MODULE I : SEAKEEPING

Topic: Regular Water Waves

Question 1

Consider a towing tank of length 150m with a wave-maker at one end. For a regular water wave of period 2 sec. and amplitude 0.2m being generated by the wave maker, determine:

(i) the circular frequency and wave length
(ii) the maximum horizontal and vertical fluid particle velocities
(iii) the maximum dynamic pressure 0.5m below still water level
(iv) phase velocity and group velocity
(v) time taken for the wave front to reach the other end of the tank

Assume water depth is sufficiently high to consider the generated wave a deep water wave.

Answer:

The velocity potential for a linear deep water wave advancing along +ve x axis can be written as:

$$\varphi = \frac{gA}{\omega} e^{kz} \sin(kx - \omega t)$$

(i) Circular frequency $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s.

From dispersion relation: $\omega^2 = gk = \frac{2\pi g}{\lambda}$,

Wave length $\lambda = \frac{2\pi g}{\omega^2} = \frac{2\pi g}{\pi^2} = \frac{2g}{\pi} = 6.24$ m

(ii) Horizontal velocity $u = \frac{\partial \varphi}{\partial x} = \frac{gkA}{\omega} e^{kz} \cos(kx - \omega t) = \omega A e^{kz} \cos(kx - \omega t)$

Vertical velocity $w = \frac{\partial \varphi}{\partial z} = \frac{gkA}{\omega} e^{kz} \sin(kx - \omega t) = \omega A e^{kz} \sin(kx - \omega t)$

Thus, $u_{\max} = w_{max} = \omega A = 0.2\pi = 0.628$ m/s

(iii) Linear dynamic pressure is given by : $p = -\rho \frac{\partial \varphi}{\partial t} = \rho g A e^{kz} \cos(kx - \omega t)$

Thus, at $z = -0.5$ m, $p_{max} = \rho g A e^{-0.5k}$, and $k = 2\pi / \lambda = 1$.

Inserting the values, $p_{\max} = (1.0)(9.8)(0.2)\exp(-0.5) = 1.18\,\text{kN/m}^2$

(iv) Phase velocity is given by : $c = \lambda / T = 2\pi / 2 = \pi = 3.14\,\text{m/s}$
Group velocity $c_g = 0.5c$ for deep water = 1.57m/s

(v) The wave front travels at group speed. Thus the time taken for the wave front to reach the other end of the tank is 200/1.57=95.5 sec.