

Assignment 3

1. a. An end burning solid propellant rocket, having a throat diameter of 28 mm, is employed as the sustainer of a missile. It needs to provide a constant thrust of 5 KN. If a double-base propellant with a burn rate law $r = 0.004p^{0.45}$ where p is in Pa and r is in mm/s is used, determine the diameter of the propellant grain.

You can assume the specific impulse of the booster to be 1900 N-s/kg. The C^* and density of the double base propellant are 1600 m/s and 1540 kg/m^3 respectively.

b) In the above problem, if the sustainer has to operate for 200 s, determine the length of the solid propellant grain.

c) If the initial temperature of the grain gets reduced to a temperature of -40°C from the nominal value of 30°C due to storage in very cold surroundings, determine the change in the burning duration. You can assume the values of C^* , C_F and density to remain the same. The temperature sensitivity coefficient $\pi_r = d\ln(r)/dT|_p$ is $0.0020 \text{ }^\circ\text{C}^{-1}$.

2. A radial burning solid propellant rocket has a hollow cylindrical grain. The inner diameter of the grain is 30 mm and the outer diameter is 100 mm. The length of the grain is 0.5 m. The throat diameter of the nozzle is 12 mm.

If the burn rate of the propellant in mm/s is given by the law $r = 0.019 p^{0.38}$, where p is the pressure in Pa, determine:

- The thrust soon after ignition (initial thrust) and the thrust at burnout
- The maximum value of chamber pressure
- The burning duration
- The time taken for the chamber pressure to fall from the maximum to 10% of the maximum (tail-off).

You can assume the density of the propellant as 1700 kg/m^3 , the values of C^* and C_F as 1400 m/s and 1.1 respectively.

3. An end-burning propellant grain of length 0.6 metre takes 100 seconds to burn over its entire length when the ambient pressure is 3 MPa. The burn duration increases by 20 seconds to 120 seconds when it is burnt at an ambient pressure of 2 MPa. Determine the constant “a” and index “n” in the linear burn rate law: $r = ap^n$.

4. A payload of mass 100 kg is launched into orbit from the surface of the Earth, using a four-stage solid propellant rocket. The total mass of the four stages including the propellant put together is 17,000 kg. If the initial acceleration to be provided by the booster rocket is $1.5g$, where g is the acceleration due to gravity, determine for a chamber pressure of 10 MPa in the booster rocket:

- Throat diameter of the booster rocket
- Initial value of the burning surface of the booster grain

You can assume the following:

Burn rate of the propellant used in the booster is given by $r = 1.5 p^{0.3}$ where r is in mm/s and p is in atmospheres

C^* of the propellant is 1600 m/s
Density of the propellant is 1750 kg/m^3 ; C_F for the booster is 1.1.

5. A hollow cylindrical propellant grain of outer diameter 600 mm has the initial propellant surface in the shape of a diverging cone. The axis of the cone coincides with the axis of the cylindrical chamber containing the grain. The port diameter of the conical surface increases from 200 mm at the head end of the grain to 250 mm at the nozzle end. The length of the grain is 750 mm. The throat diameter is 45 mm. The ends of the grain are inhibited from burning.

- i) Determine the web thickness
- ii) The mass of the propellant sliver
- iii) The web burn time assuming that a_{70} and n in the burn rate law are $a_{70} = 5 \text{ mm/s}$ and $n=0.3$. The characteristic velocity $C^* = 1500 \text{ m/s}$ and the density of the propellant $\rho_p = 1700 \text{ kg/m}^3$.

6.a) In the simplified model of combustion of a solid composite propellant, you can assume that the final diffusion flame is at a temperature of 3000 K and stands off at a distance of $120 \mu\text{m}$ from the surface. The steady state temperature of the burning surface is 700 K. The specific heat of the propellant is 800 J/(kg K) , the thermal conductivity of the gas above the propellant is 0.05 W/(m K) and the overall heat required at the surface to bring about the pyrolysis and gas release is 50 kJ/kg . Determine the burn rate of the propellant.

The density of the propellant is 1500 kg/m^3 and the density of the gas above the propellant is 12 kg/m^3 . The temperature of the unburned propellant is 300 K. You can assume a linear profile for the temperature distribution between the flame and the propellant surface.

b) If in the above problem, the mean pressure was 1.6 MPa, and if the flame stand-off distance decreases to $100 \mu\text{m}$ from the surface at a pressure of 4 MPa, determine the exponent 'n' of the burn rate law. You can assume that the density and thermal conductivity of the gas to be directly proportional to the pressure. All other properties do not change with pressure.

7. The initial burning surface area of a cylindrical propellant grain of outer diameter 300 mm has the configuration of a square hole of dimensions 150 mm. The length of the grain is 400 mm and the grain is inhibited from burning at the ends. If the propellant has a burn rate law given by $a_{70} = 5 \text{ mm/s}$; $n=0.3$, determine:

- A) initial equilibrium pressure in the rocket chamber
- b) chamber pressure after 2 seconds of burning
- c) maximum chamber pressure,
- d) web thickness, and
- e) propellant sliver.

The C^* of the propellant is 1680 m/s and the nozzle throat is 35 mm in diameter. The density of the propellant grain is 1700 kg/m^3 .

8. A solid propellant rocket comprises a hollow cylindrical composite propellant grain having an inner diameter of 200 mm and outer diameter of 600 mm. The length of the grain is 1.5 metre. The propellant grain is inhibited from burning at both ends. The burning is radially outward from the inner cylindrical surface. A convergent divergent nozzle, attached at the aft-end of the grain, has a throat diameter of 100 mm. Determine the following:

- a) Maximum chamber pressure
- b) Burn duration of the solid propellant rocket.

The propellant data is given below:

- i. $a_{70} = 6$ mm/s
- ii. Burn rate index “n” in burn rate law $r = ap^n$ is 0.35
- iii. Characteristic velocity of the propellant is 1400 m/s
- iv. Density of propellant is 1600 kg/m^3 .

9. A long cylindrical hollow propellant grain of density 1700 kg/m^3 and burn rate given by $r \text{ (mm/s)} = 0.01p^{0.38}$ where p is the pressure in Pa is of length 10 m. The characteristic velocity of the propellant is 1600 m/s. The propellant grain has an outer diameter of 0.8 metre and is housed in a chamber and integrated with a nozzle. The port diameter is 200 mm. The ends of the grain are inhibited from burning. The throat diameter of the nozzle is 135 mm.

Determine:

- a) The static pressure at (i) the head end of the grain and (ii) the nozzle end of the grain at the start of burning
- b) The static pressure at (i) the head end of the grain and (ii) the nozzle end of the grain at the end of burning
- c) If the erosive burning in mm/s for the particular propellant is given by the law: $r \text{ (mm/s)} = 0.0093 (M p)^{0.7}$, where M is the Mach number and p is the chamber pressure in atmospheres, determine the ratio of the burn rate at the nozzle end to the head end of the grain at the start of burning.

You can assume the specific heat ratio $\gamma = 1.22$, molecular mass of the products as 22 kg/kmole and the temperature of the hot gases as 3000 K .

10. A solid propellant rocket used for injecting a spacecraft into orbit is spun on its axis at the rate of 180 rpm while it is burning. The propellant grain, having a burn rate given by $r = 0.004p^{0.45}$ where p is in Pa and r is in mm/s, is of hollow cylindrical configuration. The burning is radial from inside to outside. The inner diameter of the grain is 150 mm and the outer diameter is 450 mm giving a mean diameter to be 300 mm. If the burn rate of the propellant increases linearly from 10% to 30% as the acceleration level increases from 2g to 6g, determine the percent change in the thrust of the rocket due to spinning. State the assumptions made.