PCD_STiCM_P1

Department of Physics
Indian Institute of Technology Madras

STiCM: Select/Special Topics in Classical mechanics Assignment-1

1. When this bus is going on a curved road, what do you think will happen to the luggage on the  top of its roof? Why does the luggage tend to remain stable when the bus is going at a constant velocity, whereas it seems to get laterally thrown off when the bus is going on a curve?

2. *Newton's laws for 'dogmatix':* Suppose dogmatix jumps out of a spacecraft while it is at a point in space, far away from earth, where all the gravitational forces on the spacecraft due to

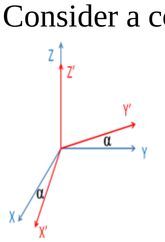
all external objects balance each other, and the net external force is zero. What do you think will be the trajectory of the dog? Does it matter with what speed they throw the dog out? What if they just place it outside, with zero speed? In the first place, what will be the nature of the motion of the spacecraft at such a location?

3. *Time for* a *short* BREAK ______________________________ _

A one dimensional medium has a its dielectric property changed halfway. The dielectric constant is ε_0 for $0 \le x \langle L$ (medium 1) and has a different value ε for $L \le x \le 2L$ (medium 2). What is the potential energy of two charges q1 and q2 placed at a distance *d* apart when (a) both are in medium 1, and (b) both are in medium 2. Would it be possible to hold them at the same distance in medium 2 as in medium 1 without doing any work (*Note that the relative displacement between the charges remains zero, with the two charges maintained at the same relative distance between each other. Also, remember that* $\delta W = \vec{F} \bullet \overrightarrow{\delta s}$).

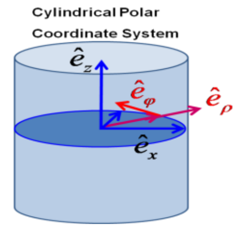
4. Set up the equation of motion for a 'damped' one-dimensional linear harmonic oscillator 'driven' by a 'damped harmonic force' $F_d(t) = F_0 e^{-\alpha t} \cos \omega t$, and describe the displacement of this oscillator as a function of time.

5. Consider a coordinate system: (x,y,z). Write the transformation matrix for a rotation of the



primed (x',y',z') coordinate system through an angle α about z-axis. Now consider a third coordinate system: (x", y", z") obtained by rotating the primed coordinate system about y' by an angle β. What will be the transformation matrix from the first (x, y, z) to the third (x", y", z") system?

6. Show that the parity operation (reflection through the origin) on a point (ρ, φ, z) relative to fixed (x, y, z) axes consists of the transformation: ρ → ρ   φ → φ ± π   z → −z.



Show that the unit vectors of the cylindrical polar coordinate system $\hat{e}_\rho$ and $\hat{e}_\phi$ have odd parity while $\hat{e}_z$ has even parity.

7. (a) A particle moves along a path given by $\rho = ae^{b\phi}$ where b is a positive constant of appropriate dimensions. If the angular component of acceleration is known to be zero, then write the angular speed in terms of ρ and sketch the path of the particle.

(b) An ant crawls on the surface of a ball of radius b in a manner such that the ant's motion is given in spherical coordinate system by the equation:

$$r = b \qquad \phi = \omega t \qquad \theta = \frac{\pi}{2}\left[1 + \frac{1}{4}\cos(4\omega t)\right]$$

Find the speed of the ant as a function of the time t. What sort of path is represented by above equation?

8. Determine whether magnetic field vector is 'polar' or 'axial' by examining the Lorentz force on a charge q given by the expression $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$.

9. A particle of mass m moves in response to a central force as, $m\ddot{\vec{r}} = \hat{e}_r f(r)$.

Show that $\vec{r} \times \dot{\vec{r}} = \vec{c}$, a constant and that the geometric interpretation of this leads to Kepler's second law.

10. (a) For a central force field prove that Laplace-Runge-Lenz (LRL) vector $\vec{A} = \vec{p} \times \vec{L} - mk\frac{\vec{r}}{r}$ is constant in time *if* <u>and</u> *only if* the force varies as $\frac{1}{r^2}$.

(b) Obtain the trajectory of Kepler orbit *<u>from the properties of LRL vector</u>*.

(c) If the velocity or the momentum of a particle is translated so as to start from the center of force, then the head of the vector traces out a particle's *hodograph*. By taking the cross product of $\vec{L}$ with LRL vector $\vec{A}$ show that the hodograph for elliptical Kepler motion is, in terms of momentum, a circle of radius $\frac{mk}{L}$ with origin on the y axis a distance A/L displaced from the center of force.