.Ghosh
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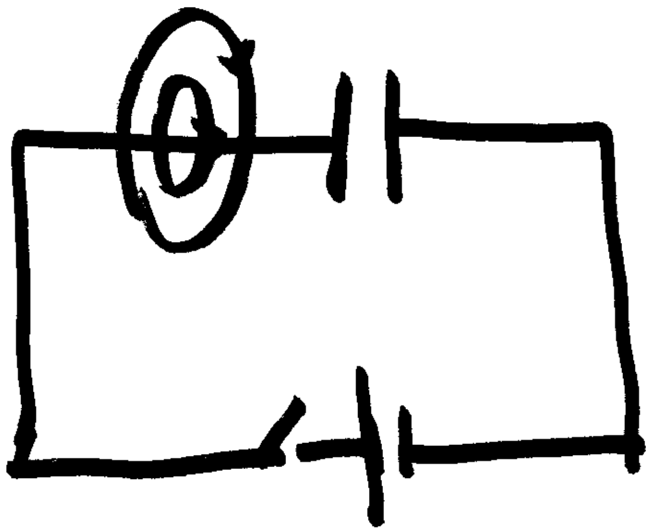
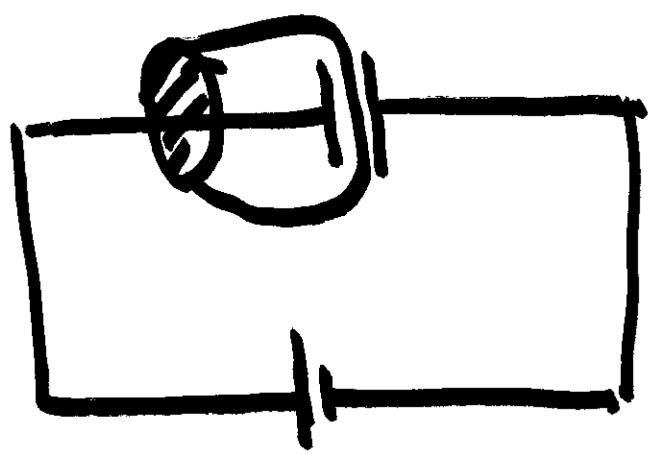


$$\oint B.dl = \mu_0 I.$$



$$\oint B.dl = 0$$

$$\Phi_E = \int \vec{D} \cdot \vec{dS}$$

$$\frac{d\Phi_E}{dt} = \frac{d}{dt} \int \vec{D} \cdot \vec{dS}$$

$$= \frac{d}{dt} \int \vec{\nabla} \cdot \vec{D} \, d^3r$$

$$= \frac{d}{dt} \int \rho \, d^3r$$

$$= \frac{dQ}{dt}$$

$$\boxed{i_d = \frac{d\Phi_E}{dt}}$$

$$\Phi_E = \int \vec{E} \cdot \vec{dS} = E \cdot A$$

$$= \frac{Q}{\epsilon_0 A} \cdot A = \frac{Q}{\epsilon_0}.$$

$$I_d = \cancel{\frac{1}{\epsilon_0}\frac{dQ}{dt}}.$$

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt}.$$

$$\oint B \cdot dl = \mu_0 \left(I + \epsilon_0 \frac{\partial \phi}{\partial t} \right)$$

$$\int (\vec{\nabla} \times \vec{B}) \cdot dS = \mu_0 \left(\int \vec{J} \cdot \vec{dS} + \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot \vec{dS} \right)$$

$$\nabla \times B = \mu_0 \vec{J} + \underline{\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}.$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

$$\vec{J} = \mu_0 \left(\overbrace{\vec{J}_{free} + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{P}}{\partial t}} \right) + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J}_f + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{P}}{\partial t} \right) + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f + \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

$$= \vec{J}_f + \frac{\partial}{\partial t} \left(\underbrace{\epsilon_0 \vec{E} + \vec{P}} \right)$$

$$= \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla}\cdot\vec{D} = \rho_{free}.$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = J_f + \frac{\partial\vec{D}}{\partial t}.$$

4 Eqns. in
6 quantities
E_x, E_y, E_z
B_x, B_y, B_z.

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Linear Electric/Magnetic material

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{E} = -\nabla V$$ — electrostatics.

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{A})$$

$$\cancel{\nabla \times \left[\nabla V + \nabla \times A \right]} =$$

$$\nabla \times \left[-\nabla V + \frac{\partial \vec{A}}{\partial t} \right] = 0$$ — e.s.

$$\nabla \times \left[\underbrace{\vec{E} + \frac{\partial \vec{A}}{\partial t}} \right] = 0$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2 V + \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}}$$

Gauss' Law Electrostat.

Faraday's Law.

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \mu_0 \vec{J} + \underline{\mu_0 \epsilon_0} \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial \vec{A}}{\partial t}\right)$$

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\left(\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}\right) - \nabla\left(\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t}\right) = 0$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -\frac{\rho}{\epsilon_0}$$

$$\left(\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}\right) - \nabla\left(\underbrace{\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t}}_{= 0}\right) = \overset{-\mu_0\vec{J}}{\cancel{0}}$$

$$\vec{A} \rightarrow A' = A + \nabla\psi$$

$$\vec{E} = -\nabla V - \frac{\partial\vec{A}}{\partial t}$$

$$V \rightarrow V' = V - \frac{\partial\psi}{\partial t}$$

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\rho/\epsilon_0$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J}$$

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0$$ Lorentz Gauge.

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = f(\vec{r},t) \neq 0$$

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla\psi$$

$$V \rightarrow V' = V - \frac{\partial\psi}{\partial t}$$

$$W_{el} = \frac{1}{2}\int d^3x \, \rho(x)\, \varphi(x)$$

$$= \frac{1}{2}\int d^3x \, \varphi(x)(\vec{\nabla}\cdot\vec{D})$$

$$= -\frac{1}{2}\int d^3x \, \vec{D}\cdot(\nabla\varphi) \qquad \nabla\cdot(\varphi\vec{D})$$

$$= \frac{1}{2}\int(\vec{D}\cdot\vec{E})\, d^3x$$

$$\cancel{u} \quad u_E = \frac{\vec{D}\cdot\vec{E}}{2}$$

$$W_{mag} = \frac{1}{2}\int \vec{A}\cdot\vec{J}\, d^3x$$

$$= \frac{1}{2}\int \vec{A}\cdot(\vec{\nabla}\times\vec{H})\, d^3x$$

$$= \frac{1}{2}\int \vec{H}\cdot(\nabla\times\vec{A})\, d^3x$$

$$= \frac{1}{2}\int \vec{B}\cdot\vec{H}\, d^3x$$

$$u_{mag} = \frac{\vec{B}\cdot\vec{H}}{2} = \frac{|B|^2}{2\mu}$$

$$u_{elec} = \frac{\vec{E}\cdot\vec{D}}{2} = \frac{\epsilon}{2}|E|^2$$

$$u = \frac{\epsilon}{2}|E|^2 + \frac{1}{2\mu}|B|^2$$

$$P_{mech} = \int \vec{F}\cdot\vec{v}\, d^3x$$

$$= \int \rho(\vec{E}+\vec{v}\times\vec{B})\cdot\vec{v}\, d^3x$$

$$= \int \vec{E}\cdot\vec{J}\, d^3x$$

$$u = \frac{\epsilon}{2}|E|^2 + \frac{|B|^2}{2\mu}.$$

$$\frac{dW}{dt} = \frac{1}{2}\int\left(2\epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \frac{2}{\mu}\,\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right)d^3x$$

$$= \int \epsilon\,\vec{E}\cdot\left(\frac{1}{\epsilon}\nabla\times H - \frac{1}{\epsilon}\vec{J}\right) + \frac{2}{\mu}\mu\,\vec{H}\cdot(-\vec{\nabla}\times\vec{E})\; d^3x$$

$$= \int\Big[\underbrace{\vec{E}\cdot(\vec{\nabla}\times H) - \vec{H}\cdot(\vec{\nabla}\times\vec{E})}\Big]d^3x - \int \vec{E}\cdot\vec{J}\, d^3x$$

$$= -\int \nabla\cdot(\vec{E}\times\vec{H})\, d^3x - \int \vec{E}\cdot\vec{J}\, d^3x.$$

$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = -\vec{E} \cdot \vec{J} \, .$$

$$\vec{S} = \vec{E} \times \vec{H}$$ Poynting Vector.