MODULE I : SEAKEEPING

Topic: Uncoupled Heave, Roll and Pitch

Question 1

(i) Write the expressions for undamped natural periods of heave, roll and pitch motions.

(ii) Determine the scale factors for these natural periods between model and prototype.

Answer:

(i)

Undamped natural heave period $T_z = 2\pi\sqrt{\dfrac{m + a_{zz}}{\rho g A_{WP}}}$

Undamped natural roll period $T_\varphi = 2\pi\sqrt{\dfrac{I_{xx} + a_{\varphi\varphi}}{\rho g \nabla GM_T}}$

Undamped natural pitch period $T_\theta = 2\pi\sqrt{\dfrac{I_{yy} + a_{\theta\theta}}{\rho g \nabla GM_L}}$

where,

m	=	mass of the ship
I_{xx}	=	rigid body mass moment of inertia about longitudinal (x) axis
I_{yy}	=	rigid body mass moment of inertia about transverse (y) axis
∇	=	submerged volume of hull
A_{WP}	=	waterplane area
GM_T	=	transverse metacentric height
GM_L	=	longitudinal metacentric height
a_{zz}	=	heave added mass
$a_{\varphi\varphi}$	=	roll added moment of inertia
$a_{\theta\theta}$	=	pitch added moment of inertia

(ii)

Let the geometric length-scale factor between model and prototype is λ, i.e. $\lambda = L_m / L_s$ (subscripts m and s represent model and prototype-ship respectively, L=length)

$$\frac{(T_z)_m}{(T_z)_s} = \sqrt{\frac{(m+a_z)_m}{(m+a_z)_s}\frac{(A_{WP})_s}{(A_{WP})_s}} = \sqrt{\frac{1}{\lambda^3}\frac{\lambda^2}{1}} = \sqrt{\frac{1}{\lambda}}$$

$$\frac{(T_\varphi)_m}{(T_\varphi)_s} = \sqrt{\frac{(I_{xx}+a_{\varphi\varphi})_m}{(I_{xx}+a_{\varphi\varphi})_s}\frac{(\nabla GM_T)_s}{(\nabla GM_T)_s}} = \sqrt{\frac{1}{\lambda^3\lambda^2}\frac{\lambda^3\lambda}{1}} = \sqrt{\frac{1}{\lambda}}$$

$$\frac{(T_\theta)_m}{(T_\theta)_s} = \sqrt{\frac{(I_{yy}+a_{\theta\theta})_m}{(I_{yy}+a_{\theta\theta})_s}\frac{(\nabla GM_L)_s}{(\nabla GM_L)_s}} = \sqrt{\frac{1}{\lambda^3\lambda^2}\frac{\lambda^3\lambda}{1}} = \sqrt{\frac{1}{\lambda}}$$

Natural periods in all three modes therefore scale as $1/\sqrt{\lambda}$. Thus for a 1:100 scale model, the prototype natural periods will be 10 times the model natural periods.

Question 2

Consider a pontoon which has a uniform rectangular cross-section. The dimensions of the pontoon are: length L=100m., breadth B=18m. and draft T = 4m. Also given is KG=5m. Estimate the natural heave, roll and pitch period of the pontoon. Use strip-theory to determine the added masses, assuming that the sectional added masses can be estimated as the $\rho\pi B^2/8$. Roll added moment of inertia can be ignored. For estimating rigid body moments of inertia, the transverse and longitudinal radii of gyration can be taken as $0.4B$ and $0.25L$ respectively.

Answer:

For heave mode:

Underwater volume of the pontoon is, $\nabla = LBT = (100)(18)(4) = 7200\,m^3$

Water-plane area: $A_{WP} = (100)(18) = 1800\,m^2$

Added mass in heave $a_z = \frac{\rho \pi B^2}{8} \int_{-L/2}^{L/2} dx = \frac{\rho \pi}{8} B^2 L = \frac{\rho \pi}{8} (18^2)(100) = 12723.45 \rho$

$$T_z = 2\pi \sqrt{\frac{m + a_{zz}}{\rho g A_{WP}}} = 2\pi \sqrt{\frac{\nabla + a_{zz} / \rho}{g A_{WP}}}$$

Inserting the values, $T_z = 6.68\ s$.

For Roll Mode:

$I_{xx} = \rho \nabla (0.4B)^2 = 373248 \rho$, $a_{\varphi\varphi} \approx 0$

$$BM_T = \frac{(I_{WP})_{xx}}{\nabla} = \frac{B^3 L / 12}{LBT} = \frac{B^2}{12T} = \frac{18^2}{(12)(4)} = 6.75\,m$$

$KB = 0.5T = (0.5)(4) = 2\,m$

$GM_T = KB + BM_T - KG = 2 + 6.75 - 5 = 3.75\,m$

$$T_\varphi = 2\pi \sqrt{\frac{I_{xx} + a_{\varphi\varphi}}{\rho g \nabla GM_T}} = 2\pi \sqrt{\frac{373248}{(9.8)(7200)(3.75)}} = 7.46\,s$$

For pitch mode:

$I_{xx} = \rho \nabla (0.25L)^2 = 45 \times 10^5 \rho$

$$a_{\theta\theta} = \frac{\rho \pi B^2}{8} \int_{-L/2}^{L/2} x^2 dx = \frac{\rho \pi B^2}{8} \left[\frac{x^3}{3} \right]_{-L/2}^{L/2} = \frac{\rho \pi B^2}{8} \frac{L^3}{12} = 10602875 \rho$$

$$BM_L = \frac{(I_{WP})_{yy}}{\nabla} = \frac{L^3 B / 12}{LBT} = \frac{L^2}{12T} = \frac{100^2}{(12)(4)} = 208.33\,m$$

$GM_L = KB + BM_L - KG = 2 + 208.33 - 5 = 205.33\,m$

$$T_\theta = 2\pi \sqrt{\frac{I_{yy} + a_{\theta\theta}}{\rho g \nabla GM_L}} = 2\pi \sqrt{\frac{4500000 + 10602875}{(9.8)(7200)(205.33)}} = 6.41\,s$$

Question 3

If the pontoon of question 5 advances in a regular wave-field in head wave conditions at a speed of 6 knots, then determine the wave lengths (λ / L values) for which large heave, roll and pitch motions are expected.

Answer:

Large motions will occur at resonance frequency, i.e. when natural frequencies coincide with the encounter frequency.

We have, $\omega_e = \omega + \dfrac{\omega^2 V}{g} = \omega + 0.315\omega^2$

For large heave, $\omega + 0.315\omega^2 = 2\pi / 6.68 = 0.94$
This gives $\omega = 0.7586$ rad/s, from which $\lambda = 2\pi g / \omega^2 = 107$m, or $\lambda / L = 1.07$

For large roll, $\omega + 0.315\omega^2 = 2\pi / 7.46 = 0.8422$
This gives $\omega = 0.6916$ rad/s, from which $\lambda = 2\pi g / \omega^2 = 129$m, or $\lambda / L = 1.29 \approx 1.3$

For large pitch, $\omega + 0.315\omega^2 = 2\pi / 6.41 = 0.98$
This gives $\omega = 0.7855$ rad/s, from which $\lambda = 2\pi g / \omega^2 = 99.8$m, or $\lambda / L = 1$

(note that for this pontoon heave and pitch periods are very close, therefore it is expected to have large and unfavorable motions in the vertical plane)

Question 4

Consider a circular cylinder with a radius R , floating upright with a draft T in water with a density ρ. Give an approximation of the wave length in deep water at which resonance in heave can occur (make suitable and realistic assumption for the heave added mass).

Answer:

We have, $\nabla = \pi R^2 T$ **,** $A_{WP} = \pi R^2$

The heave added mass can be estimated by the mass of a hemispherical volume of water of radius R . Thus, using $a_{zz} = \rho \dfrac{2}{3} \pi R^3$, we have

$$T_z = 2\pi\sqrt{\frac{m + a_{zz}}{\rho g A_{WP}}} = 2\pi\sqrt{\frac{\pi R^2 L + (2/3)\pi R^3}{g\pi R^2}} = 2\pi\sqrt{\frac{L + (2/3)R}{g}} = 2\pi\sqrt{\frac{R}{g}\left(\frac{L}{R} + \frac{2}{3}\right)}$$

This shows that a body with increasing L/R, i.e. for a deeper draft cylinder of same radius, the natural heave period will increase. This is one of the reasons for the evolution of spar platform in offshore oil exploration and production where the aim is to push T_z upwards.