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$$\Phi(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$$

$$\vec{E}(r,\theta)\Big|_{r\to\infty} = E_0\hat{k}$$

$$\boxed{\Phi(r,\theta)\Big|_{r\to\infty} = -E_0 r\cos\theta + C}$$

$$r = R \qquad \Phi = \Phi_0 .$$

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta^3(r). \implies B_0 = 0$$

$$\phi(r,\theta) = C - E_0(r\cos\theta). + \sum_{\ell=1}^{\infty}\frac{B_\ell}{r^{\ell+1}}P_\ell(\cos\theta)$$

$$E_0 R\cos\theta = \frac{B_0}{R}\cos\theta \rightarrow \underline{\underline{B_0 = E_0R^2}}$$

$$\Phi(r,\theta) = \Phi_0 - E_0\left(1 - \frac{R^3}{r^3}\right) r\cos\theta .$$

$$\phi(r,\theta) = \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}} \right) P_\ell(\cos\theta)$$

$$\Phi(R,\theta) = \begin{cases} \phi_0 & 0 \le \theta < \frac{\pi}{2} \\ -\phi_0 & \frac{\pi}{2} < \theta \le \pi \end{cases}.$$

$B_\ell = 0$ For each ℓ.

$$\nabla^2 = \frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left(\varrho\frac{\partial}{\partial\varrho}\right) + \frac{1}{\varrho^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

$$\Phi(\varrho,\theta,z) = R(\varrho).Q(\theta)Z(z)$$

$$\frac{1}{R}\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left(\varrho.\frac{\partial}{\varrho}\right) + \frac{1}{Q}\frac{1}{\varrho^2}\frac{\partial^2 Q}{\partial\theta^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = 0$$

$$\boxed{\frac{\partial^2 R}{\partial x^2} + \frac{1}{x}\frac{\partial R}{\partial x} + \left(1 - \frac{\nu^2}{x^2}\right)R = 0}$$

Bessel Equation

$$\nabla^2 \Phi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Phi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0$$

$$\Phi(\rho \to \infty) = -E_0 x = -E_0 \rho \cos\theta .$$

$$\Phi(\rho, \theta) = \sum_{n=1}^{\infty}\left(C_n \rho^n + \frac{D_n}{\rho^n}\right)\left(A_n \cos n\theta + B_n \sin n\theta\right)$$