*\right)$$

$$S - Cu^{*2} = 2Cu^*\left(u - u^*\right)$$

$$S = -Cu^{*2} + 2Cu^*u$$

Since u^* is negative,

$$S_p = 2Cu^*$$

Otherwise, if u^* itself fluctuates during iterations and changes its sign, then the entire source term may be dumped on S_c.