Assignment 6

1. Oscillations in thrust of fairly large amplitudes have been observed in very large segmented solid propellant rockets such as in the very large P230 solid propellant rocket used in Arianne V. The amplitude of the thrust oscillations is much greater than the pressure oscillations. The thrust oscillations are observed to be in the fundamental axial (longitudinal) mode; however, the oscillations are absent in the fist harmonic corresponding to the longitudinal mode. It is also noticed that the oscillations are significant only after certain duration of the burning of the solid propellant rocket with the magnitude of thrust oscillations growing as the burning progresses.
   a. Plot the standing wave mode of oscillations in the fundamental and first harmonic corresponding to the longitudinal mode.
   b. If the amplitude at the pressure anti-node is $\hat{p}$, derive an expression for the maximum amplitude of the thrust oscillation $\hat{F}$ corresponding to (a.) the fundamental (b.) the first harmonic. You can assume the throat diameter to be A_t m^2, the port diameter of the grain to be A_p m^2 and the characteristic velocity to be C* m/s. The nominal chamber pressure is $\bar{p}$ N/m^2.
   c. Show using the results from part (b.) above, that the magnitude of thrust oscillations will grow as the diameter of the grain to be A_p m^2 increases.
   d. Using the results in part (b.) above, also show that the oscillations in thrust would exist in the fundamental; however, in the first harmonic the magnitude of the oscillations would be relatively insignificant.
   e. Determine the magnitude of the transfer function between the thrust and pressure oscillation $[(\hat{F}/\bar{F})/(\hat{p}/\bar{p})]$/ Here $\bar{F}$ and $\bar{p}$ denote the mean thrust and pressure corresponding to the maximum thrust and pressure amplitude of $\hat{F}$ and $\bar{p}$ respectively.
   f. Comment on the magnitude of thrust oscillations in the second and third harmonic in the longitudinal mode.

2. A thrust chamber has an inner diameter of 0.25 m and a length of 0.9 m between the injector and the nozzle throat. It operates at a pressure of 10 MPa. The temperature of the hot gases in the chamber is 3200 K. The molecular mass and specific heat ratio of the gases are 22 kg/kmole and 1.22 respectively. Determine the fundamental frequency of the oscillations in the longitudinal, tangential and radial modes.
If a quarter wave tube of diameter 5 mm and length 0.11 m is connected to the chamber at the head end and the composition and temperature of the gas in this tube are the same as in the thrust chamber, determine the frequencies of the fundamental and first harmonics of the quarter wave tube.

3. A non-aluminized composite solid propellant rocket uses ammonium perchlorate (AP) having particle sizes of 300 and 30 µm. If the mean burn rate at which L* instability is observed is 3 mm/s, what is the likely frequency of L* oscillations?

4. A pressure-fed liquid propellant rocket with radiation and film-cooling of the thrust chamber operates at a chamber pressure of 0.8 MPa. The pressure drop across the injector is 0.3 MPa. If the residence time of the propellants within the chamber is 3 ms and the combustion delay time of the propellants is 10 ms, is it likely that bulk mode of oscillations would occur in the thrust chamber?

5. a.) The growth constant of a solid propellant is to be determined in a T-burner at a frequency of 600 Hz. Determine the length of the T-burner given that the sound velocity of the gases in the T burner is 500 m/s.
   b.) Determine the growth constant of the propellant if the maximum pressure amplitude of the pressure oscillations measured in the T-burner at two instants of time of 0.3 and 0.6 seconds are 0.02 and 0.025 MPa respectively.

6. The amplitude of pressure perturbation of an isentropic wave in ambient atmosphere at a pressure and temperature of 0.1 MPa and 27^{o}C is 50 Pa. The specific heat ratio of air is 1.4. Determine the temperature perturbation in ^{o}C due to the pressure perturbation.

7. A vertically-held tube, open at both ends, generates oscillatory pressures at the fundamental frequency of the tube when heat is released in the tube. This configuration of the tube, generating sound by heat release, is known as Rijke tube. Show that the amplitude of the oscillations is a maximum when the heat source is located at one-fourth the length of the tube from the lower end of the tube.