MODULE I : SEAKEEPING

Topic: Ship Motion in Irregular Waves

Question 1

The energy spectrum of a wave is as follows:

ω	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$S_w(\omega)$	0.0	1.0	4.0	12.0	20.0	16.0	10.0	2.0	1.0	0.1

For a ship speed of 20knots in head waves, the heave RAO's (defined as the heave amplitude / wave amplitude) are as follows:

ω_e	$\leq$0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.6	2.0
RAO	1.0	1.05	1.20	1.80	1.60	1.30	0.9	0.4	0.1	0.01

Find the heave response spectrum, and the 1/3rd significant heave displacement amplitude.

Answer:

First the encountered wave spectrum is determined using the relations: $\omega_e = \omega\left(1 - \frac{\omega V \cos\beta}{g}\right)$, which for the present case of V =20knots and β=180 deg. is $\omega_e = \omega(1 + 1.05\omega)$.

Encountered wave spectral ordinates are given by: $S_w^e(\omega_e) = \frac{S_w(\omega)}{1-(2\omega V/g)\cos\beta}$, which here is $S_w^e(\omega_e) = \frac{S_w(\omega)}{(1+2.1\omega)}$

The encountered spectrum is calculated as follows:

ω (rad/s)	$S_w(\omega)$ (m^2-s)	ω_e (rad/s)	$S_w^e(\omega_e)$ (m^2-s)

0.1	0.0	0.1105	0.0
0.2	1.0	0.242	0.704
0.3	4.0	0.3945	2.454
0.4	12.0	0.568	6.522
0.5	20.0	0.7625	9.756
0.6	16.0	0.978	7.080
0.7	10.0	1.2145	4.049
0.8	2.0	1.472	0.746
0.9	1.0	1.7505	0.346
1.0	0.1	2.05	0.032

The heave spectrum are calculated in the following table:

ω_e (rad/s)	$S_w^e(\omega_e)$ (m^2-s) (note 1)	RAO (note 2)	$S_z(\omega_e) = RAO^2 \times S_w^e(\omega_e)$ (m^2-s)
0.1	0	1.	0.
0.2	0.48	1.	0.48
0.3	1.37	1.	1.37
0.4	2.58	1.	2.58
0.5	4.93	1.05	5.43
0.6	7.05	1.20	10.15
0.7	8.72	1.80	28.25
0.8	9.29	1.60	23.78
0.9	8.05	1.30	13.60
1.0	6.80	0.90	5.51
1.1	5.52	0.65	2.33
1.2	4.23	0.4	0.68
1.3	2.95	0.33	0.32
1.4	1.67	0.25	0.10
1.5	0.71	0.18	0.02
1.6	0.56	0.10	0.005
1.7	0.42	0.08	0.
1.8	0.29	0.05	0.
1.9	0.19	0.02	0.
2.0	0.03	0.01	0.

Note 1 : the values of this column values are determined by interpolation of $\omega_e - S_w^e(\omega_e)$ data above

Note 2: this column values are by interpolation of RAO table above.

Area under the heave spectrum, determined using simple trapezoidal rule is:

$$m_0 = 0.1\times(2\times0. + 0.48 + 1.37 + 2.58 + 5.43 + 10.15 + 28.25 + 23.78 + 13.60 + 5.51 + 2.33 + 0.68 + 0.32 + 0.10 + 0.02 + 0.005 + 0 + 0 + 0 + 2\times0)$$
$$= 9.4605\ m^2$$

Thus, 1/3rd significant heave displacement is: $2\sqrt{m_0}$ =6.15 m

Question 2

A ship motion trial is carried out in a long-crested (2D) seaway. The wave spectrum as measured at a stationary point is as follows:

$S(\omega)$ (m^2/s)	1.2	7.6	12.9	11.4	8.4	5.6
ω (rad/s)	0.3	0.4	0.5	0.6	0.7	0.8

The heave energy spectrum as measured on board the ship while moving at 12 knots on a course of 150 deg. relative to the waves is as follows:

$S_z(\omega_e)$ (m^2/s)	0.58	1.62	1.66	0.76	0.15	0.03
ω_e (rad/s)	0.40	0.50	0.60	0.70	0.80	0.90

Find the heave transfer function (defined as the ratio of heave and wave amplitudes) over the range of encounter frequencies 0.4-0.9 rad/s.

Answer:

For this problem,

$$\omega_e = \omega\left(1 - \frac{\omega V\cos\beta}{g}\right) = \omega\left(1 - \frac{\omega\,(12)(0.5144)\cos150}{9.8}\right) = \omega(1 + 0.545\omega)$$

$$S_w^e(\omega_e) = \frac{S_w(\omega)}{1-(2\omega V/g)\cos\beta} = \frac{S_w(\omega)}{1+1.09\omega}$$

The encountered spectrum is computed below.

ω (rad/s)	$S_w(\omega)$ (m^2-s)	ω_e (rad/s)	$S_w^e(\omega_e)$ (m^2-s)
0.3	1.2	0.35	0.90
0.4	7.6	0.49	5.29
0.5	12.9	0.64	8.35
0.6	11.4	0.80	6.89
0.7	8.4	0.97	4.76
0.8	5.6	1.15	2.99

Heave RAO can be obtained from the spectral ordinates as $RAO = \sqrt{\frac{S_z(\omega_e)}{S_w^e(\omega_e)}}$. Thus, we have:

ω_e (rad/s)	$S_w^e(\omega_e)$ (m^2-s) (note 1)	$S_z(\omega_e)$ (m^2-s)	RAO
0.4	2.47	0.58	0.48
0.5	5.35	1.62	0.55
0.6	8.1	1.66	0.45
0.7	7.6	0.76	0.32
0.8	6.9	0.15	0.15
0.9	5.7	0.03	0.07

Note 1: values determined from interpolation of $\omega_e - S_w^e(\omega_e)$ table above.