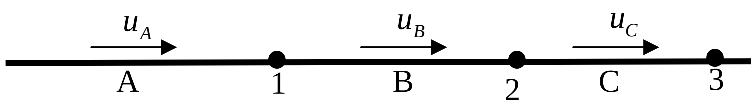
Fluid flow

Q1.

In the one-dimensional constant-density situation shown below, the momentum equations for u_B and u_c can be written as follows:

$$u_B = 5 + 2.5(p_1 - p_2)$$

$$u_c = 5 + 7.5(p_2 - p_3)$$



The boundary conditions are as: u_A = 15 (all values are given in consistent units). Obtain the values for p_1 and p_2, following the SIMPLE calculation procedure. Comment on the uniqueness of your solution.

Solution

Assume $p_1^* = p_2^* = p_3^* = 0$

Solving momentum equation, we get

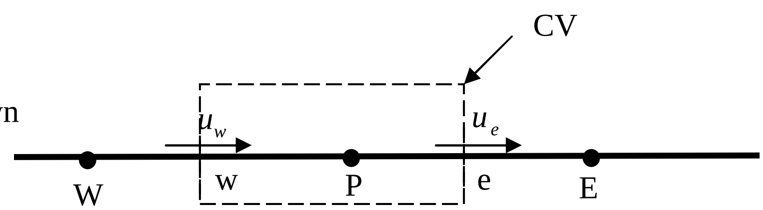
$u_B^* = 5$

Continuity equation becomes

$$\frac{du}{dx} = 0$$

Integrating within a control volume as shown

$$u_e = u_w$$



Continuity equation for grid point 1

$$u_A = u_B$$

$$u_A = 5 + 2.5(p_1' - p_2')$$

$$15 = 5 + 2.5(p_1' - p_2') \qquad (16)$$

Continuity equation for grid point 2

$$u_B = u_C$$

$$5 + 2.5(p_1' - p_2') = u_C$$

$$5 + 2.5(p_1' - p_2') = 15 \qquad (17)$$

No unique solutions of (16) and (17)

<u>Solutions are guided by initial guess for iteration</u>, depicting the relative nature of p as a numerically computed variable.

Let $p_1' = 0$

$p_2' = -4$

$p_1 = p_1^* + p_1' = 0$

$p_2 = p_2^* + p_2' = -4$

$u_B = u_B^* + 2.5(p_1' - p_2') = 5 + 2.5(0+4) = 15$

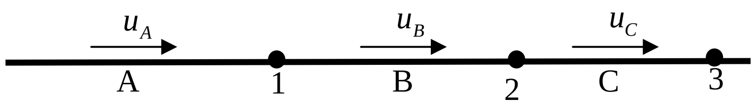
Solution is converged.

Q2.

In the one-dimensional constant-density situation below, the momentum equations for u_B and u_C can be written as follows:

$$u_B = 5 + 2.5(p_1 - p_2)$$

$$u_C = 5 + 7.5(p_2 - p_3)$$



The boundary conditions are as: $u_B = 15$, $p_3 = 10$ (all values are given in consistent units).

(i) Write the continuity equations for the regions AB and BC and hence derive the corresponding pressure correction equations.

(ii) Starting with guesses for p_1 and p_2, follow the SIMPLE procedure to obtain converged values of p_2, u_B and u_C.

Solution

Assume $p_1^* = p_2^* = 0$

Solving momentum equations, we get

$$u_B^* = 5$$

$$u_C^* = 5 + 7.5(0 - 10) = -70$$

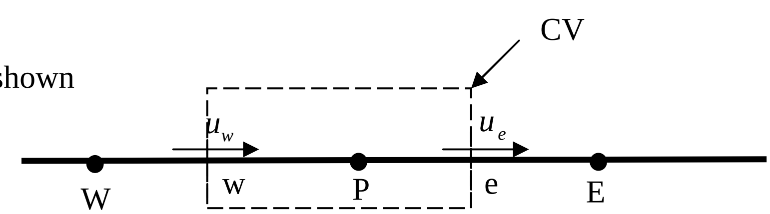
$$u_e = u^*_e + d_e(p'_P - p'_E)$$

$$u_w = u_w^* + d_w(p'_W - p'_P)$$

Continuity equation becomes

$$\frac{du}{dx} = 0$$

Integrating within a control volume as shown



$$u_e = u_w$$

$$u^*_e + d_e(p'_P - p'_E) = u_w^* + d_w(p'_W - p'_P)$$

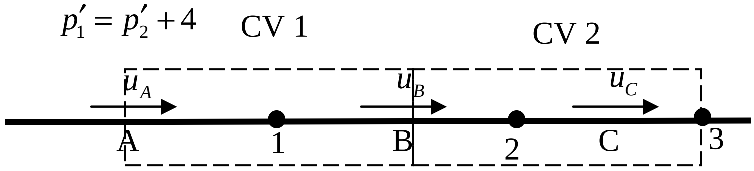
$$(d_e + d_w)p'_P = d_e p'_E + d_w p'_W + (u_w^* - u^*_e)$$

For point 1 as shown

$$(d_B + d_A)p'_1 = d_B p'_2 + \cancel{d_A}^{\,0} p'_A + (u_A^* - u_B^*)$$

$$2.5p'_1 = 2.5p'_2 + 10$$

or, $p'_1 = p'_2 + 4$ (18)



For point 2 as shown

$$(d_C + d_B)\, p'_2 = d_C\, \cancel{p'_3}^{\,0} + d_B\, p'_1 + \left(u^*_B - u^*_C\right)$$

$$10\, p'_2 = 2.5\, p'_1 + (5 + 70)$$

or, $\quad 4 p'_2 = p'_1 + 30 \qquad (19)$

Solving Eqs (18) and (19), we get

$$p'_1 = \frac{46}{3}$$

$$p'_2 = \frac{34}{3}$$

$$u_B = u^*_B + d_B\left(p'_1 - p'_2\right) = 5 + 2.5\left(\frac{46}{3} - \frac{34}{3}\right) = 15$$

$$u_C = u^*_C + d_C\left(p'_2 - p'_3\right) = -70 + 7.5\left(\frac{34}{3} - 0\right) = 15$$

$$p_1 = p^*_1 + p'_1 = \frac{46}{3}$$

$$p_2 = p^*_2 + p'_2 = \frac{34}{3}$$

$$b = u^*_w - u^*_e$$

For control volume 1

$$b = u^*_A - u^*_B = 0$$

For control volume 2

$$b = u^*_B - u^*_C = 0$$

This is the continuity satisfying velocity field.

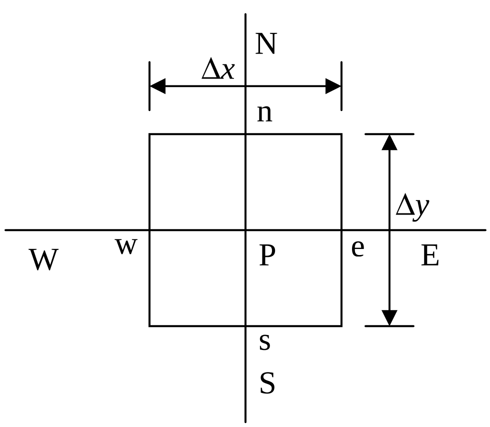
Q3.

In the two dimensional situation shown, the following quantities are given: $u_w = 50$, $v_s = 20$, $p_N = 0$, $p_E = 10$. The flow is steady and the density is uniform. The momentum equations for u_e and v_n V_N are given by:

$$u_e = d_e\left(p_P - p_E\right)$$

$$v_n = d_n\left(p_P - p_N\right)$$

where the constants d_e and d_n are given by $d_e = 1$, $d_n = 0.6$. The control volume shown has $\Delta x = \Delta y$. Use SIMPLE algorithm to obtain the values of u_e, v_n and p_P.



Solution

Let $p_W^* = 20$, $p_S^* = 20$, $p_P^* = 15$

Solving momentum equations, we get

$$u_e^* = d_e\left(p_P^* - p_E^*\right) = (1)(15-10) = 5$$

$$v_n^* = d_n\left(p_P^* - p_N^*\right) = 0.6(15-0) = 9$$

Continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Integrating within a control volume as shown

$$\int_s^n \int_w^e \frac{\partial u}{\partial x} dxdy + \int_w^e \int_s^n \frac{\partial v}{\partial y} dydx = 0$$

$$(u_e - u_w)\Delta y + (v_n - v_s)\Delta x = 0$$

$$u_e^* + d_e\left(p'_P - p'_E\right) + u_w^* + d_w\left(p'_W - p'_P\right) + v_n^* + d_n\left(p'_P - p'_N\right) + v_s^* + d_s\left(p'_S - p'_P\right) = 0$$

$$\left(d_e + d_w + d_n + d_s\right) p'_P = d_e p'_E + d_w p'_W + d_n p'_N + d_s p'_S + \left(u_w^* - u_e^*\right) + \left(v_s^* - v_n^*\right)$$

$$1.6 p'_P = d_e \cancel{p'_E}^{0} + d_w p'_W + d_n \cancel{p'_N}^{0} + d_s p'_S + (50-5) + (20-9)$$

$$p'_P = \frac{56}{1.6} = 35$$

$$u_e = u_e^* + d_e\left(p'_P - p'_E\right) = 5 + 1(35-0) = 40$$

$$v_n = v_n^* + d_n\left(p'_P - p'_N\right) = 9 + 0.6(35-0) = 30$$

$$p_P = p_P^* + p'_P = 15 + 35 = 50$$

$$b = u_w^* - u_e^* + v_s^* - v_n^* = 50 - 40 + 20 - 30 = 0$$

Solution is converged.

Q.4

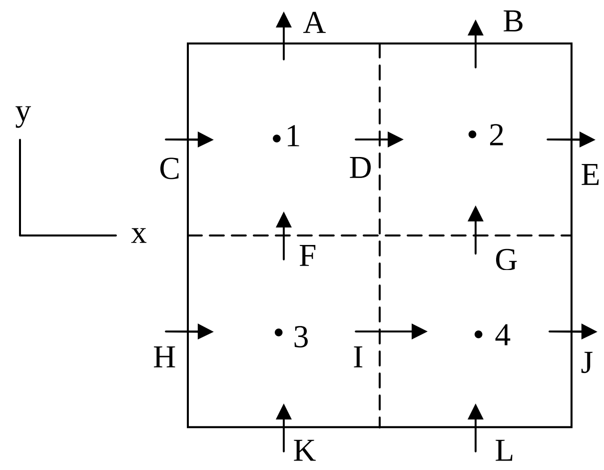
A steady, uniform-density, 2-D flow is to be calculated on the square grid shown below. The boundary velocities are given as; $v_A = 30$, $v_B = 40$, $u_C = 100$, $u_E = 50$, $u_H = 200$, $u_J = 210$, $v_K = 0$ and $v_L = 20$. Among these numbers, there is some doubt about correctness of the value of u_J. If all other numbers are correct, what should be the correct value of u_J?

The internal velocities are governed by simplified momentum equations given by:

$$u_D = 70 + 0.5\left(p_1 - p_2\right) \qquad u_I = 10 + 0.7\left(p_3 - p_4\right)$$

$$v_F = 30 + 0.5\left(p_3 - p_1\right) \qquad v_G = 18 + 0.8\left(p_4 - p_2\right)$$

Write discretized continuity equation for each control volume. Derive the discretization equation for pressure by substituting from momentum equations, following SIMPLER calculation procedure. Solve the pressure equations to obtain p_1, p_2, p_3 and p_4. Hence obtain values of u_D, u_I, v_F and v_G.



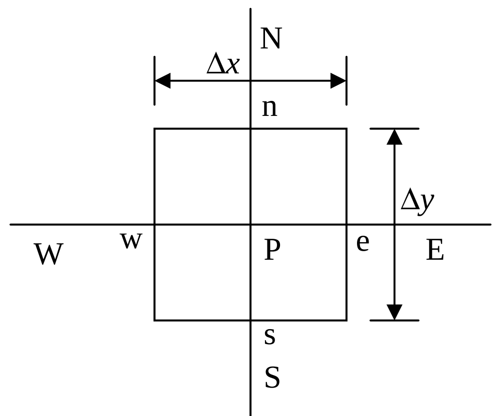
Solution

From the conservation of mass, we have

$\sum$rate of inflow=$\sum$rate of outflow

$100+200+0+20=30+40+50+u_J$

or, $u_J=200$



Continuity equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Integrating within a control volume as shown, it becomes

$$\int_s^n\int_w^e\frac{\partial u}{\partial x}dxdy+\int_w^e\int_s^n\frac{\partial v}{\partial y}dydx=0$$

$$\left(u_e-u_w\right)\Delta y+\left(v_n-v_s\right)\Delta x=0$$

Momentum equations can be expressed as

$$u_e=\hat{u}_e+d_e\left(p_P-p_E\right)$$

$$u_w=\hat{u}_w+d_w\left(p_P-p_W\right)$$

$$v_n=\hat{v}_n+d_n\left(p_P-p_N\right)$$

$$v_s = \hat{v}_s + d_s\left(p_P - p_S\right)$$

$$\left(d_e + d_w + d_n + d_s\right)p_P = d_e p_E + d_w p_W + d_s p_S + d_n p_N + \hat{u}_w - \hat{u}_e + \hat{v}_s - \hat{v}_n$$

For grid point 1

$$d_e = d_D = 0.5, d_w = d_C = 0$$

$$d_s = d_F = 0.5, d_n = d_A = 0$$

$$p_1 = 0.5p_2 + 0.5p_3 + (100-70) + (30-30)$$

$$p_1 = 0.5p_2 + 0.5p_3 + 30$$

For grid point 2

$$d_e = d_E = 0, d_w = d_D = 0.5$$

$$d_s = d_G = 0.8, d_n = d_B = 0$$

$$1.3p_2 = 0.5p_1 + 0.8p_4 + (70-70) + (18-40)$$

$$1.3p_2 = 0.5p_1 + 0.8p_4 - 2$$

For grid point 3

$$d_e = d_E = 0, d_w = d_D = 0.5$$

$$d_s = d_G = 0.8, d_n = d_B = 0$$

$$1.3p_2 = 0.5p_1 + 0.8p_4 + (70-70) + (18-40)$$

$$1.2p_3 = 0.7p_4 + 0.5p_1 + 160$$

For grid point 4

$$d_e = d_E = 0, d_w = d_D = 0.5$$

$$d_s = d_G = 0.8, d_n = d_B = 0$$

$$1.3p_2 = 0.5p_1 + 0.8p_4 + (70-70) + (18-40)$$

$$1.5p_4 = 0.7p_3 + 0.8p_2 - 188$$

Choose $p_4 = 0$ (reference)

$$p_1 = 160$$

$$p_2 = 60$$

$$p_3 = 200$$

$$u_D^* = 70 + 0.5\left(p_1^* - p_2^*\right) = 70 + 0.5(160-60) = 120$$

$$u_I^* = 10 + 0.7\left(p_3^* - p_4^*\right) = 10 + 0.7(200-0) = 150$$

$$v_F^* = 30 + 0.5\left(p_3^* - p_1^*\right) = 30 + 0.5(200-160) = 50$$

$$v_G^* = 18 + 0.8\left(p_4^* - p_2^*\right) = 18 + 0.8(0-60) = -30$$

Pressure correction equation becomes

$$\left(d_e + d_w + d_n + d_s\right)p'_P = d_e p'_E + d_w p'_W + d_s p'_S + d_n p'_N + \underbrace{\left(u_w^* - u_e^*\right) + \left(v_s^* - v_n^*\right)}_{b}$$

Top left CV:

$$b = (100-120) + (50-30) = 0$$

Top right CV:

$$b = (120 - 50) + (-30 - 40) = 0$$

Bottom left CV:

$$b = (200 - 150) + (0 - 50) = 0$$

Bottom right CV:

$$b = (150 - 200) + (20 + 30) = 0$$

This is the converged solution.