Lecture 8 – 9:

Plastic deformation of pure metal: mechanisms(slip & twin), critical resolved shear stress, single crystal tensile test (fcc), theoretical strength of ideal crystal

1. S11, S44 & S12 of tungsten are 0.257, 0.660 & -0.073 (unit: 10^{-11} m^2/N) respectively. Check is this isotropic?
2. C11, C44 & C12 of a cubic crystal with respect to its crystal axes are 267, 82.5 & 161 GPa respectively. Estimate its elastic compliances and Young's modulus along [100]. Will this be the same along [110] or [111]?
3. In which mode of plastic deformation atomic displacement could be less than inter atomic spacing?
4. Estimate the magnitude of shear strain for (111) $[11\bar{2}]$ twin in fcc lattice.
5. What is the effect of tensile stress on lattice spacing?
6. Show with schematic diagram resolved shear stress versus shear strain diagram of fcc crystal if the tensile axes were (a) [123] (b) [001]
7. What is the difference between simple shear & pure shear? Under which category will you place plastic deformation by slip?
8. What is the effect of plastic deformation on lattice parameter?
9. Draw a standard [001] projection showing all possible slip planes & directions for a bcc crystal. Assume slip can take place only on {110} planes.
10. When does a polycrystalline material have same yield strength along all possible direction?
11. Estimate the ideal cleavage strength and shear strength of pure iron. Given E = 211 GPa and G = 83 GPa.

Answers:

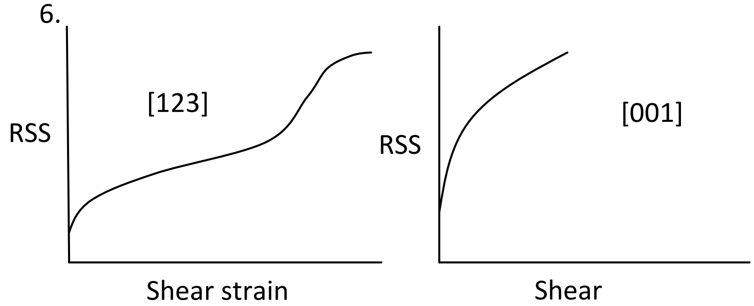
1. If the material is isotropic then $\frac{2(S_{11}-S_{12})}{S_{44}} = 1$. On substituting the numerical values it is found that $\frac{2(0.257+0.073)}{0.66} = 1$Therefore it is isotropic.

2. $S_{11} = \frac{(C_{11}+C_{12})}{(C_{11}-C_{12})(C_{11}+2C_{12})} = \frac{(267+161)}{(267-161)(267+322)} = 0.00866$ GPa^{-1}, $S_{12} = \frac{-C_{12}}{(C_{11}-C_{12})(C_{11}+2C_{12})} = \frac{-161}{(267-161)(267+232)} = -0.00258$ GPa^{-1} & $S_{44} = \frac{1}{C_{44}} = \frac{1}{82.5} = 0.0117$ GPa^{-1} Young's modulus = $1/S_{11}$ = 1/0.00866 = 146 GPa. No it represents modulus along cube directions only. Modulus along [110] & [111] will be different.

3. Twin.

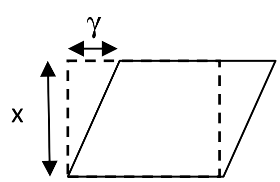
4. The distance between twin plane = $\frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a}{\sqrt{3}}$ and the magnitude of slip =$\frac{a}{6}[11\bar{2}] = \frac{a\sqrt{1^2+1^2+(-2^2)}}{6}$ = $\frac{a}{\sqrt{6}}$ (this represents the distance between two atoms along$[11\bar{2}]$). Shear strain is the ratio of magnitude of slip to the distance between the two planes $= \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} = 0.71$

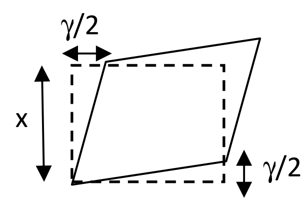
5. Lattice spacing increases with tensile stress till it reaches its elastic limit. Elastic strain is equal to the ratio of change in lattice spacing along the tensile axis to its original value =$\left(\frac{\Delta d}{d}\right)$.

6. 

In case of [123] initially resolved shear stress reaches its critical value for a single slip system. Therefore all 3 stages of deformation are seen. In case of [001] RSS reaches CRSS simultaneously on several slip system (8 to be precise). Therefore 3 distinct stages are not seen.

7. Simple shear represents displacement or slip on a plane along a specified direction. This is schematically represented as follow:



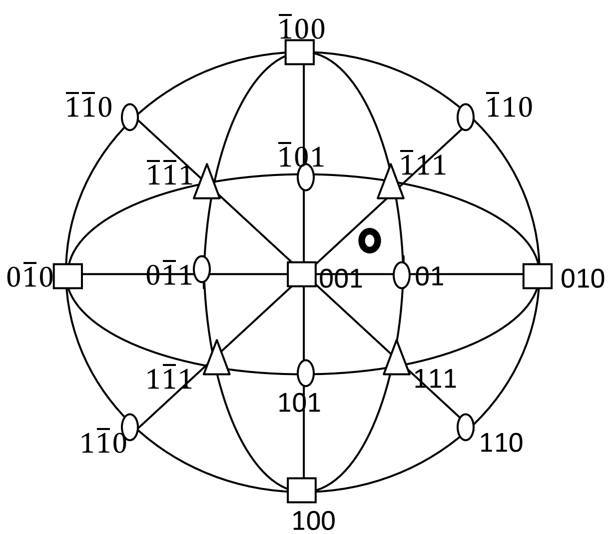


Simple shear: x is normal to plane on which shear has taken place & y is displacement. Displacement gradient is: $e_{xy} = \frac{y}{x} = \gamma$. Using the notation used it is e_{12}. All other components are zero. The matrix is not symmetric. Slip is a simple shear

Pure shear: simple shear (e_{ij}) is equal to sum of pure shear (ε_{ij}) and rotation (ω_{ij}) as shown above. This makes the strain matrix symmetric. $e_{ij} = \epsilon_{ij} + \omega_{ij}$

$$\begin{bmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \gamma/2 & 0 \\ \gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \gamma/2 & 0 \\ -\gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. Plastic deformation does not alter crystal structure or its dimension. Lattice parameter after deformation is still the same.

9. 

Slip systems of bcc crystal are {101}$<\bar{1}11>$There are 12 such system. These are shown in standard projection. For a pole lying within a stereographic triangle slip plane & direction having maximum resolved shear stress lie in the adjacent triangle. This is shown for one case where the pole is marked as a circle. The slip plane is $(\bar{1}01)$ and slip direction is [111].

10. Strength of a crystal may vary with the direction of loading. Polycrystalline metals may have a large number of grains. Its properties will be a function of the entire group. If these are randomly oriented then one would expect its properties to be isotropic.

11. Ideal shear strength $\tau_{max} = \frac{G}{2\pi} = \frac{83}{2\pi} = 13.21\ GPa$. For estimating cleavage strength please see problem 15 of chapter 1. This gives $\sigma_{max} = \sqrt{\frac{E\gamma}{a_0}}$ & $\gamma = \frac{a^2}{\pi^2}\frac{E}{a_0} \approx \frac{Ea_0}{10}$ since ***a*** is nearly equal to a_0. Thus $\sigma_{max} = \sqrt{\frac{E\gamma}{a_0}} \approx \frac{E}{\sqrt{10}} = \frac{211}{3.16} = 67\ GPa$. These are nearly two orders of magnitude higher than the real strength of iron.