Vibrations of Structures

Module III: Vibrations of Beams

Exercises

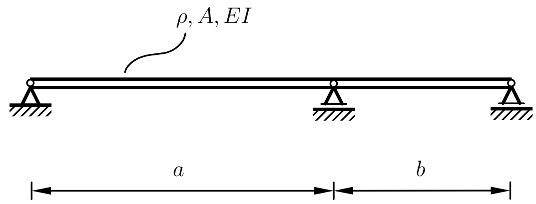

Figure 1: Exercise 1

1. A uniform Euler-Bernoulli beam of length $a+b$, flexural stiffness EI and linear density ρA is supported, as shown in Fig. 1. Plot the variation of the first few eigenfrequencies for variation of b in the range $(0, a)$.

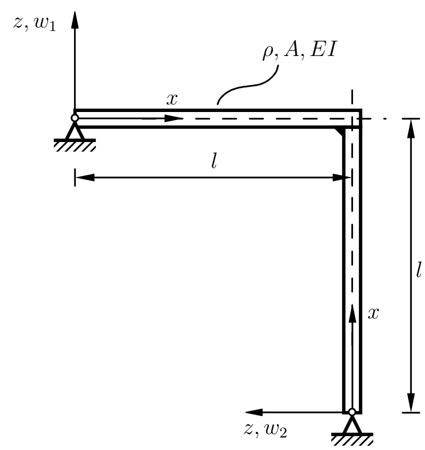

Figure 2: Exercise 2

2. The frame shown in Fig. 2 is made by welding two uniform homogeneous Euler-Bernoulli beams infinitely stiff in tension. Determine the first few eigenfrequencies and the corresponding modes of vibration of the frame.

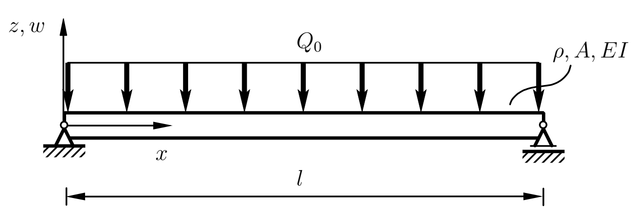

Figure 3: Exercise 3

3. A simply-supported uniform beam is loaded by a constant distributed force $q(x,t) = Q_0$, as shown in Fig. 3. Determine the motion of the beam when the force is suddenly removed at $t = 0$.

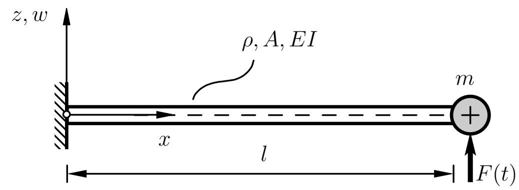

Figure 4: Exercise 4

4. Determine the steady-state response of a uniform cantilever beam with end-mass when excited by a force $F(t) = F_0 \cos \Omega t$, as shown in Fig. 4.

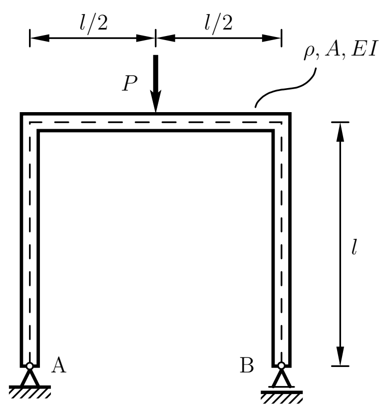

Figure 5: Exercise 5

5. A frame shown in Fig. 5 is made of three uniform beams each having flexural rigidity EI, linear density ρA. A constant force P acts at the center of the horizontal beam as shown. Estimate the first eigenfrequency of the frame $\omega_1(P)$ as a function of the force P. Re-calculate the first eigenfrequency if the frame is pinned at the support B.

6. A uniform beam, pinned at one end, is released from rest from a horizontal position, as shown in Fig. 6. The free-end A of the beam hits a rigid edge when the beam reaches a vertical position. Determine the reaction forces at the ends A and B as functions time.

7. A simply-supported beam of rectangular cross-section has a constant width b, while the height varies from $h/2$ to h over its length l. The material has density ρ, and Young's modulus E. Estimate the first two modal frequencies and modes of vibration.

8. A uniform Euler-Bernoulli beam is fixed to a small hub rotating at a constant angular speed Ω in the vertical plane, as shown in Fig. 7, has a Lagrangian

$$\mathcal{L} = \frac{1}{2} \int_0^l \left[\rho A[(\Omega x + w_{,t})^2 + \Omega^2 w^2] - T w_{,x}^2 - EI w_{,xx}^2 \right] \mathrm{d}x,$$

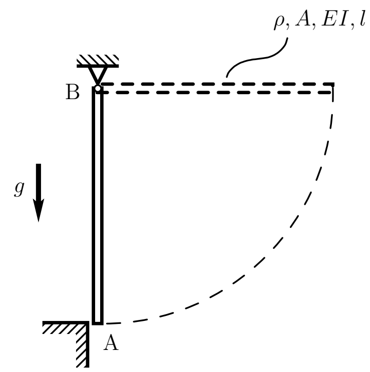

Figure 6: Exercise 6

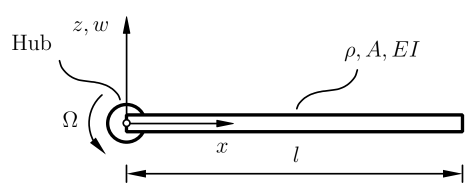

Figure 7: Exercise 8

where $\mathrm{d}T/\mathrm{d}x = \rho A \Omega^2 x$. Derive the equation of motion for transverse vibrations of the beam. Using the Ritz method, estimate the first few eigenfrequencies of the beam, and plot their variation with Ω.