Department of Physics
Indian Institute of Technology Madras

STiCM: Select/Special Topics in Classical mechanics Assignment-2

1. *"Can Spring be far behind" ... how far?* : Professor Calculus claims that he can determine the frequency of small oscillations of a mass-spring system even if he knows neither the spring constant nor the mass of the system. All he needs to know, he claims, is how far the spring stretches in the Earth's gravitational field under the effect of the mass attached. Do you believe his claim is justified?

2. *Inducing a capacity to resist*: Consider a simple circuit which has an inductance L and capacitance C in series with one another. A current $i(t)$ flows through the circuit (all components of which are assumed resistance less) in such a way that the current has a value I_0 at time t=0. Obtain the Lagrangian and Hamiltonian for this circuit and interpret your results in terms of generalized momentum. Check the dimensions of action in this case.

3. Consider two positive charges of equal magnitude Q fixed at the points separated by a distance d. Now another positive charge of the same magnitude is kept exactly at the middle of the line joining the two charges.

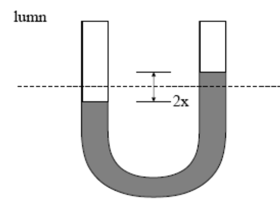
   a) Why will it be possible at all to keep the third charges as stated? Will the configuration be stable?
   b) What would happen if the third charge were displaced by a very small amount along the line joining the first two charges, to either of the sides initially?
   c) How will the situation be different if the third charge is displaced by a very small amount along a direction perpendicular to the line joining the first two charges?

4. *The not-so-simple pendulum:* It is quite easy to write a program (or use a math software package) to simulate the motion of pendulum on a computer. You only have to solve a differential equation of the kind $m\ddot{x} = -kx$ where the symbols have their usual meaning. But real pendulum is not always *simple*. Make such a program and see what would happen at large amplitudes. Of course x in the RHS will be replaced by $sin(x)$. In the simulation you can set initial amplitude to be very, very large and see the consequence. What will be the phase orbit? Can you explain the results on the basis of a phase diagram?

5. ‘*Chaltee ka naam gaaadi*’ : An automobile can be considered to be mounted on four identical springs as far as the vertical oscillations are concerned. The springs of a certain car (of mass 1450 kg) are adjusted such that these vibrations are at 3 Hz. Find the spring constant. Further, find the frequency of oscillations if 5 passengers averaging 73 kg each rides the car.

6. Analyze the problem of the LC circuit without the help of Lagrangian method. Now include also a resistance R in series. Write down an expression for the voltage in the circuit at any time t in terms of the given quantities. Suggest a solution to this differential equation of motion under typical initial conditions. Bring out the analogy between electrical and mechanical oscillating systems.

7. *Watch this on U-tube*: A ‘U’ shaped glass tube has a uniform cross section area of A and contains a volume V of mercury. The mercury is displaced slightly and released. Disregarding friction, find the oscillation time period in terms of V, A and g.



8. Consider the Morse potential function $V(r) = D_e[1- \exp\{-a (r-r_e)\}]^2$ for a molecule in stable equilibrium with an internuclear separation r_e and D_e and a are constants. Find the period of small molecular oscillations near r_e.

a) Sketch this potential as a function of the internuclear distance.
b) Identify the main features of the plot and their physical significance.
c) Under what condition would the molecule be stable?
d) Discuss the features of motion for internuclear separations close to equilibrium. Show that it can be considered as harmonic potential. Derive the expression for k.

9. The equation of motion of an oscillator is given by $x(t) = A\sin(\omega t)$ where the symbols have their usual meaning. What will be the mathematical relation between the speed v and the displacement x at any instant of time?

10. A particle of mass m moves along the x-axis under the influence of a potential given by $U(x) = \frac{kx^2}{2}\left\{1 - \frac{x^2}{2a^2}\right\}$ where k and a are real and positive constants of appropriate dimensions. Identify all possible points of equilibrium and the nature of each one o these.

11. A particle of mass *m* moves under the potential $U(x) = \frac{cx}{a^2 + x^2}$, where *c* and *a* are positive constants of appropriate dimensions.
    a) Find all points of equilibrium and their nature.
    b) Determine the frequency of small oscillations around the point of stable equilibrium.
    c) Sketch the potential as a function of *x*, indicating the relevant points in the plot.

12. A particle of mass m moves in a potential given by $U(y) = Aye^{-By}$ where A and B are positive constants of appropriate dimensions. Find the period of small oscillations about the point of stable equilibrium

13. *The incredible melting pendulum*: Consider a pendulum with its bob made out of a big ice crystal attached to a mass less string being set in to oscillation with amplitude *A* and has a period of 1 sec (length of the sting is much longer compared to the dimensions of the ice crystal). Now the ice is melting in such a way that the rate of loss of mass due to dripping water is constant (can you guess how you should achieve this? This skill is very useful in making solid rocket boosters! Why? Don't worry; this part is an extreme exploration!). Describe what will happen. Now consider the case where the rate of mass loss is proportional to the mass of ice at that instant. Describe what happens. Have you seen anything like this?

14. Consider an electron moving in a constant magnetic field pointing in some direction, neither parallel not perpendicular to the initial velocity of electron. Derive the equation of motion using Lorentz force equation $q(\vec{v} \times \vec{B})$. Now over this, add an electric filed parallel to the magnetic field. What would be the trajectory of the electron in these two cases?

15. Consider an ideal pendulum hung from the roof of a car moving on a circular track with uniform angular speed. Describe nature of the motion of the pendulum. How would this be different if the car were to be going up along a spiraling driveway with constant pitch and diameter?

16. Squaring the circle? With a little bit of programming or use of simple mathematical/graphical software, you will be able to see how a square wave can be thought of to be made up of several sine waves of different frequencies and amplitudes. Build up with one wave, then two, then three etc., with frequencies ω, 2ω, 3ω etc. Add the terms each time and plot on the screen. Adjust the amplitudes till the output waveform starts resembling a square wave. This becomes easier if you look at any text book and see the actual terms of the Fourier series expansion of a square wave and sketch the component waves one by one.

17. *The not-so-simple pendulum:* It is quite easy to write a program (or use a math software package) to simulate the motion of pendulum on a computer. You only have to solve a differential equation of the kind $m\ddot{x} = -kx$ where the symbols have their usual meaning. But real pendulum is not always *simple*. Make such a program and see what would happen at large amplitudes. Of course x in the RHS will be replaced by $sin(x)$. In the simulation you can set initial amplitude to be very, very large and see the consequence. What will be the phase orbit? Can you explain the results on the basis of a phase diagram such as the one discussed in the text of this chapter?