Department of Physics
Indian Institute of technology Madras
Select/Special Topics in Classical Mechanics
Self-Assessment-2 (Questions)

NOTE: Symbols/notations used in this question paper have their usual meanings, as used in our course.

1. *State whether the following statements are 'TRUE' or 'FALSE' and give reason. The reason should be short, but as rigorous as you can provide.*

   a. For a particle of mass m moves in a region of space where the potential is described by $U(x,y) = -U_0 \exp\left[-\frac{(x^2+y^2)}{2L^2}\right]$, the point (x=0, y=0) is a '*saddle point*' (given: U_0 & L are positive constants).

   b. If a vector field $\vec{A}$ is both irrotational $\left(\vec{\nabla}\times\vec{A}=\vec{0}\right)$ and solenoidal $\left(\vec{\nabla}\bullet\vec{A}=0\right)$, then it must be identically equal to the *null vector*.

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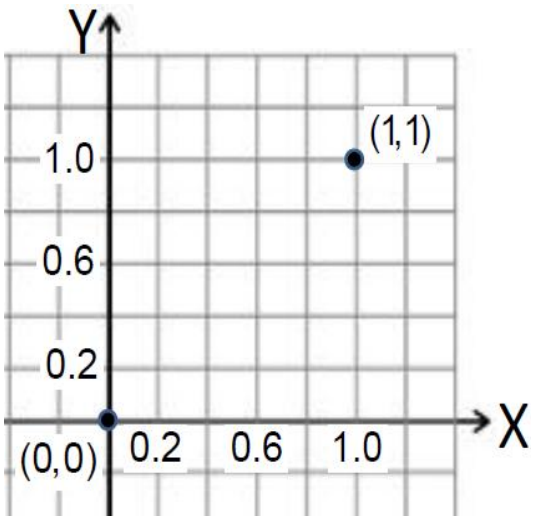
2. A position-dependent force field is given by the expression $\vec{F} = A(x-y)\hat{e}_x + (x+y)\hat{e}_y$. It is given that $|A| = +1$.



(a) What is/are the dimension(s) of A?

(b) The given force acts on a particle, moving it along a closed path described by the two curves:

$y = x^2$, *traversed from (0,0) to (1,1),*

and

$y^2 = x$ *traversed from (1,1) to (0,0).*

Sketch the closed path on the graph sheet in the margin. Write your name here:_____ and **submit this graph** along with the rest of the answer book when you finish the exam.

(c) Determine the work $\oint \vec{F}\bullet\vec{dl}$ done by the above force over the closed path described above.

(d) Without determining the curl of this force $\left(i.e.\text{ without finding } \vec{\nabla}\times\vec{F}\right)$, can you tell if the force is irrotational or not? **Explain how!**

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3. A scalar field $\psi(x, y)$ is given by the expression $\psi(x, y) = \psi_0 \exp(x^2 + y^2 - 4x - 8y)$,

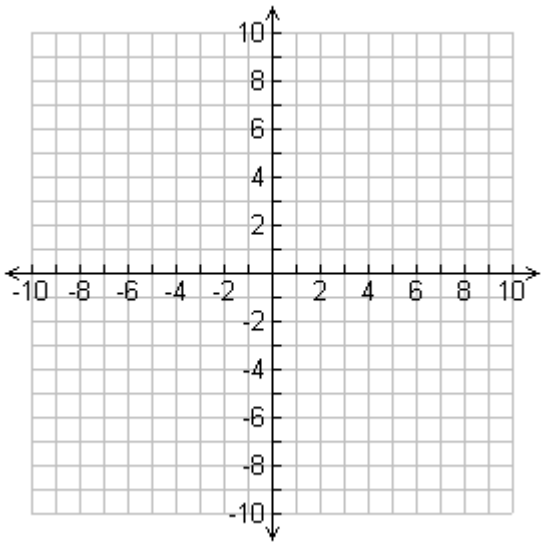
where ψ_0 is a constant having suitable dimensions.

(a) Obtain the equipotential curve for $\psi = \psi_0$.

(b) Sketch the vector field $\vec{\nabla}\psi$ at $\psi = \psi_0$.

Write your name here:_____

and **<u>submit this graph</u>** along with the rest of the answer book when you finish the exam.



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4 (a) Determine the divergence of the vector point function described by:

$$\vec{A}(\hat{r}) = (r\cos\theta)\,\hat{e}_r + (r\sin\theta)\,\hat{e}_\theta + (r\sin\theta\cos\varphi)\hat{e}_\phi$$

(b) Find the flux of the above vector field over a *closed* surface that encloses a hemisphere of radius R resting on the xy-plane, with its center at origin and located in the region $z \geq 0$.

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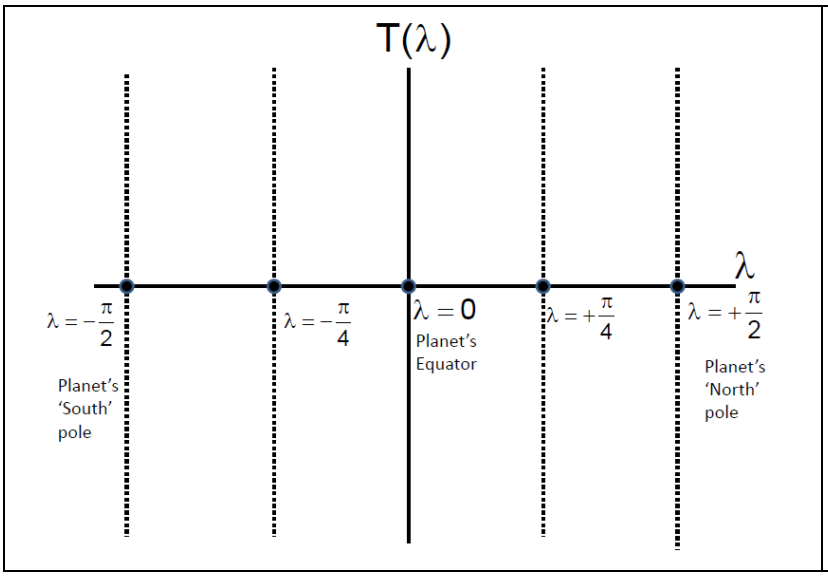
5 A planet in a remote galaxy rotates rapidly about its own axis. It completes one full rotation in one second. Sketch $T(\lambda)$ *vs* λ for this planet, where $T(\lambda)$ is the time period for the rotation of a Foucault pendulum set in motion on this planet, λ *is the latitude*; $-\frac{\pi}{2} \leq \lambda \leq \frac{\pi}{2}$.



Write your name here:

and
submit this sheet having the graph along with the rest of the answer book when you finish the exam.