

## Physics of Materials

- 1) The total energy of a KCl pair is given by:

$$E = (-e^2/4\pi\epsilon_0 R) + (B/R^{10}) + 8.00 \times 10^{-20} \quad \text{J}$$

What is the equilibrium spacing between the ions?

- 2) Assuming the classical free electron model, calculate the number of free electrons per cubic meter for silver. Assume that, on average, there are 1.4 free electrons per Ag atom. The electrical conductivity and density of Ag are  $7 \times 10^7 (\Omega\text{m})^{-1}$  and  $10.5 \text{ g/cm}^3$ , respectively. Also calculate the mobility of electrons in Ag
- 3) Briefly explain, using appropriate diagrams, how the Sommerfeld model overcomes the limitation of the Drude model, with respect to estimating  $C_v^e$
- 4) Using a diagram, indicate the variation of resistivity of a well annealed pure metal, with temperature. On the same diagram show the variation of resistivity with temperature of the same metal after alloying it with another metal, the alloying addition being 5% and the alloy being a substitutional solid solution. Also on the same diagram, indicate the variation of resistivity with temperature of the same alloy after extensive plastic deformation. **It is sufficient of the diagram is a schematic diagram. The trend in each variation and relative positions of the variations are important.** Clearly indicate which curve/line represents each of the above cases.
- 5) At room temperature, the electrical conductivity for Silver is  $7 \times 10^7 (\Omega\text{m})^{-1}$ . Assume the electron mobility in Silver, at this temperature, is  $3.0 \text{ m}^2/\text{Vs}$ .
- a) What is the number of free electrons per Silver atom?
- b) Assuming the Wiedemann Franz law is obeyed, what is the thermal conductivity of Silver at room temperature (assume 30 C, and the Wiedemann Franz law constant to be  $2.4 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$ )
- 6) A hypothetical superconductor has a  $H_{C2}$  value of 1.5 T. What 'type' of super conductor is it? Explain your answer.
- 7) A Si based extrinsic semiconductor has a room temperature conductivity of  $45 \Omega^{-1}\text{m}^{-1}$ . Assuming  $\mu_e = 0.05 \text{ m}^2/\text{V}$  and  $\mu_h$  is  $0.03 \text{ m}^2/\text{V}$ , what concentration of a group VA dopant could have resulted in this? What concentration of a group IIIA dopant could have resulted in this? Assume material is in extrinsic region.
- 8) In the case of free electrons, energy is related to wave vector as indicated below:

$$E(\mathbf{k}) = \hbar^2 \mathbf{k}^2 / 2m$$

In the case of solids, this relationship breaks down, however its general form can be maintained using:

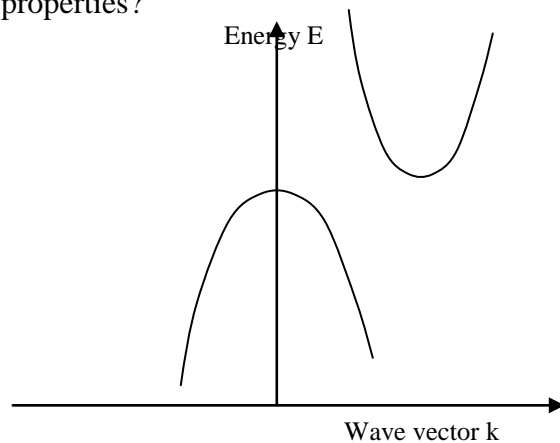
$$E(k) = \hbar^2 k^2 / 2m^*$$

Where  $m^*$  is an adjustable parameter, which we call the effective mass. The effective mass can be defined by:

$$m^* = \hbar^2 / (d^2 E / dk^2)$$

Assuming that the solid behaviour is consistent with free electrons subject to a weak periodic potential, **schematically plot** the variation of effective mass  $m^*$  as a function of wave vector in the first Brillouin zone, in the region that would correspond to the first band (assume a simple one dimensional lattice with lattice spacing 'a')

- 9) Make a plot (to scale) of the first two Brillouin zones for a rectangular two dimensional lattice which has unit vectors in the real space of along the x and y directions of  $a = 0.2$  nm and  $b = 0.4$  nm
- 10) A crystal has a real space lattice that has a unit cell defined by the vectors  $(a/2)(i + j)$ ,  $(a/2)(j + k)$ ,  $(a/2)(k + i)$ , where  $i, j, k$ , are unit vectors in three mutually perpendicular directions. What are the unit cell vectors corresponding to the reciprocal lattice of this real space lattice?
- 11) For Cu at 1000 K, if the Fermi energy is 7.06 eV, what is the energy at which the probability that an electron state will be occupied, is 70%? What is the probability of occupancy at the Fermi energy?
- 12) The band structure of a material is as shown below. What can you say about its use for its optical properties?



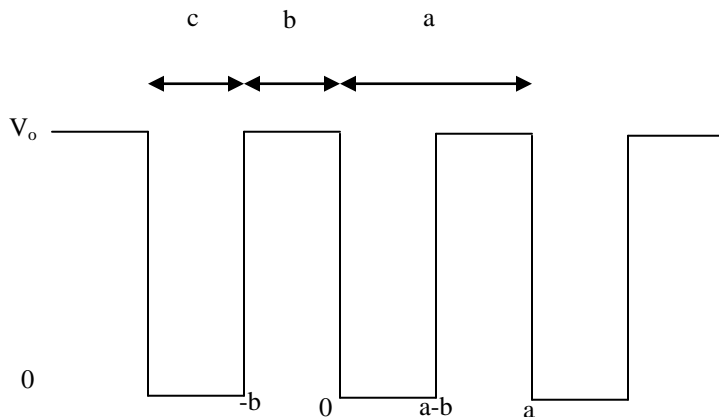
- 13) From a reciprocal lattice perspective what can you say about the differences in diffraction patterns obtained from a single crystal using a typical laboratory X-ray diffraction

equipment and an electron microscope operating at 200 kV? (The crystal is mounted in the same fixed orientation in both cases with respect to incident beam)

14) When you construct the Wigner Seitz cell for an arbitrary two dimensional, periodic, lattice, what is the most general shape (number of sides) that will result? Under what conditions will the number of sides decrease relative to that obtained in the most general case.

15) Derive the expression for Bose-Einstein Statistics. Clearly show all steps, indicate all assumptions, and approximations and justify the same.

- 16) Assuming an electron experiences the simplified square periodic potential shown in the figure below, write and solve the Schrodinger wave equation for the regions where the potential is  $V_0$  as well as for the regions where the potential is zero. Assuming Bloch periodicity condition is valid, what relationships can you expect between the solution at  $-b$  on the  $V_0$  potential side, and the solution at  $a-b$  on the potential 0 side? (expand the relationships completely, you need not write or solve the determinant)



- 17) Indicate using appropriate, fully labeled sketches, how paramagnetism is explained using the quantum mechanical nearly free electron theory.
- 18) What are Cooper pairs? What can you say about their  $\mathbf{k}$  vectors and spin?
- 19) How do Si and GaAs compare with respect to their potential use for opto-electronic applications. Explain using appropriate diagrams.
- 20) The top of the valence band of a hypothetical pure elemental semiconductor is at 4.4 eV, while the conduction band starts at 6.0 eV and the Fermi energy of the semiconductor is at 5.2 eV. What frequencies of light will be absorbed by this pure elemental semiconductor?

Mass of electron =  $9.11 \times 10^{-31}$  kg  
 Mass of proton =  $1.67 \times 10^{-27}$  kg  
 Charge on an electron =  $1.6 \times 10^{-19}$  C  
 Speed of light =  $3 \times 10^8$  m/s  
 $h = 6.63 \times 10^{-34}$  Js  
 $k = 1.38 \times 10^{-23}$  J/K  
 Avogadro Constant =  $6.02 \times 10^{23}$  mol $^{-1}$