FAQs of Plasma Physics

by Prof. V.K. Tripathi , IIT Delhi, New Delhi.

Q.1: What is ambipolar diffusion?

Ans: Under any external force or pressure gradient force, when electrons of plasma tend to move away from the ions, the space charge field thus created inhibits the motion of electrons and induces ions to follow electrons. As a result electron and ions move together with mutual separation less than a Debye length. This process of diffusion is called ambipolar diffusion.

A good example is the redistribution of plasma density under the radiation pressure force of a laser. We shall learn later that a laser of finite spot size exerts a radiation pressure force (ponderomotive force) on electrons radially outward (across the laser axis) As the electrons are pushed away a strong space charge field is created that pulls the ions along with the electrons. The time scale for ambipolar plasma density redistribution is $\tau_d \sim r_o/c_s$ where r_o is the laser spot size, $c_s \sim (T_e/m_i)^{1/2}$ is the ion sound speed , T_e is the electron temperature and m_i is the ion mass.

Q2: What is the procedure of linearization?

Ans: The fields (electric, magnetic or gravitational) that exist in a plasma in equilibrium may be called zeroth order fields and the quantities like density, drift velocity, temperature and distribution function of different species as zeroeth order quantities. We may designate them by subscript zero, e.g.; n_o, $\overrightarrow{v_o}$, T_0,f_o. When one launches a wave of low amplitude in plasma or there exists a small fluctuation in the plasma, the additional fields , $\vec{E}$, $\vec{B}$ that appear in the plasma are perturbed fields or first order fields, corresponding changes in plasma quantities called the response or perturbations may be designated by a subscript 1, e.g. n_1, $\overrightarrow{v_1}$ etc. In solving the equations of motion and continually we substitute the fields, density and drift velocity as the sum of equilibrium and perturbed quantities. The terms in the equations that have products of perturbed quanties (e.g. ; $n_1 \overrightarrow{v_1}$, $\overrightarrow{v_1} \times \overrightarrow{B_1}, \overrightarrow{v_1} .\nabla, \overrightarrow{v_1}$) are dropped. This is called the linearization approximation.

Q3: What is the physical significance of complex conductivity?

Ans: Conductivity is merely a quantity that connects two physically observable quantities , current density and electric field that are the real parts of their complex representations. Complex conductivity $\sigma = \sigma_r + i\, \sigma_i$ can be written as

$$\sigma = (\sigma_r^2 + \sigma_i^2)^{1/2} \; e^{i\varphi},$$

$$\varphi = \tan^{-1}(\sigma_i/\sigma_r),$$

where φ denotes the phase difference between current density and electric field. If elective field is $\vec{E} = \vec{A}\cos(\omega t)$, the current density would be

$$\vec{J} = (\sigma_r^2 + \sigma_i^2)^{1/2} \;\; \vec{A}\cos(\omega t - \varphi\,),$$

In a collisionless plasma $\sigma_r = 0$, $\varphi = \pi/2$, i.e. the current density is $\frac{\pi}{2}$ out of phase with the electric field (in steady state).In the other limit when frequently ω is much smaller than the collision frequency (ω<<ν) $\sigma_i \approx 0$, φ=0 and the current density is in phase with the electric field. For arbitrary ω and v, $\varphi = \tan^{-1}(\omega/\nu)$.

Q4: A cold collisionless plasma slab of electron density n_o and width d (-d/2 < x < d/2) is placed between two capacitor plates (x=-d_o/2. d_o/2) with surface charge densities $\boldsymbol{\sigma_s}$ and $-\boldsymbol{\sigma_s}$ where $\boldsymbol{\sigma_s = \sigma_0\, exp(-i\omega t)}$. Obtain the electric field inside the plasma in electrostatic approximation.

Ans: The charge on capacitor plates produces an electric field $\overrightarrow{E_1} = (\boldsymbol{\sigma_s}/\boldsymbol{\epsilon_0})\hat{x}$. Under this field as the electron slab moves with respect to the ion slab, surface charge appers at x= -d/2, d/2. Let the displacement of the electron slab due to the net electric field $\vec{E}$ be $\Delta\hat{x}$. Then the surface charge density at x=-d/2 is $n_o e\,\Delta$ and at x=d/2 is -$n_o e\Delta$, where as $\vec{E}$ is the sum of fields due to the surface charges on capacitor plates and plasma slab,

$$\vec{E} = \frac{\sigma_s}{\epsilon_0}\,\hat{x} + \frac{n_o e}{\epsilon_0}\,\Delta\,\hat{x}$$

$\vec{\Delta}$ is governed by the equation of motion,

$$\frac{d^2\vec{\Delta}}{dt^2} = -\,\frac{e\,\sigma_s}{\epsilon_0 m}\,\hat{x} - \omega_p^2\vec{\Delta}$$

where $\omega_p = (n_o e^2/m\epsilon_0)^{1/2}$. Replacing d/dt by –iω one obtains

$$\vec{\Delta} = \hat{x}\,\frac{e\sigma_S}{m\varepsilon_0\ \omega^2 - \omega_p^2}$$

$$\vec{E} = \hat{x}\,\frac{e\sigma_S}{\varepsilon_o\ 1 - \omega_p^2/\omega^2}.$$

This is the same field as one would get by treating plasma as a dielectric of effective relative permittivity $\epsilon_{eff} = 1 - \omega_p^2/\omega^2$. The field is resonantly large at $\omega = \omega_p$.

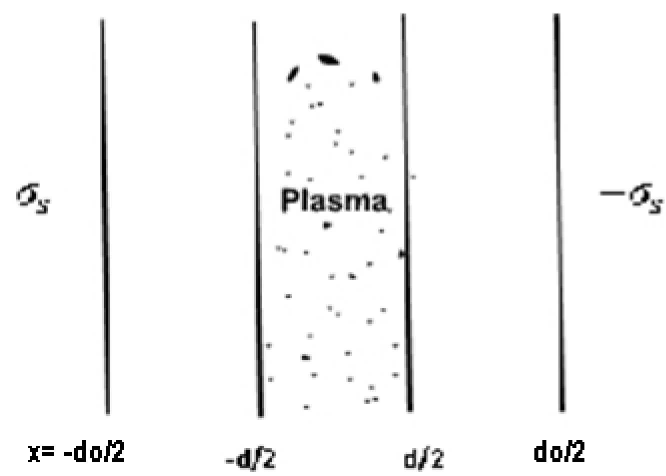

Fig : plasma slab inside a capacitor

Q.5: A cold collisionless magnetized plasma slab of width d (-d/2 < x < d/2), electron density n_0 and static magnetic field $B_S\,\hat{z}$ is placed between two capacitor plates (x = -

$d_0/2$, $d_0/2$) with surface change densities σ_S and $-\sigma_S$ w where $\sigma_S = \sigma_o \exp(-i\omega t)$. Obtain the electric field inside the plasma in electrostatic approximation.

Ans: Let the electric field inside the plasma of effective plasma permittivity tensor $\underline{\underline{\in}}$ *be* $\vec{E}$ .In the free space region $d_0/2 > |x| > d/2$, the electric field is $\vec{E}_0 = (\sigma_s / \in_o)\,\hat{x}$. Continuity of parallel components of electric field at the plasma surface (x = -d/2) gives $E_y = 0$, $E_Z = 0$. Continuity of normal component of displacement vector at x = -d/2 gives

$$E_x = \frac{E_o}{\in_{xx}} = \frac{\sigma_s}{\in_o \in_{xx}},$$

$$\in_{xx} = 1 - \frac{\omega_P^2}{\omega^2 - \omega_c^2} - \frac{\omega_{Pi}^2}{\omega^2 - \omega_{ci}^2}.$$

The field inside the plasma is resonantly enhanced at $\omega = \omega_{UH} = (\omega_p^2 + \omega_c^2)^{1/2}$ and $\omega^2 = \omega_{ci}^2 + \omega_{pi}^2 / (1 + \omega_p^2 / \omega_c^2)$.

Q.6 Why can`t we have ion acoustic wave in a cold plasma?

Ans: In a cold plasma, the electron drift velocity under the electric field of the wave would be opposite to that of the ions and of much larger magnitude. Thus only electrons prevail and their natural frequency of oscillation is ω_P and electrostatic wave with $\omega = \omega_P$ only exists. In a hot plasma electron pressure gradient force, at low frequencies, almost neutralizes the electric force, hence the electron and ion drift velocities are of comparable magnitude and in the same phase. This leads to $\omega < \omega_{pi}$, i.e. the existence of ion acoustic wave below the ion plasma frequency.

Q.7 Why do we neglect the effect of magnetic field of an electromagnetic wave in obtaining the electron response at low amplitude of the wave?

Ans: The magnetic force an electrons due to an electromagnetic wave is $-e\vec{v}\times\vec{B} = -e\vec{v}\times(\vec{k}\times\vec{E})/\omega$ where as the electric force is $-e\vec{E}$ Thus the magnetic force is smaller by a factor $\vec{k}.\vec{v}/\omega \simeq v/v_{ph}$ with respect to the electric force and can be neglected when the drift velocity ν is much less than the phase velocity ν_{ph} of the wave. At high amplitude when ν becomes comparable to ν_{ph} or c, the magnetic force gives rise to fascinating nonlinear phenomena like harmonic generation and parametric instabilities.

Q.8 Why can`t electromagnetic waves be excited by a co-moving electron beam in an unmagnetized plasma?

Ans: An electron beam can excite a wave when (i) the wave moves with phase velocity less than the velocity of the beam ($\omega/k < \nu_b$). (ii) the wave electric field has a component in the direction of beam, (iii) the beam must bunch in the retarding phases of the wave. Conditions (i) and (ii) are not satisfied in an unmagnified plasma as $\omega/k > c$ and $\vec{E}$ is perpendicular to the velocity $\vec{v}_b$ of the co-moving beam.

Q.9 What is the reason, the phase velocity of an electromagnetic wave in a plasma is greater than c while it is less than c in a dielectric?

Ans: Phase velocity of an electromagnetic wave, $v_{ph} = c / \in_{eff}^{1/2}$ depends on effective relative plasma permittivity. In dielectrics $\in_{eff} = \in_r = 1 + x$ where x is the dielective susceptibility that connects polarization $\vec{P}$ to electric field $\vec{P} = \in_o x \vec{E}$. $\vec{P}$ is the induced dipole moment per unit volume that equals $-n_a e \vec{\Delta}$ where $\vec{\Delta}$ is the displacement of electron cloud of an atom with respect to the nucleus and n_a is the atomic density. For an electron rotating round the nucleus with angular frequency ω_0, the displacement due to an electric field $\vec{E}$ of frequency ω (in the limit when atom density of the dielectric is low) is

$$\vec{\Delta} = \frac{e\vec{E}}{m(\omega^2 - \omega_0^2)}, hence$$

$$\in_r = 1 - \frac{\omega_{pa}^2}{\omega^2 - \omega_0^2}$$

where $\omega_{pa} = (n_a e^2 / m \in_0)^{1/2}$. In a plasma $\in_{eff} = 1 - \frac{\omega_P^2}{\omega^2}$. Thus the basic difference in dielectric and plasma is the appearance of frequency of revolution of the bound electrons around the nucleus. ω_0 falls in the ultraviolet. For $\omega < \omega_0$, $\in_r > 1$ and $\nu_{ph} < c$ in the dielectric. In plasma, electrons are free and move $\pi/2$ out of phase with the electiric field, introducing a phase difference of π between the current density and the displacement current. That makes $\in_{eff} < 1$ *and* $\nu_{ph} > c$.