Engineering Fracture Mechanics

Assignment # 3

Crack-tip field equations

1 The Airy's stress function for Westergaard's solution of Mode-I crack problem is

$$\phi = \mathrm{Re}\bar{\bar{Z}} + y\mathrm{Im}\bar{Z}$$

where, $\bar{Z} = \int Z\, dz$; $\bar{\bar{Z}} = \int \bar{Z}\, dz$

The Westergaard stress function is $Z = \dfrac{\sigma z}{\sqrt{z^2 - a^2}}$

(a) Verify whether the above mentioned stress function models a crack in an infinite plate subjected to uni-axial loading or bi-axial loading? Substantiate your results.

(b) Determine the stress field.

2. What do you understand by Stress Intensity Factor? How is it mathematically defined?

3. What is the contribution of Irwin in analyzing finite body problems? Substantiate your answer with results from Photoelasticity. Can his solution be used for long cracks?

4. What is the deformed shape of a Mode-I crack. Justify your result by an appropriate mathematical equation.

5. (a) What prompted Sanford for developing *Generalised Westergaard Equation*? (Hint: Look at the results of photoelasticiy for short and long cracks).

(b) Comment on Williams stress function approach and the multi-parameter stress field equation of Atluri-Kobayashi.

6. Explain with neat sketches how photoelasticity has contributed to the development of stress field equations in fracture mechanics. Wherever necessary support your discussions with appropriate mathematical equations/derivations.

7. Discuss the elegance of Westergaard complex functions in solving the stress field in Mode –I, Mode-II and Mode-III loadings.
