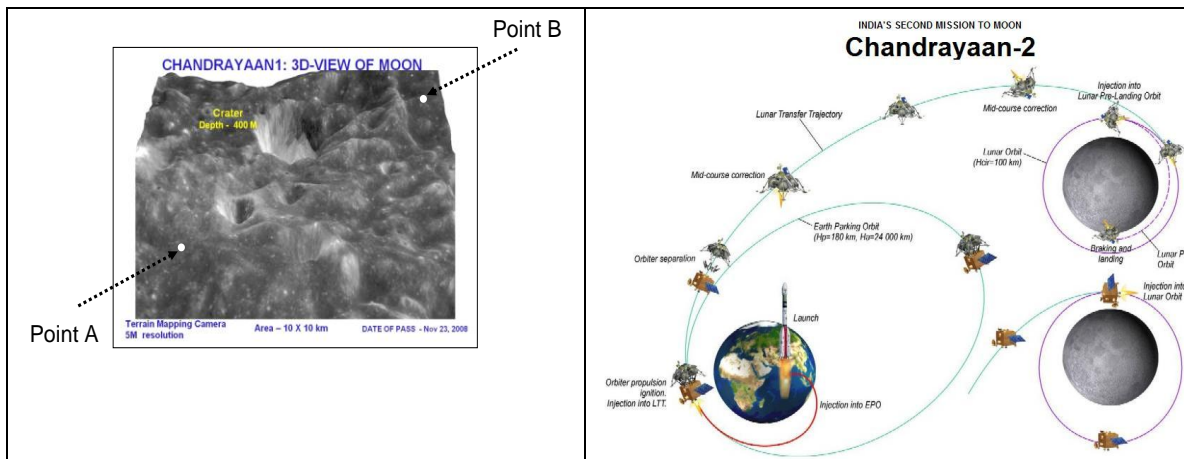


Department of Physics
Indian Institute of Technology Madras

STiCM: Select/Special Topics in Classical mechanics

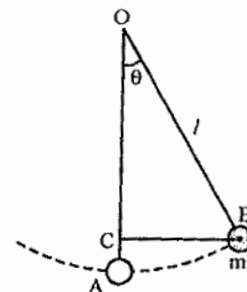
Assignment-3

1. *How to rush in where angels fear to tread:* The photograph given below is a 5 meter resolution 3D map (height versus position in 2D) of a '10 km by 10' km section on the moon's surface produced by Chandrayaan 1 <http://www.isro.gov.in/satellites/spacemissions.aspx>. Several details can be seen in the picture. Now suppose you have to design a moon rover which is to be launched on Chandrayaan 2. Suppose the rover would land on the moon's surface at point A. You are to program the rover to go to point B and perform some tests to find out if there is water in the form of ice. Of course you do not want the rover to fall in the valleys in the terrain, get stuck in the small pot holes or hit a tall mountain. Let the figure be in xy plane; explain how you will program the rover to find the *safest* but the *fastest* path using this pictorial data from Chandrayaan 1. NOTE! The purpose of this exercise is not to make you actually find the solution, but just think about what the solution would entail and discover for yourself what mathematical techniques you would need to solve the problem.

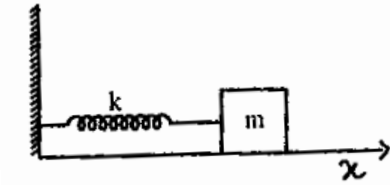


- 2 Write down (a) the Lagrangian, (b) the generalized momentum & (c) the Hamiltonian for a simple pendulum of mass m and length l in a uniform gravitational field of intensity \vec{g} .

$$(b) p_{\theta} = \frac{\partial L}{\partial \dot{\theta}} = ml^2 \dot{\theta}$$



- 3 Write down (a) the Lagrangian, (b) the generalized momentum & (c) the Hamiltonian for a linear harmonic oscillator moving along x-axis and having a restoring force constant k .

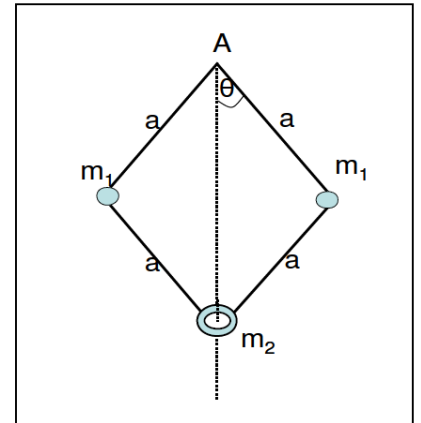


Q4: For a point mass 'm' in a central field potential

$V = -\frac{k}{\rho}$, find (a) the Lagrangian, (b) the generalized momentum and (c) the Hamiltonian.

Use the plane polar coordinates (ρ, φ) .

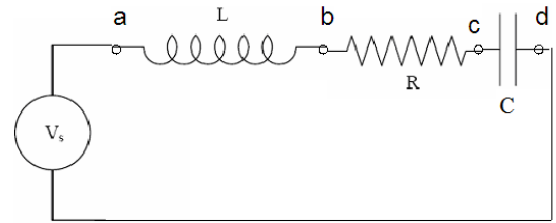
5. Show that the Euler-Lagrange equations in the case of (i) Q2, (ii) Q3 and (iii) Q4 are equivalent to Newton's equation of motion.
6. Consider a system comprising of two masses m_1 each and a ring of mass m_2 as shown in the figure. The ring moves on a vertical axis and whole system rotates about this axis with a constant angular velocity Ω . Write down the Lagrangian for this system. Comment on the result.



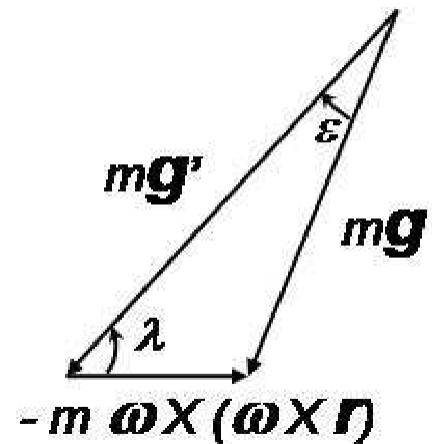
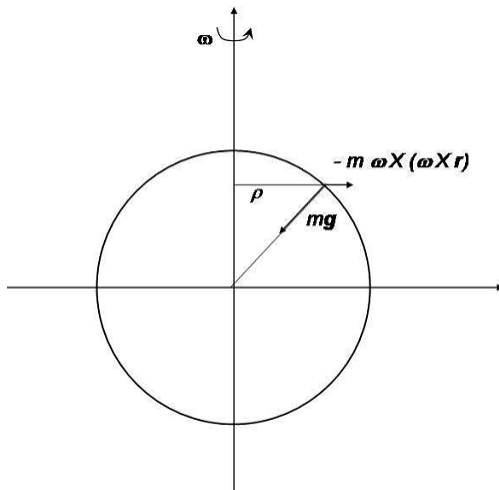
7. *Inducing a capacity to resist:*

Consider a simple electrical LC circuit which has just an inductance L and capacitance C in series with one another. A current $i(t)$ flows through the circuit. We shall assume that all connectors between L & C offer no resistance to the current; the only impedance to current would be from L & C . The current has a value I_0 at time $t=0$. Obtain the Lagrangian and Hamiltonian for this circuit and interpret your results in terms of generalized momentum. Check the dimensions of 'action' in this case. Use the symbol \tilde{L} for the Lagrangian to distinguish it from the symbol L used for the inductance.

8. An alternating (AC) LCR circuit. Construct the electromechanical analogues and get the solution using Lagrangian formulation by comparison with the solution of a mechanical damped and driven oscillator.



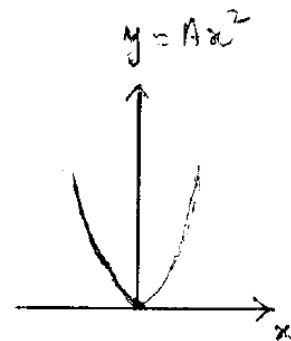
9. *Humbug forces?*: A bug crawls outwards with constant speed v along the spoke of a wheel which is rotating with constant angular velocity about a vertical axis. Find all the apparent forces acting on the bug.
10. *Pisa Hut*: Consider a heavy bob at the end of a plumbline being used for some construction work of a building in city situated at a certain geocentric latitude (an angle which varies between zero at Equator and $\pm 90^\circ$ at poles). Does the plumbline deviate from the true vertical? If yes, then calculate how much and where will the deviation be maximum.



11. *Force of the match*: In a cricket match at the Firoz Shah Kotla Stadium, New Delhi (latitude $\approx 30^\circ$ N), Sachin is at the crease. Muralidharan is bowling and tries a googly, but anticipating the ball Sachin quickly moves down the pitch and hits a powerful straight drive over the bowlers head. The ball starts from the lower part of the bat at an elevation angle of 15° and hits the ground at a distance of 75 m just crossing the boundary line.

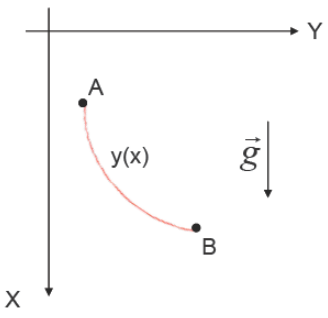
Neglecting air resistance, determine the amount of deflection due to Coriolis force alone. (Assume that the stroke is played in the eastward direction.)

12. Consider the Lorentz force (in an inertial frame) acting on a charged particle due to electric field \vec{E} and magnetic field \vec{B} . Show that by choosing a rotating coordinate system with $\vec{\omega} = \frac{-q}{2m} \vec{B}$, the term involving \vec{B} in the force equation can be eliminated. This result is known as Larmors theorem.
13. *Mumbai Express*: The latitude of Mumbai city is about 19° N. What is the period of precession of a Foucault pendulum there?
14. A centrifuge operates at an angular velocity $\vec{\omega} = \omega \hat{e}_z$. Consider the origin to be at the geometric center of the centrifuge. Find the centrifugal force experienced by a particle in the centrifuge at a point with position vector $a\hat{e}_\rho + b\hat{e}_z$ (where a and b are positive constants of appropriate dimensions).
15. An ultracentrifuge has a rotational speed of 500 rps.
 - a) Find the centrifugal force on a one-microgram particle in the sample chamber if the particle is 5cm from the rotational axis.
 - b) Express the result as the ratio of the centrifugal force to the weight of the particle.
16. A particle moves in a horizontal plane on the surface of the earth. Show that the magnitude of the horizontal component of the Coriolis force is independent of the direction of the motion of the particle.
17. A particle of mass 'm' slides on a smooth surface, the shape of which is given by $y = Ax^2$ where A is a positive constant of suitable dimensions and y is measured along the vertical direction. The particle is moved slightly away from the position of equilibrium and then released. Write down the equation of motion of the particle



18. Consider a force of magnitude $F = -kx$ acting on a particle of mass m where k is a positive constant of appropriate dimensions and x is the instantaneous displacement.
- Reason out a possible equation of motion which provides the position of the particle at any instant t .
 - What will be the time-dependence of displacement if it is known that the displacement is zero at $t=0$?
 - Will this change if $x=A$ (the position corresponding to the maximum displacement) at $t=0$?
 - What will be the value of the momentum at any time t in each case?
 - Sketch the momentum as a function of the instantaneous position.

OPTIONAL problem (slightly advanced, for those amongst you who wish to do something 'more':

	<p>The brachistochrone problem: If A and B are two points in a vertical plane, and a mass M having only a point dimension moves under its own weight, what would be the path taken by this mass in moving from the point A to B that would take the least time? You are thus required to find the shape of the curve along which a bead sliding from rest and accelerated by gravity will slip (without friction) from one point to another in the least time. In Greek, 'brachistos' means 'the shortest' and 'chronos' means 'time', hence the name 'brachistochrone'. <i>The solution was provided by Newton within a day since the problem was posed by Johann Bernoulli in 1696.</i></p>
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