STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 31

Unit 10 : **Classical Electrodynamics**

Classical Electrodynamics

Charles Coulomb 1736-1806

Carl Freidrich Gauss 1777-1855

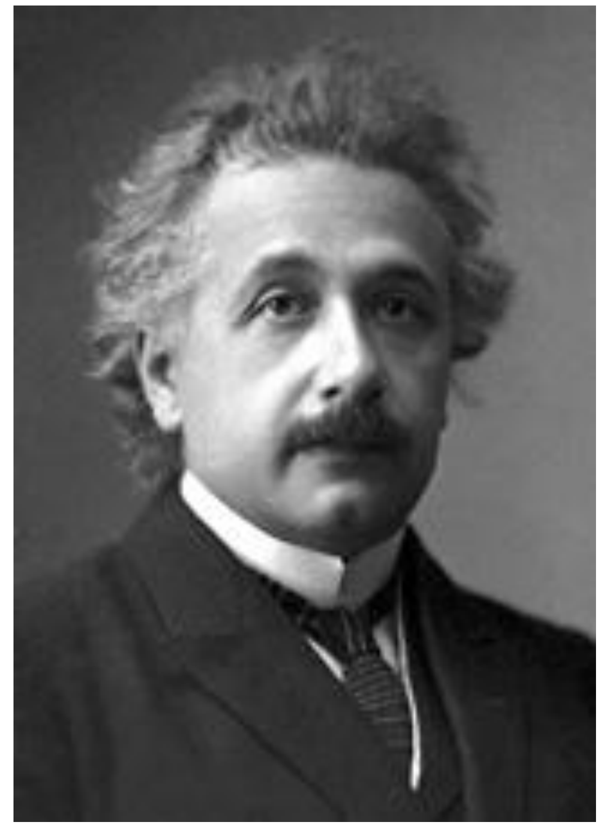
Andre Marie Ampere 1775-1836

Michael Faraday 1791-1867

Electrodynamics & STR

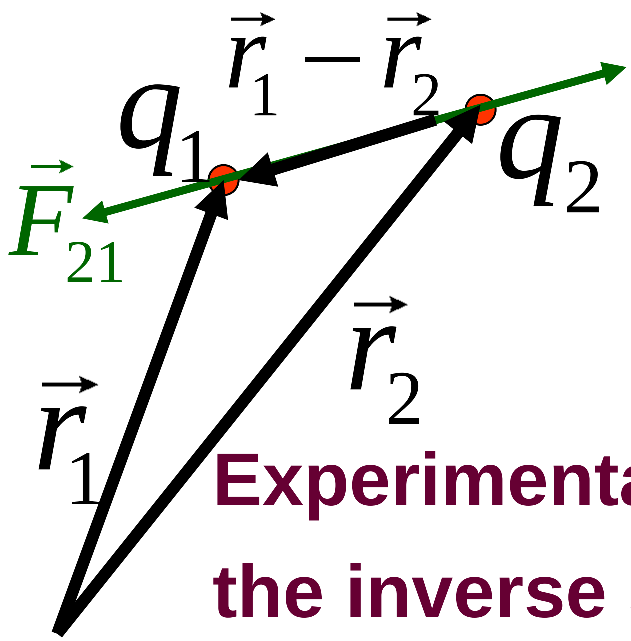
The special theory of relativity is intimately linked to the general field of electrodynamics. Both of these topics belong to 'Classical Mechanics'.

James Clerk Maxwell
1831-1879



Albert Einstein
1879 - 1955

Foundations of classical electrodynamics



$$\vec F_{12} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec r_1 - \vec r_2}{\left|\vec r_1 - \vec r_2\right|^3}$$

Experimental recognition of the inverse square law:

Priestly (1767)

Robinson (1769)

Cavendish (1771)

Coulomb (1785)

Coulomb also advanced the view that negative charges exist, that they did not merely represent absence of a positive charge.

Linear Superposition

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}$$

$$\vec{F}_{on\ q} = \frac{q}{4\pi\varepsilon_0} \sum_i q_i \frac{\vec{r} - \vec{r}_i}{\left|\vec{r} - \vec{r}_i\right|^3}$$

$$\vec{F}_{on\ q} = \frac{q}{4\pi\varepsilon_0} \iiint \frac{\rho(\vec{r}\,')\left(\vec{r} - \vec{r}\,'\right) d^3\vec{r}\,'}{\left|\vec{r}_1 - \vec{r}\,'\right|^3}$$

Since force on a particle is proportional to its charge q, it is fruitful to define the proportionality as the electric field $\vec{E}$:

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q} = \frac{q'}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}'}{\left|\vec{r} - \vec{r}'\right|^3}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i q_i \frac{\vec{r} - \vec{r}_i}{\left|\vec{r} - \vec{r}_i\right|^3}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\vec{r}')\left(\vec{r} - \vec{r}'\right) d^3\vec{r}'}{\left|\vec{r}_1 - \vec{r}'\right|^3}$$

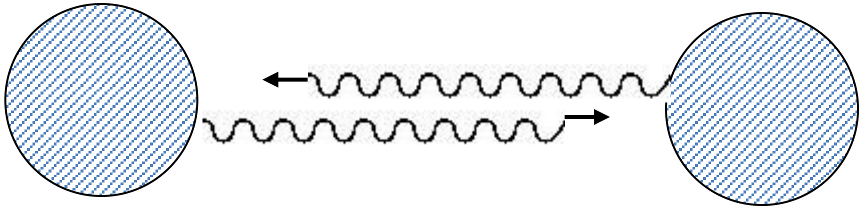
What is the confidence level in our contention that the force goes as inverse-square of the distance between the charges?

Inverse force requires: $V(r) \sim \dfrac{1}{r},$

so that the force would vary as: $\dfrac{1}{r^2}.$

Why can't the potential be: $V(r) \sim \dfrac{e^{-r/\lambda}}{r}$ (Yukawa) ?

The force/interaction can originate from an exchange of particles – like ping-pong balls thrown back and forth between the charges, thus binding them.



λ: some fundamental length

$$V(r) \sim \frac{1}{r}$$

or

$$V(r) \sim \frac{e^{-r/\lambda}}{r} \quad ?$$

$$\lambda = \frac{h}{\mu c}$$

dimension of $\frac{h}{\mu c}$

$$\frac{\left[L \times MLT^{-1}\right]}{MLT^{-1}} = L$$

$$V(r) \sim \frac{e^{-\frac{r}{h/\mu c}}}{r}$$

$$V(r) \sim \frac{e^{-\frac{r\mu c}{h}}}{r}$$

μ: mass of the 'ping-pong' messenger carrier

$\rightarrow$ photon mass

$$V(r) \sim \frac{1}{r}; \quad \text{or} \quad V(r) \sim \frac{e^{-\frac{r\mu c}{h}}}{r} \;?$$

Note that $\mu \to 0 \quad \Rightarrow$ Coulomb.

Inverse force requires: $V(r) \sim \frac{1}{r},$

so that the force would vary as: $\frac{1}{r^2}.$

Thus, the question of the interaction potential being Coulomb or Yukawa is bound to the value of μ, the photon mass.

The question thus translates to what is our confidence level in knowing the mass of the photon?

"Because classical Maxwellian electromagnetism has been one of the cornerstones of physics during the past century, experimental tests of its foundations are always of considerable interest. Within that context, one of the most important efforts of this type has historically been the search for a rest mass of the photon….."

The mass of the photon

Liang-Cheng Tu, Jun Luo and George T Gillies

Rep. Prog. Phys. 68 (2005) 77–130

The uncertainty principle, puts an ultimate upper limit:

$$\mu \langle \frac{\hbar}{c^2 \Delta t}$$

$$\langle\ 10^{-66}\ gms$$

$$\mu \langle\, 10^{-66}\, gms$$

Consequences of even this tiny mass:

- a wavelength dependence of the speed of light in free space,

- deviations from exactness in Coulomb’s law and Amp`ere’s law,

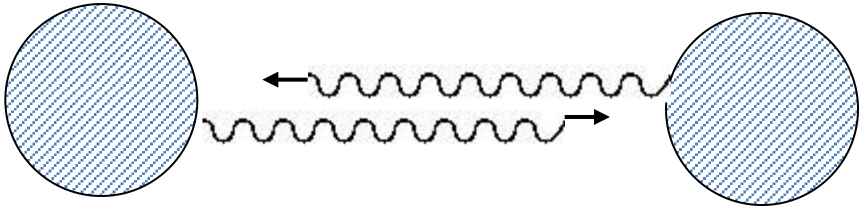
- the existence of longitudinal electromagnetic waves,

- the addition of a Yukawa component to the potential of magnetic dipole fields, ……

The mass of the photon
Liang-Cheng Tu, Jun Luo and George T Gillies
Rep. Prog. Phys. 68 (2005) 77–130

Range of the Coulomb interaction:

$$R: \quad c\Delta t \sim c\frac{\hbar}{\Delta E} \sim \frac{\hbar c}{\mu c^2}$$



$$\mu \rightarrow 0$$

$$\mathrm{R} \rightarrow \infty$$

$$V(r) \sim \frac{e^{-\frac{r}{h/\mu c}}}{r}; \qquad \text{i.e.} \qquad V(r) \sim \frac{e^{-\frac{r\mu c}{h}}}{r}$$

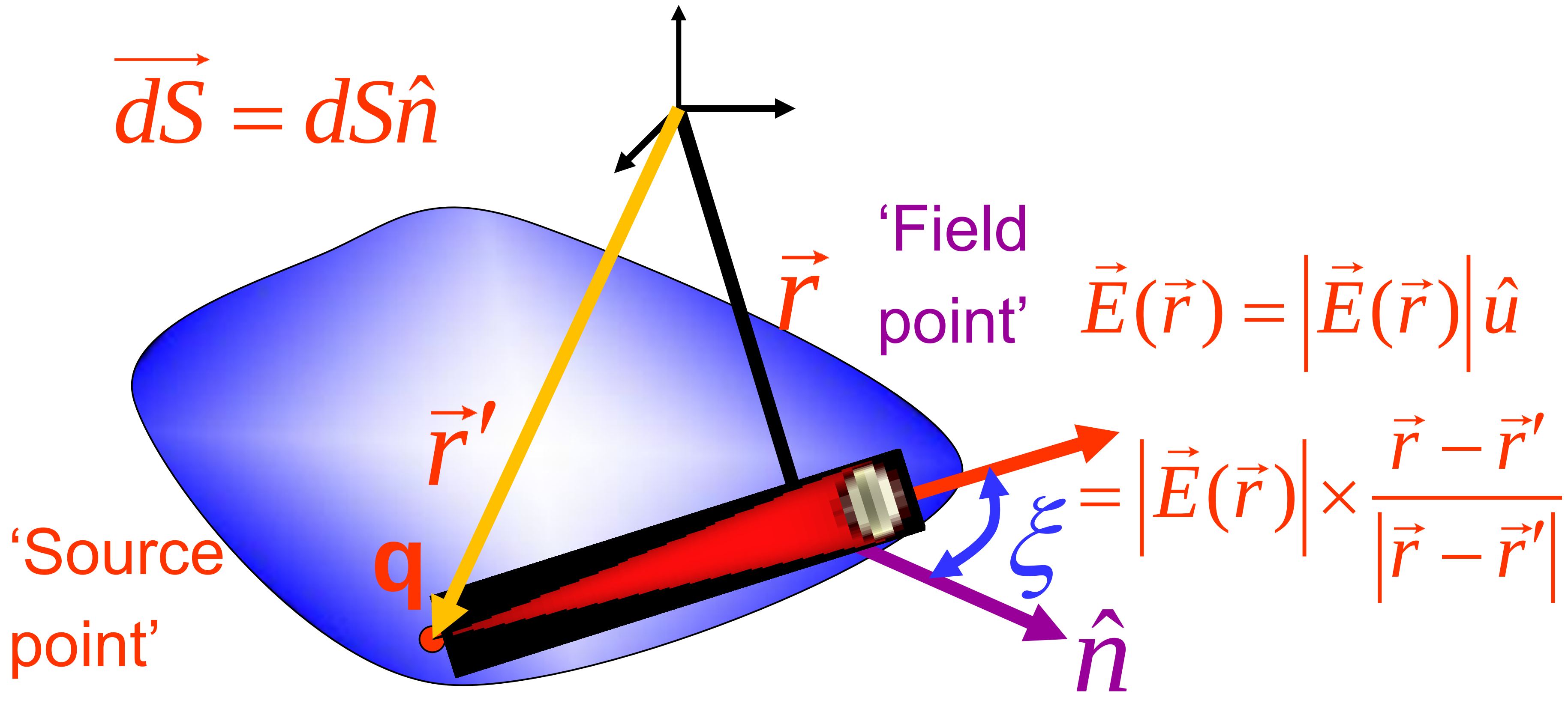
$\mu \rightarrow 0 \quad \Rightarrow$ Coulomb.

Rest mass of the photon

$$\frac{1}{(\text{distance})^2}$$

Range of the Coulomb potential

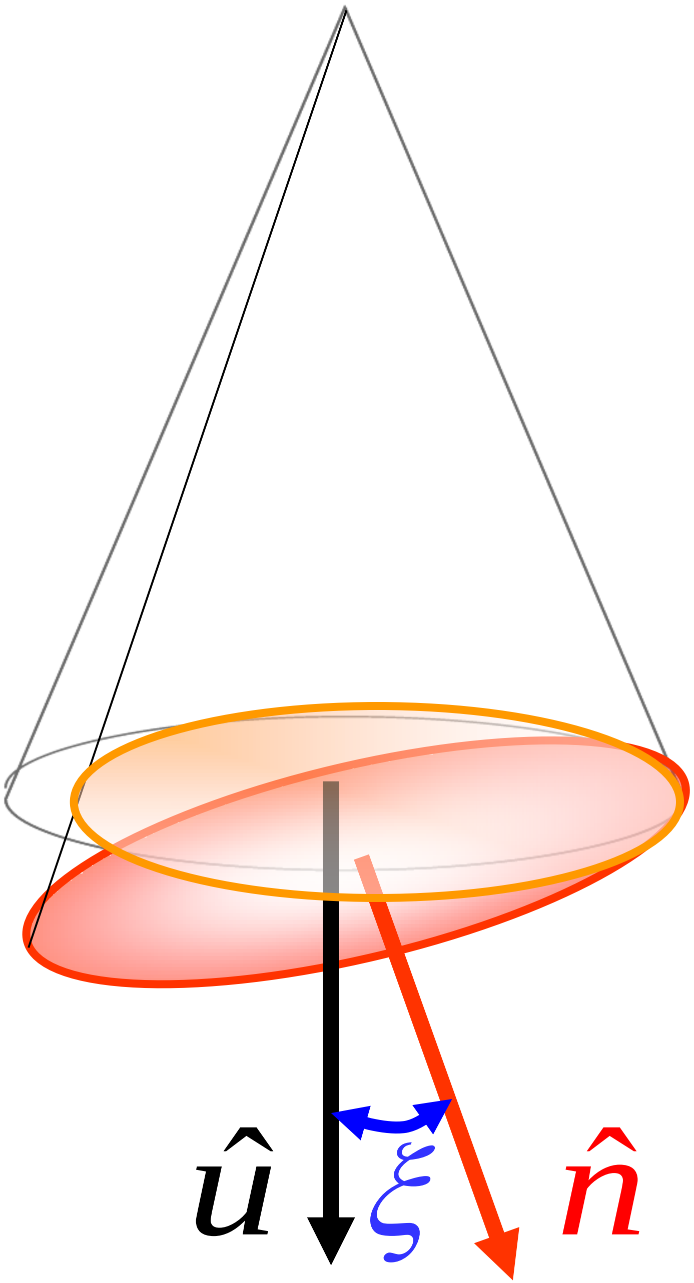
At what rate does the potential between two charges diminish with distance?



Consider the 'source' charge to be in a 3-dimensional space bounded by a closed surface having arbitrary shape.

Position vectors with prime: source points

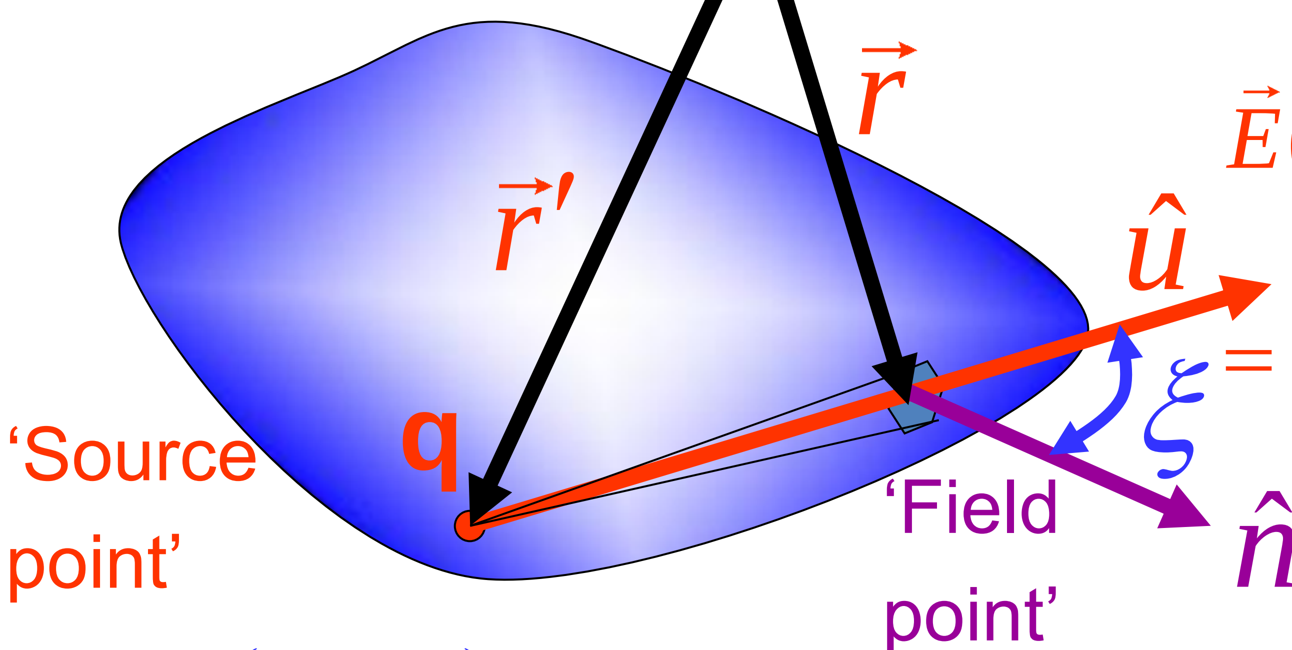
Without prime: field points



$$d\Omega = \left(\frac{\hat{u}}{\left|\vec{r}-\vec{r}'\right|^2}\right) \bullet \overrightarrow{dS}$$

$$= \left(\frac{1}{\left|\vec{r}-\vec{r}'\right|^2}\right) dS \cos\xi$$

$$\overrightarrow{dS} = dS\hat{n}$$



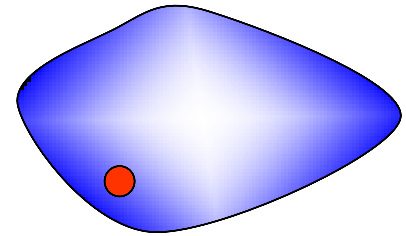
$$\vec{E}(\vec{r}) = \left|\vec{E}(\vec{r})\right|\hat{u} = \left|\vec{E}(\vec{r})\right| \times \frac{\vec{r}-\vec{r}'}{\left|\vec{r}-\vec{r}'\right|}$$

$$d\Omega = \left(\frac{\hat{u}}{\left|\vec{r}-\vec{r}'\right|^2}\right) \bullet \overrightarrow{dS} = \left(\frac{1}{\left|\vec{r}-\vec{r}'\right|^2}\right) dS\cos\xi$$

$$dS\cos\xi = d\Omega\left|\vec{r}-\vec{r}'\right|^2$$

Independent of shape!

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oiint \left(\frac{q}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}'}{\left|\vec{r}-\vec{r}'\right|^3} \right) \bullet \overrightarrow{dS}$$



$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oiint \left(\frac{q}{4\pi\varepsilon_0} \frac{\hat{u}}{\left|\vec{r}-\vec{r}'\right|^2} \right) \bullet \overrightarrow{dS}$$

$$dS \cos\xi = d\Omega \left|\vec{r}-\vec{r}'\right|^2$$

Independent of shape!

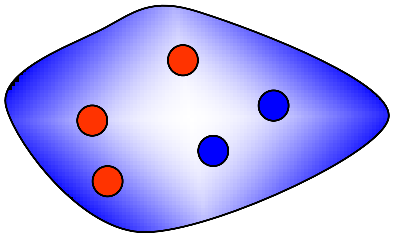
$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oiint \left(\frac{q}{4\pi\varepsilon_0} \frac{dS \cos\xi}{\left|\vec{r}-\vec{r}'\right|^2} \right)$$

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oiint \left(\frac{q}{4\pi\varepsilon_0} \frac{d\Omega \left|\vec{r}-\vec{r}'\right|^2}{\left|\vec{r}-\vec{r}'\right|^2} \right)$$

$$= \frac{q}{\varepsilon_0}$$

Also, the result is completely independent of just where inside the arbitrary region is the charge placed!

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{q_{inside}}{\varepsilon_0}$$



Independent of shape!

The result is completely independent of just where inside the arbitrary region the charge is placed!

Hence principle of linear superposition must hold!

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{q_{total\ \text{charge}\ inside}}{\varepsilon_0}$$

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{q_{total\ \text{charge}\ inside}}{\varepsilon_0}$$

$$= \frac{\sum_i q_{i,\ inside}}{\varepsilon_0}$$

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{\iiint \rho(\vec{r}\,')d^3\vec{r}\,'}{\varepsilon_0}$$

Gauss' divergence theorem

$$\iiint \vec{\nabla} \bullet \vec{E}(\vec{r})d^3\vec{r} = \frac{\iiint \rho(\vec{r}\,')d^3\vec{r}\,'}{\varepsilon_0}$$

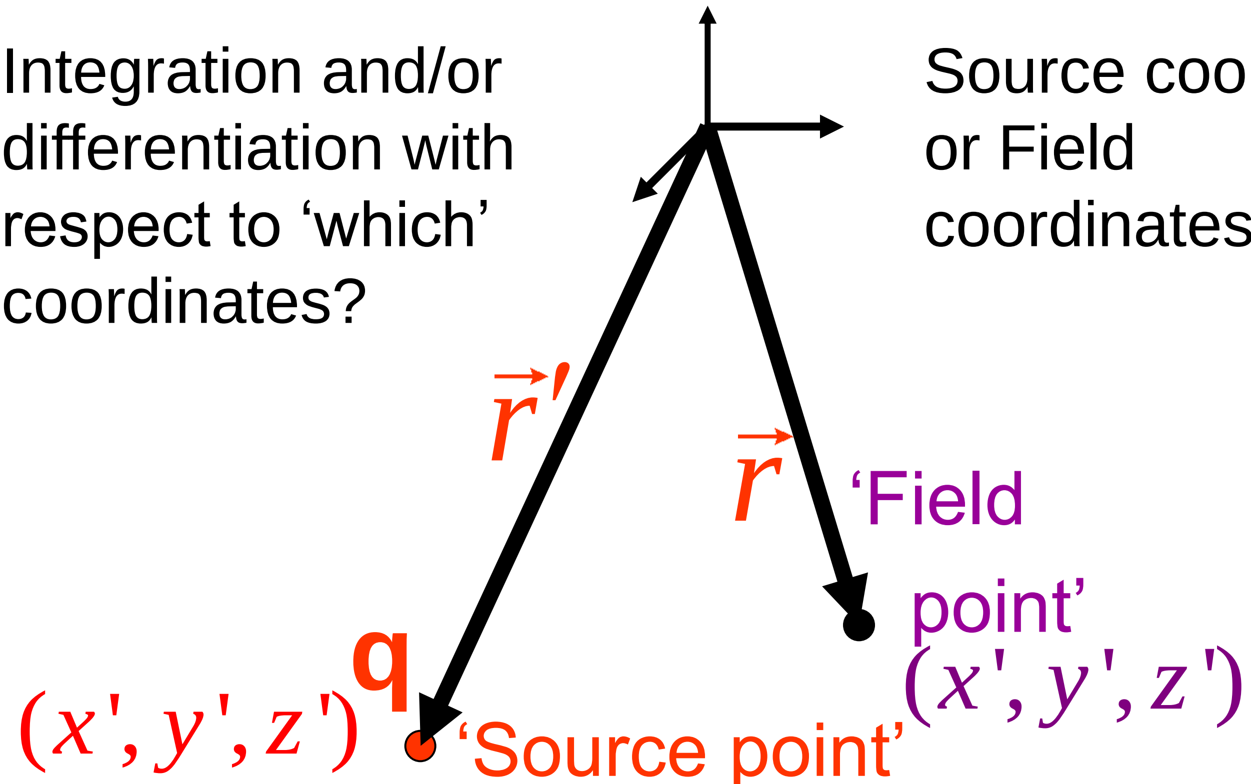
Differential and Integral forms of Gauss' law.

Here, $\vec{r}$ and $\vec{r}\,'$ are dummy labels; they get integrated out.

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

Integration and/or differentiation with respect to 'which' coordinates?

Source coordinates, or Field coordinates?



$$\frac{\vec{r}-\vec{r}'}{\left|\vec{r}-\vec{r}'\right|}$$

$$\vec{\nabla}' = \hat{e}_x \frac{\partial}{\partial x'} + \hat{e}_y \frac{\partial}{\partial y'} + \hat{e}_z \frac{\partial}{\partial z'}$$

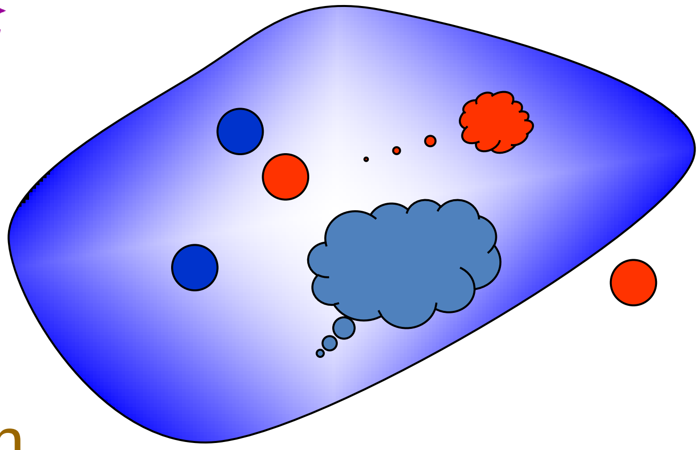
$$\vec{\nabla} = \hat{e}_x \frac{\partial}{\partial x} + \hat{e}_y \frac{\partial}{\partial y} + \hat{e}_z \frac{\partial}{\partial z}$$

Primed/Unprimed variables:

Integration/differentiation with respect to source/field coordinates

$$\iiint \vec{\nabla} \bullet \vec{E}(\vec{r}) d^3\vec{r} = \frac{\iiint \rho(\vec{r}) d^3\vec{r}}{\varepsilon_0}$$

$$= \oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS}$$



The result is completely independent of :

- shape of the region.
- where the charge/charges of charge-distributions is/are located,

- and also irrespective of these charge distributions being in any state of motion.

– as long as they remain inside the region under our consideration.

Continuous charge distributions:

charge density $\rho(\vec{r}) = \lim_{\delta V \to 0} \frac{\delta q}{\delta V}$

$$q = \iiint \rho(\vec{r}) d^3\vec{r}$$

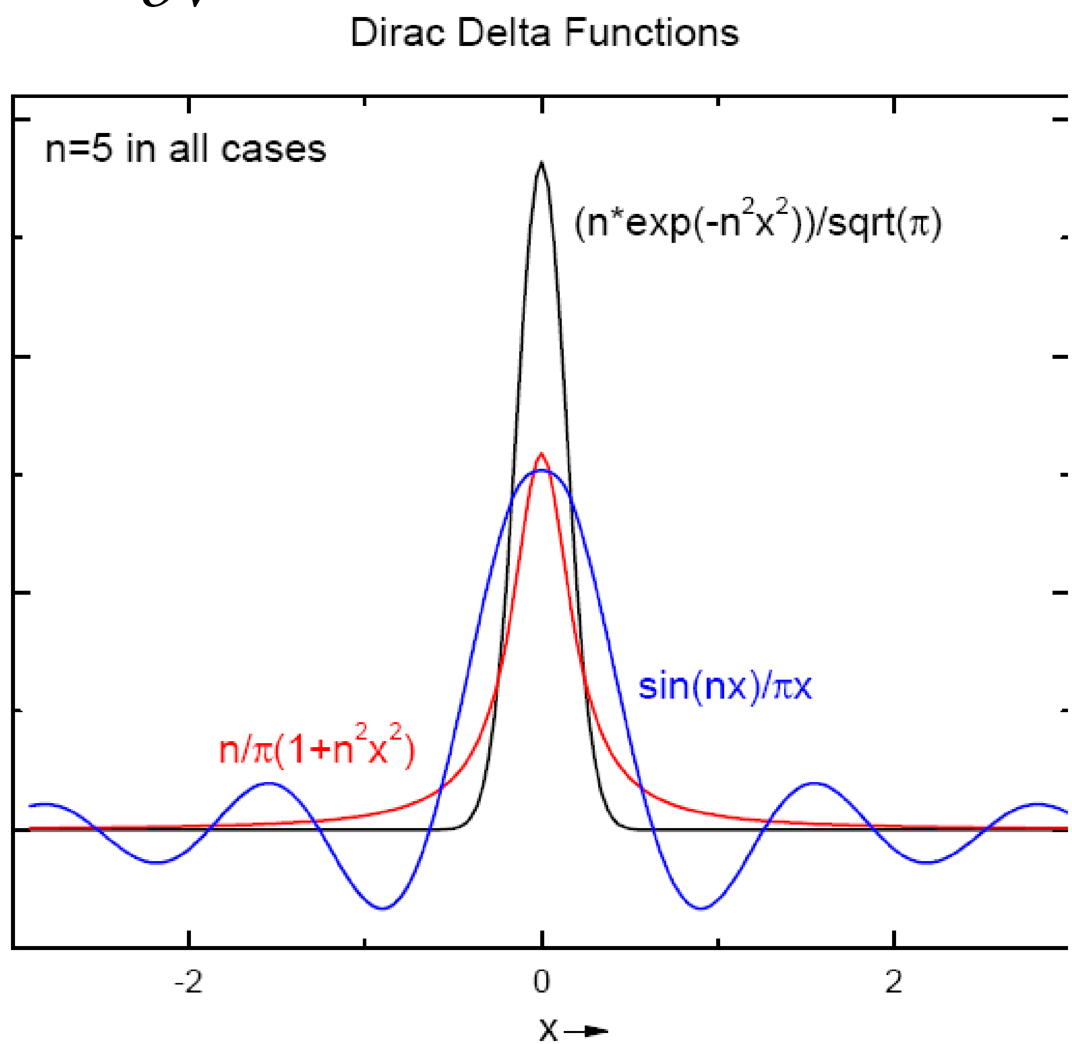
$$f(0) = \int_{-\infty}^{+\infty} f(x)\delta(x)dx$$

$$f(a) = \int_{-\infty}^{+\infty} f(x)\delta(x-a)dx$$

$$1 = \int_{-\infty}^{+\infty} \delta(x-a)dx$$

$\delta(x-a)$ has a spike at x=a

DIRAC
δ
'function'



$$\iiint \vec{\nabla} \bullet \vec{E}(\vec{r}) d^3\vec{r} = \frac{\iiint \rho(\vec{r}) d^3\vec{r}}{\varepsilon_0}$$

$$= \oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS}$$

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

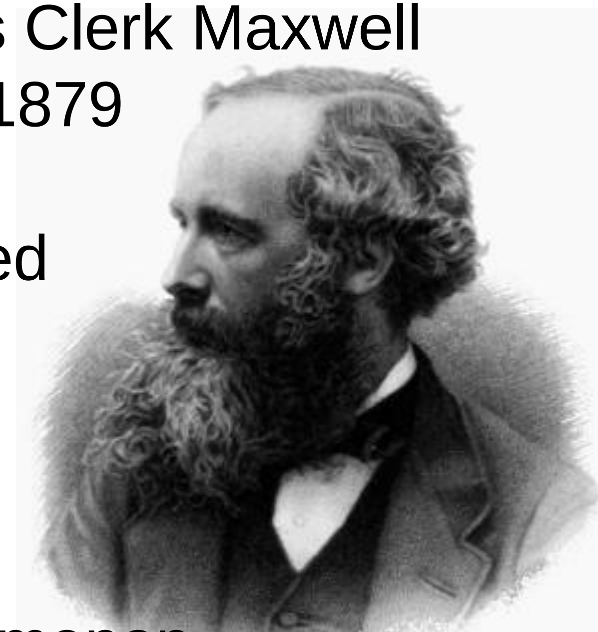
Integral and Differential form of Gauss' law:

First Equation in 'Maxwell's Equations'

Carl Friedrich Gauss formulated the law in 1835; published in 1867

James Clerk Maxwell 1831-1879

Showed that light is EM phenomenon



We shall take a break here…….

Questions ?

Comments ?

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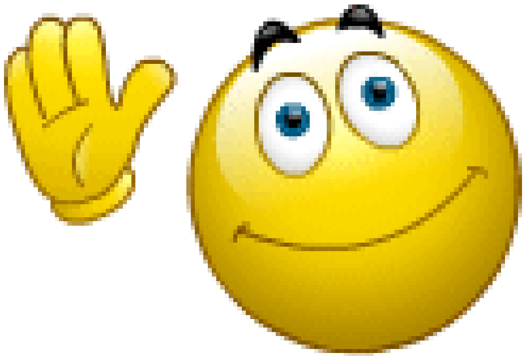

Next: L32

Unit 10 – Oersted-Ampere-Maxwell law

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STiCM Lecture 32

Unit 10 : **Classical Electrodynamics**

Oersted-Ampere-Maxwell Law

How shall we write the electric field due to a point charge as gradient of a scalar function?

$$\vec{E}(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]=$$

$$=\left(\hat{e}_x\frac{\partial}{\partial x}+\hat{e}_y\frac{\partial}{\partial y}+\hat{e}_z\frac{\partial}{\partial z}\right)\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-1/2}$$

$$=\hat{e}_x\frac{\partial}{\partial x}\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-1/2}+\hat{e}_y\frac{\partial}{\partial y}[..]^{-1/2}+\hat{e}_z\frac{\partial}{\partial z}[..]^{-1/2}$$

$$=\hat{e}_x(-\frac{1}{2})\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2}\left\{\frac{\partial}{\partial x}\left[(x-x')^2+(y-y')^2+(z-z')^2\right]\right\}+\ldots+\ldots$$

$$=\hat{e}_x(-\frac{1}{2})\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2}\left[2(x-x')\right]+\ldots+\ldots$$

$$\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]=-\frac{\vec{r}-\vec{r}'}{\left\{|\vec{r}-\vec{r}'|^2\right\}^{3/2}}=-\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right] = -\frac{\vec{r}-\vec{r}'}{\left\{|\vec{r}-\vec{r}'|^2\right\}^{3/2}} = -\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$= -\frac{q}{4\pi\varepsilon_0}\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

'FIELD',
as
negative
gradient
of
'POTENTIAL'

Curl of gradient is identically zero.

The electric field is conservative.

$$\vec{E}(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$= -\frac{q}{4\pi\varepsilon_0}\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$\vec{\nabla}\times\vec{E}(\vec{r}) = \vec{0}$$

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E}(\vec{r}) = \vec{0}$$

$$\vec{\nabla} \bullet \left(-\vec{\nabla}\phi\right) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\vec{\nabla} \bullet \vec{\nabla}\phi(\vec{r}) = \nabla^2\phi(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0}$$

Poisson's equation

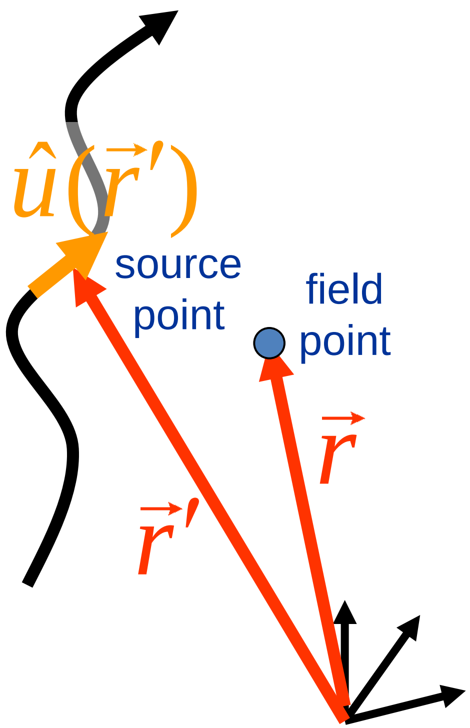
Siméon Denis Poisson
1781-1840

"Life is good for only two things, discovering mathematics and teaching mathematics."

- Poisson

Magnetic field $\vec{B}(\vec{r})$ does not originate from magnetic ‘charges’ / ‘poles’

Electric charges, when in motion, constitute a ‘current’ which generates magnetic field.



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{dl\hat{u}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{\left|\vec{r}-\vec{r}\,'\right|^3}$$

$$= \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')d^3\vec{r}\,'}{\left|\vec{r}-\vec{r}\,'\right|^3}$$

Biot & Savart 1820

Empirical law, based on experimental observations.

The primary definition of the magnetic field

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{dl\hat{u}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$= \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')d^3\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

gives the field's divergence and curl

$$\vec{\nabla}\bullet\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

This is not hard to see by using elementary vector calculus. A useful result in this regard is the following:

$$\vec{\nabla}\bullet\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = 4\pi\delta(\vec{r}-\vec{r}')$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Stokes' theorem

$$\Rightarrow$$

$$\mu_0 \iint \vec{J} \bullet \overrightarrow{dS} = \iint \vec{\nabla} \times \vec{B} \bullet \overrightarrow{dS} = \oint \vec{B} \bullet \overrightarrow{dl}$$

$$\Rightarrow$$

$$\mu_0 I = \oint \vec{B} \bullet \overrightarrow{dl}$$

Oersted-Ampere's law

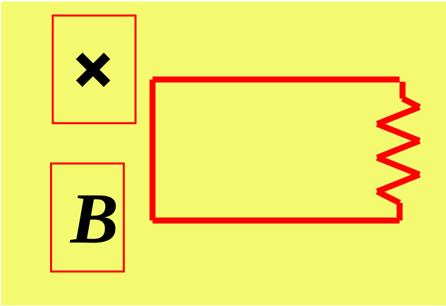
Source of electromotive force.

What is it that can have an influence on an electric charge?

- Electric field generated by another charge.

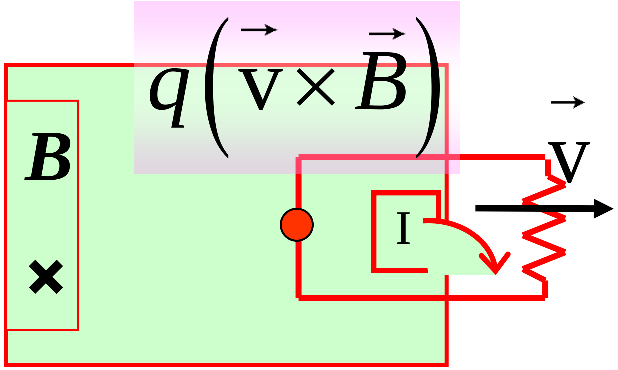
- An influence due to a changing magnetic field.

Loop : Stationary Lorentz force predicts:



(a) Clockwise Current
(b) Counterclockwise Current
√(c) No Current

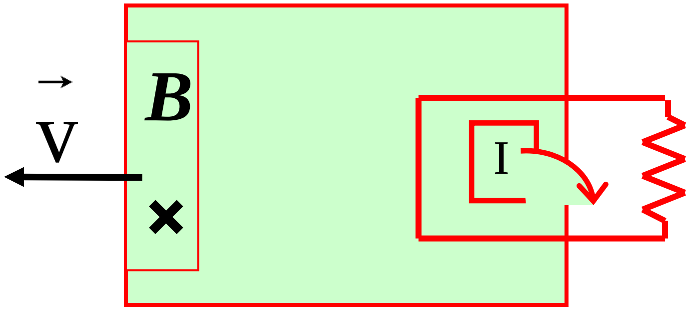
Loop : Dragged to the right.



Lorentz force predicts:

√(a) Clockwise Current
(b) Counterclockwise Current
(c) No Current

Faraday's experiments

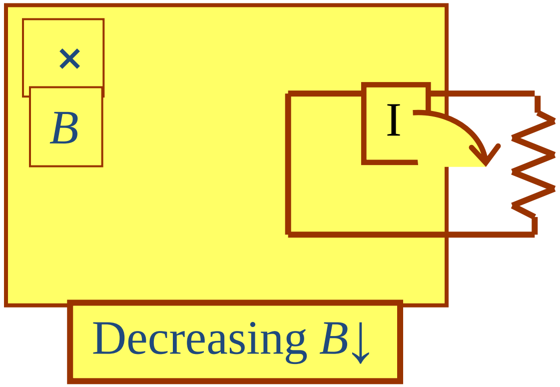


Loop held fixed; Magnet field dragged toward left.

NO Lorentz force. $q\left(\vec{v}\times\vec{B}\right)$

Current: identical!

Strength of B *decreased*.
Nothing is moving,
but still, current seen!!!



$$I \propto \frac{dB}{dt}$$

'Motion of Charged Particles
in
Electromagnetic Fields
and
Special Theory of Relativity'

P. Chaitanya Das, G. Srinivasa Murty, *K. Satish Kumar, T A. Venkatesh* and P.C. Deshmukh

Resonance, Vol. 9, Number 7, 77-85 (2004)

http://www.ias.ac.in/resonance/July2004/pdf/July2004Classroom3.pdf

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}'|}\right] = -\frac{q}{4\pi\varepsilon_0}\vec{\nabla}\left[\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$\iint\left(\vec{\nabla}\times\vec{E}\right)\bullet\overrightarrow{dS} = \iint\left(-\frac{\partial \vec{B}}{\partial t}\right)\bullet\overrightarrow{dS}$$

$$\oint \vec{E}\bullet\overrightarrow{dl} = -\frac{\partial}{\partial t}\iint \vec{B}\bullet\overrightarrow{dS} = -\frac{\partial \Phi_B}{\partial t};$$

Φ_B : magnetic flux crossing the surface

FARADAY – LENZ Law

Empirical laws of Classical Electrodynamics

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

Coulomb, Gauss

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday, Lenz

$$\oint \vec{E} \bullet \overrightarrow{dl} = -\frac{\partial \Phi_B}{\partial t}$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

No magnetic 'charges'/ 'monopoles'

$$\oiint \vec{B}(\vec{r}) \bullet \overrightarrow{dS} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Oersted, Ampere

$$\oint \vec{B} \bullet \overrightarrow{dl} = \mu_0 I_{enclosed}$$

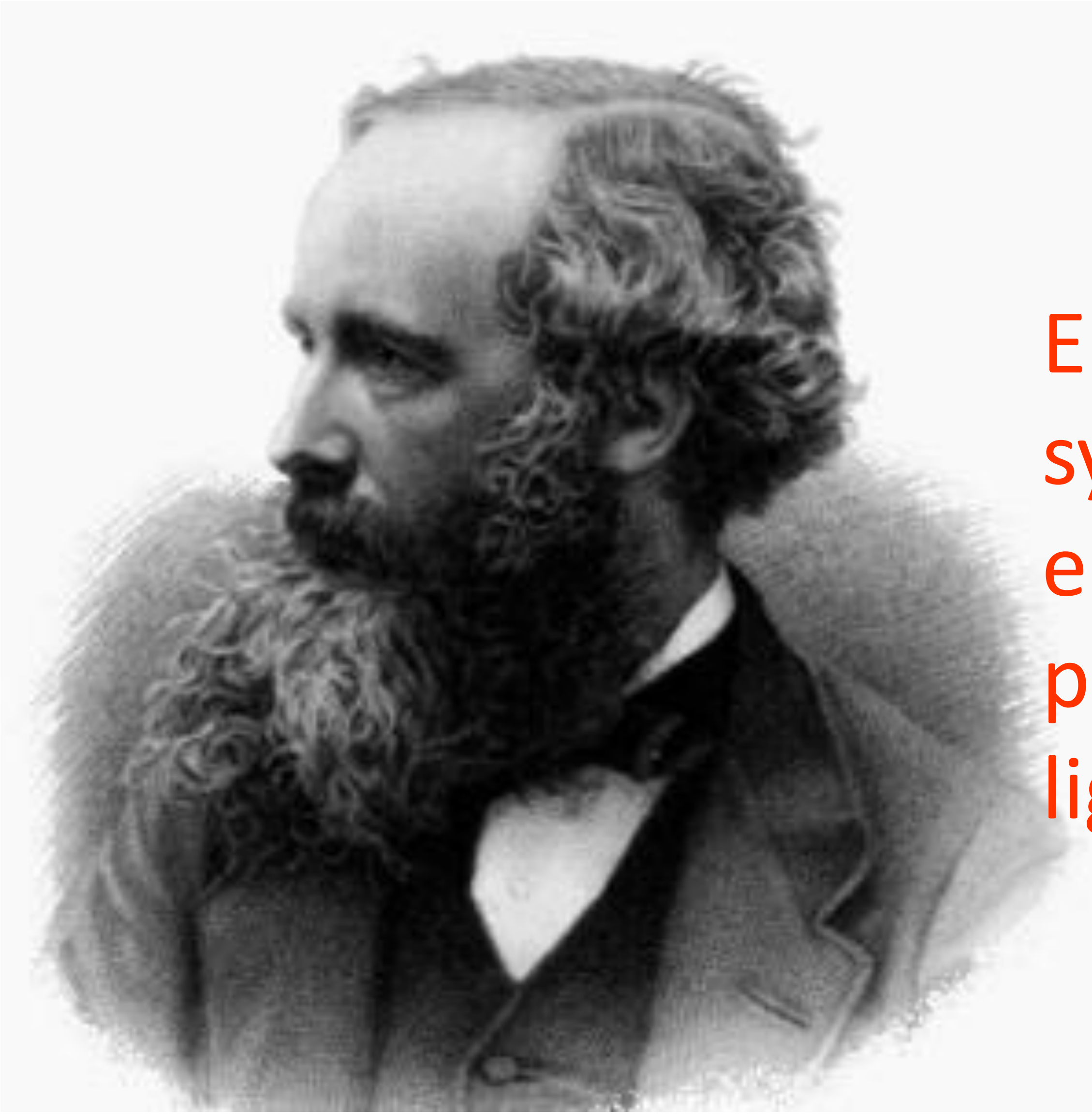
Empirical laws
of
Classical Electrodynamics

Charles
Coulomb
1736-1806

Carl Freidrich
Gauss
1777-1855

Andre Marie
Ampere
1775-1836

Michael
Faraday
1791-1867



Electrodynamics: synthesis of electromagnetic phenomena and light/optics.

James Clerk Maxwell
1831-1879

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Oersted, Ampere

$$\oint \vec{B} \bullet \overrightarrow{dl} = \mu_0 I_{enclosed}$$

$$\oint \vec{E} \bullet \overrightarrow{dl} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t} \iint \vec{B} \bullet \overrightarrow{dS}$$

← Faraday, Lenz

Maxwell added a term corresponding to changing electric flux, similar to the term for changing magnetic flux of Faraday-Lenz law

$$\oint \vec{B} \bullet \overrightarrow{dl} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint \vec{E} \bullet \overrightarrow{dS}$$

Oersted,Ampere - Maxwell

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

The equations of James Clerk Maxwell

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E} \bullet \overrightarrow{dl} = -\frac{\partial \Phi_B}{\partial t}$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

$$\oiint \vec{B}(\vec{r}) \bullet \overrightarrow{dS} = 0$$

SYMMETRY!

SYMMETRY!

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\oint \vec{B} \bullet \overrightarrow{dl} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint \vec{E} \bullet \overrightarrow{dS}$$

Take the curl of the following vector: $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=-\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)$$

Work this out, it is easy : $\boxed{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=\vec{\nabla}\left(\vec{\nabla}\bullet\vec{E}\right)-\left(\vec{\nabla}\bullet\vec{\nabla}\right)\vec{E}}$

$$\vec{\nabla}\left(\vec{\nabla}\bullet\vec{E}\right)-\left(\vec{\nabla}\bullet\vec{\nabla}\right)\vec{E}=-\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}\bullet\vec{E}\right)-\left(\vec{\nabla}\bullet\vec{\nabla}\right)\vec{E}=-\frac{\partial}{\partial t}\left(\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\overrightarrow{E}}{\partial t}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}\bullet\vec{E}\right)-\left(\vec{\nabla}\bullet\vec{\nabla}\right)\vec{E}=-\frac{\partial}{\partial t}\left(\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\left(\frac{\rho}{\varepsilon_0}\right)-\nabla^2\vec{E}=-\frac{\partial}{\partial t}\left(\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)=-\mu_0\frac{\partial\vec{J}}{\partial t}-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

In vacuum: $\nabla^2\vec{E}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$

Likewise (show!): $\nabla^2\vec{B}=\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$

Second-order homogeneous partial differential equation

Wave equations

$$\mathrm{v}=\sqrt{\frac{1}{\mu_0\varepsilon_0}}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\frac{\omega}{k} = \mathrm{v} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c$$

$$\vec{E}(\vec{r}, t) = \left\{ \left| \vec{E}_0 \right| \hat{u} \right\} e^{i\left(\vec{k} \bullet \vec{r} - \omega t\right)}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c} \hat{k} \times \vec{E}(\vec{r}, t)$$

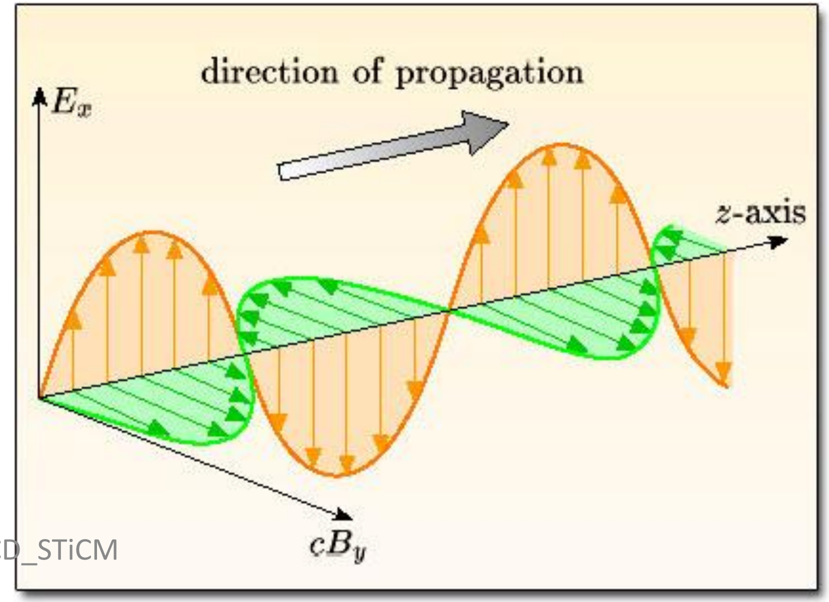
$$\hat{u} \bullet \hat{k} = 0$$

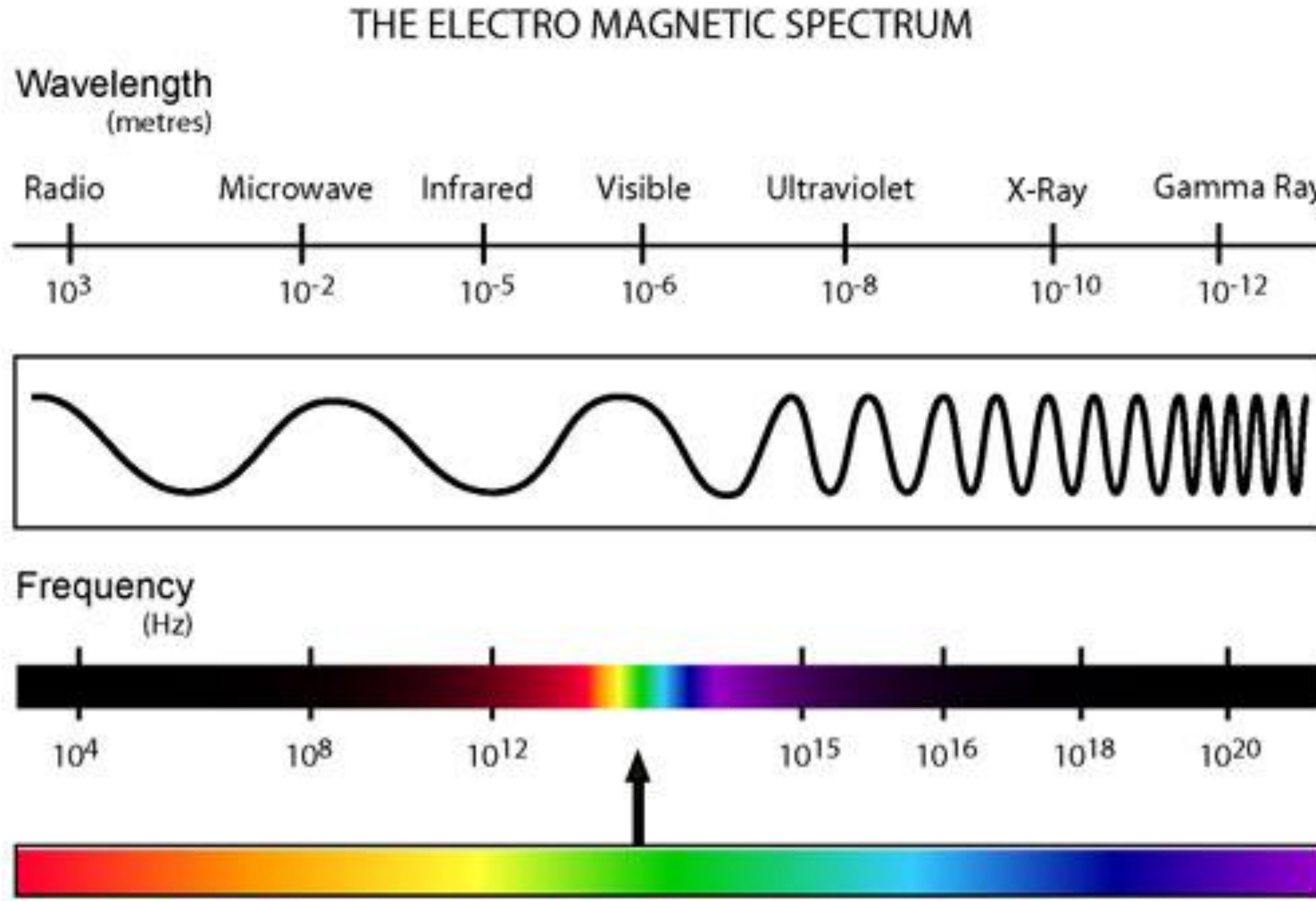
$$v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c = 2.9979 \times 10^8 \, m/s$$

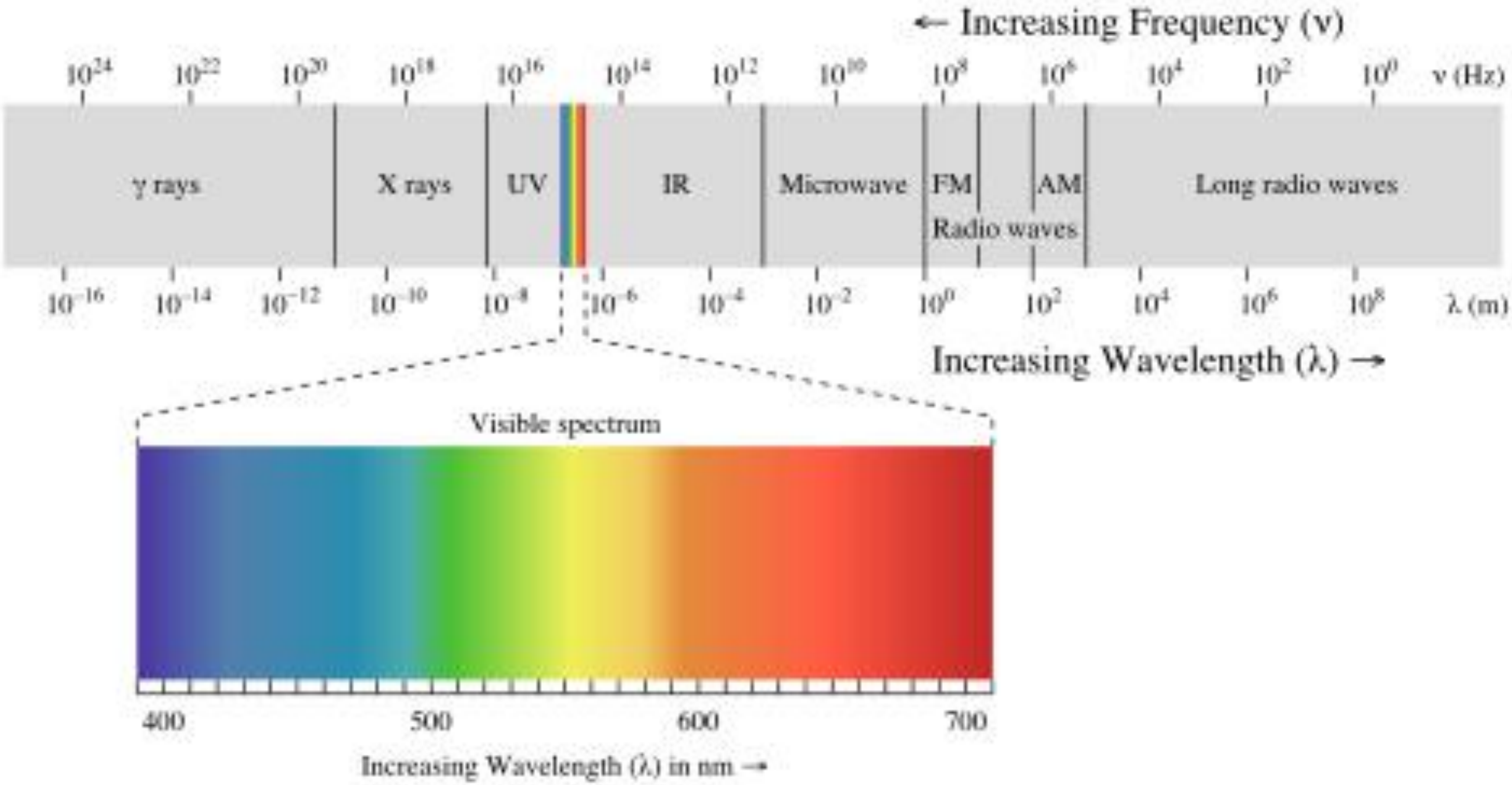
Maxwell observed that v obtained as above agreed with the speed of light.

He therefore concluded:

"light is an electromagnetic disturbance propagated through the field according to electromagnetic laws"




THE ELECTRO MAGNETIC SPECTRUM
Wavelength
(metres)
Radio
Microwave
Infrared
Visible
Ultraviolet
X-Ray
Gamma Ray
10^3
10^{-2}
10^{-5}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Frequency
(Hz)
10^4
10^8
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}


← Increasing Frequency (ν)
10^{24} 10^{22} 10^{20} 10^{18} 10^{16} 10^{14} 10^{12} 10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Radio waves
Long radio waves
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400
500
600
700
Increasing Wavelength (λ) in nm →

We shall take a break here…….

Questions ? Comments ?

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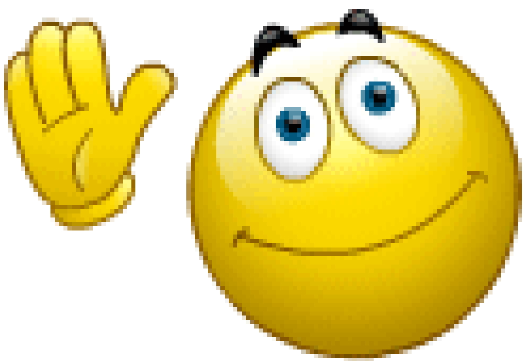
pcdeshmukh@iitmandi.ac.in



Next: L33

Unit 10 – Electrodynamics & STR

STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 33

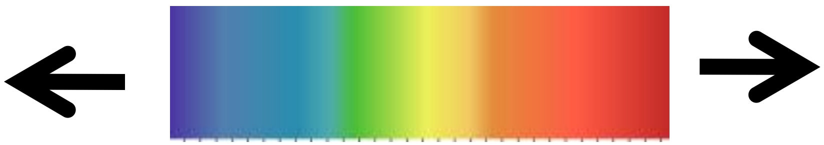
Unit 10 : **Classical Electrodynamics**

Electrodynamics & Special Theory of Relativity

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

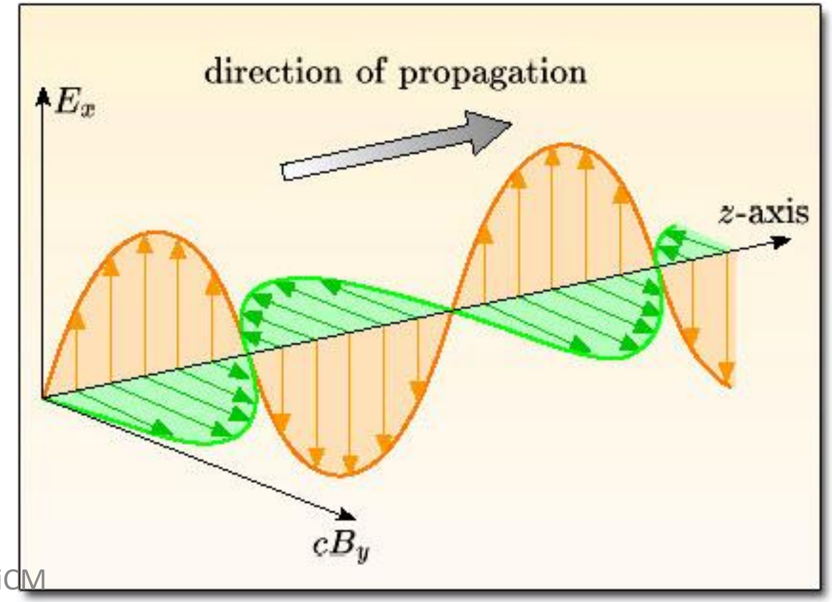
$$\frac{\omega}{k} = \mathrm{v} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c$$

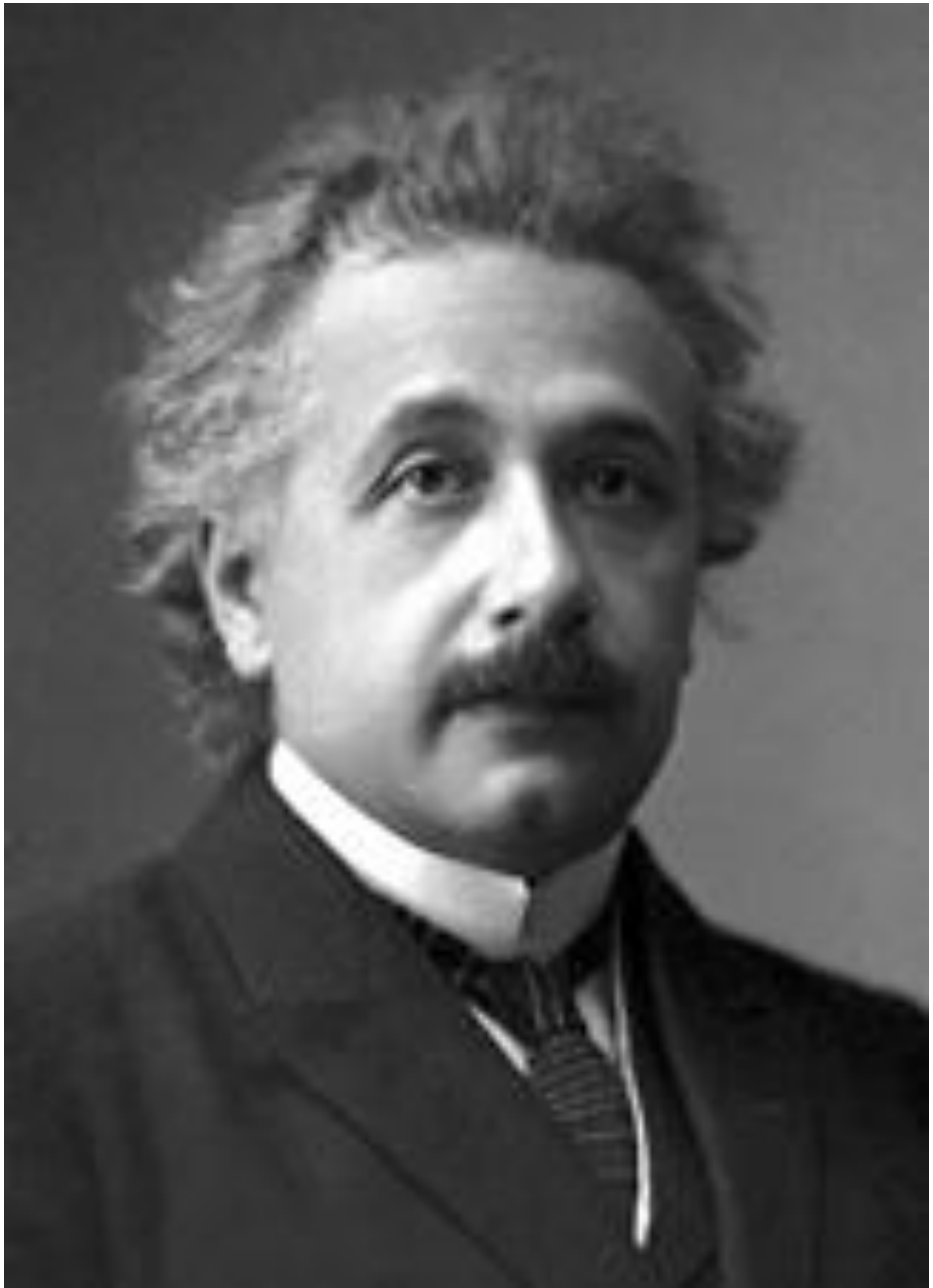


$$\vec{E}(\vec{r},t) = \left\{ \left| \vec{E}_0 \right| \hat{u} \right\} e^{i\left(\vec{k} \bullet \vec{r} - \omega t\right)}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c} \hat{k} \times \vec{E}(\vec{r},t)$$

$$\hat{u} \bullet \hat{k} = 0$$





Electrodynamics & STR

The special theory of relativity is intimately linked to the general theory of electrodynamics.

Both of these topics belong to ‘Classical Mechanics’.

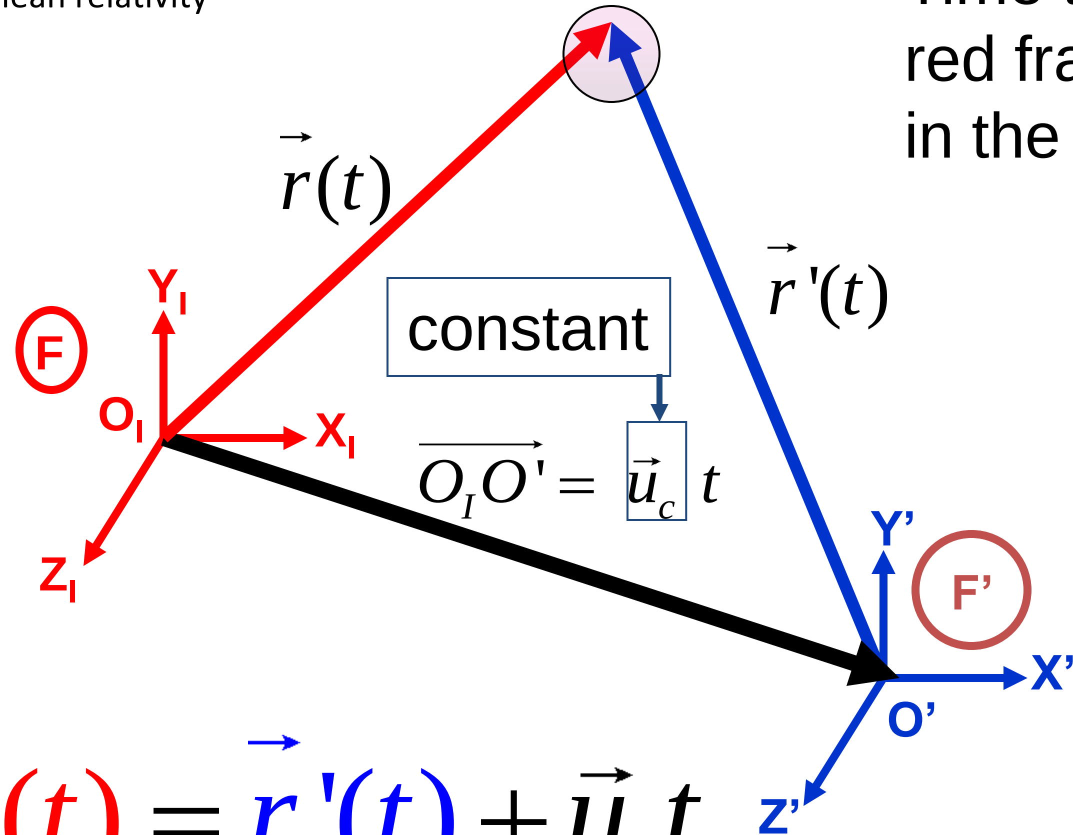
Albert Einstein
1879 - 1955

Galilean Relativity

What is the velocity of the oncoming car?

… relative to whom?

Time t is the same in the red frame and in the blue frame.



$$\vec{r}(t) = \vec{r}\,'(t) + \vec{u}_c t$$

$$\frac{d\vec{r}}{dt} - \vec{u}_c = \frac{d\vec{r}\,'}{dt}$$

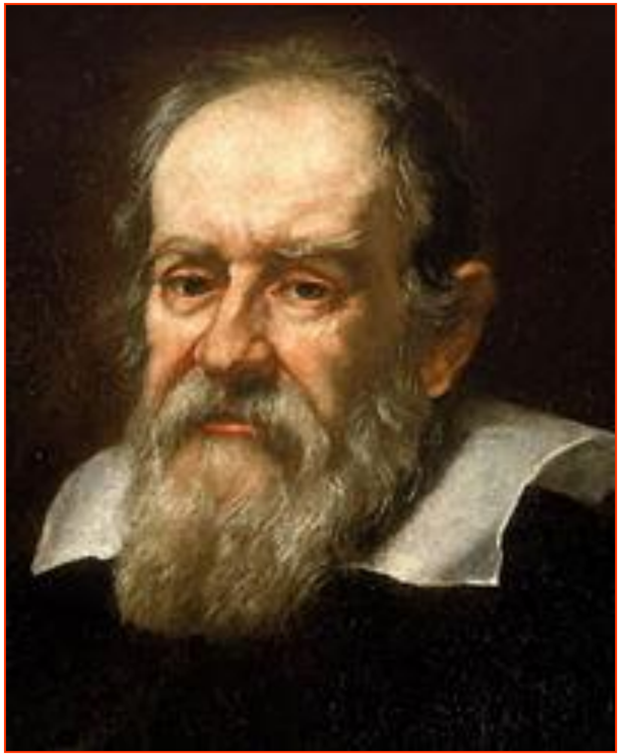
$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \vec{u}_c$$

What would happen if the object of your observations is light?

$$\frac{d\vec{r}}{dt} - \vec{u}_c = \frac{d\vec{r}\,'}{dt}$$

Speed of light ?

Galilean & Lorentz Transformations.
Special Theory of Relativity.



Galileo Galilei
1564 - 1642

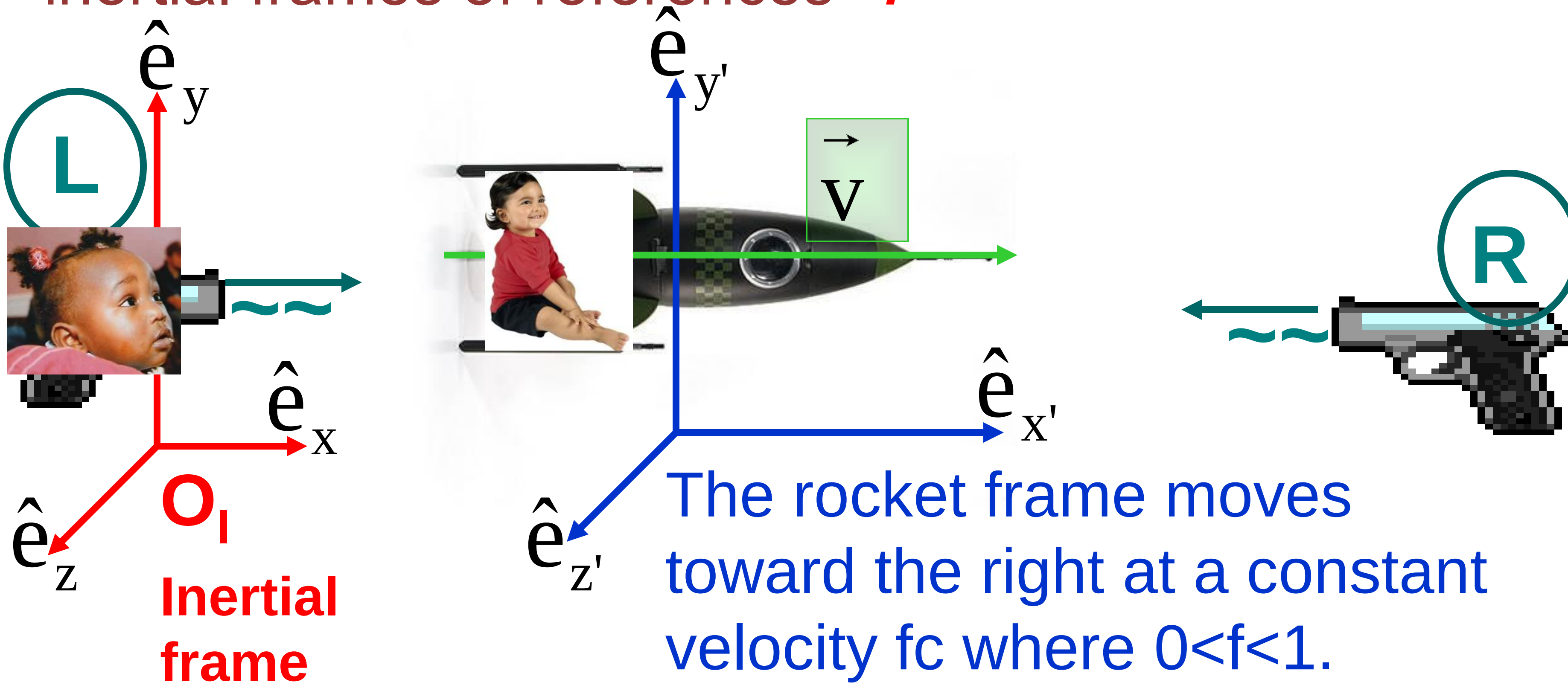
Hendrik Antoon Lorentz
1853-1928

Smoking is injurious to health!

Albert Einstein
1879-1955

Just what does it mean to say that

"Light (EM waves) travels at the constant speed in all inertial frames of references" ?

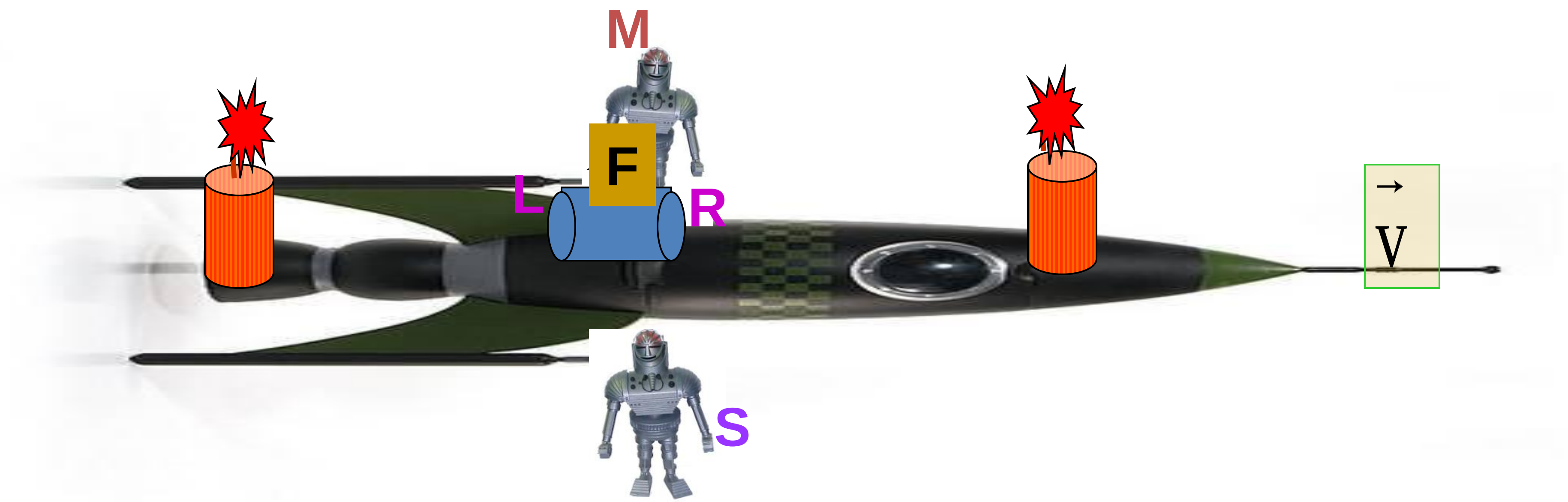


The rocket frame moves toward the right at a constant velocity fc where 0<f<1.

COUNTER-INTUITIVE ?

Speed of light in a vacuum is a universal constant for all observers regardless of the motion of the observer or of the light source

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$



(1)S detects both the flashes simultaneously.

(2)Light from both explosions travels at equal speed toward S/M.

(3) M would expect his sensor to record light from the right-cracker, *before* it senses light from the one on our left side.

Events that seem SIMULTANEOUS

to the stationary observer do not seem

to be so to the moving observer – who

also is in an inertial frame *!*

So, let us, in all humility, reconsider our notion of TIME and SPACE !

1. Maxwell's equations are correct in all inertial frames of references.

2. Maxwell's formulation predicts : EM waves travel at the speed $c = \sqrt{\dfrac{1}{\mu_0 \varepsilon_0}}$.

3. HENCE, light (EM waves) travels at the constant speed in all inertial frames of references.

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

Notion of TIME itself would need to change

Einstein was clever enough, & bold enough, to stipulate just that!

What happens to our notion of space & time ?

$$speed = \frac{\text{distance}}{time}$$

Time Dilation

Length Contraction

Hendrik Antoon Lorentz
1853-1928

1902 Nobel Prize in Physics

"in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena"

Pieter Zeeman
1865-1943

Lorentz contraction!

Lorentz moving up!

Lorentz moving to right!

http://www.bun.kyoto-u.ac.jp/~suchii/lorentz.tr.html

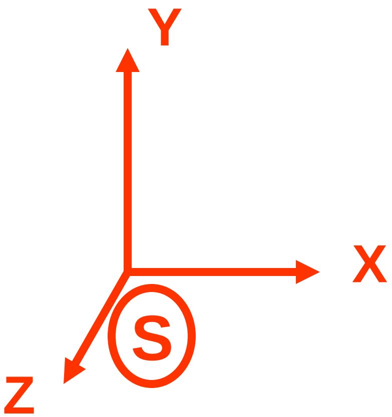
LORENTZ transformations (x,y,z,t) to (x',y',z',t')

Requirements:

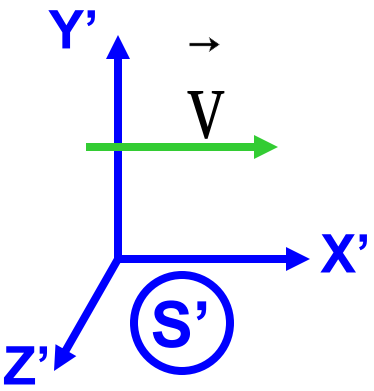
Ensure that speed of light is same in all inertial frames of references.

Transform both space and time coordinates.

Transformation equations must agree with Galilean transformations when v<<<c.



Origins O and O' of the two frames S and S' coincide at $t=0$ and $t'=0$.



$$x' = \gamma (x - vt) \qquad x = \gamma (x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \qquad t = \gamma \left(t' + \frac{vx'}{c^2} \right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

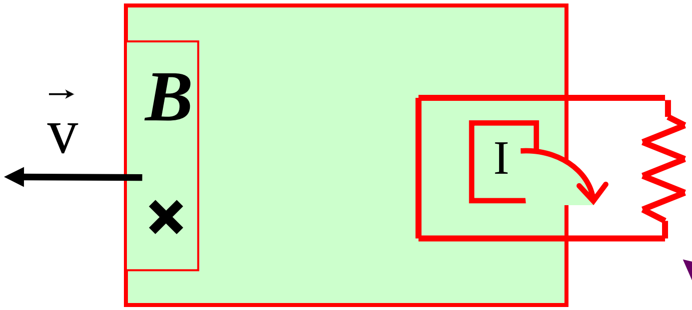
$$= \frac{1}{\sqrt{1 - \beta^2}}$$

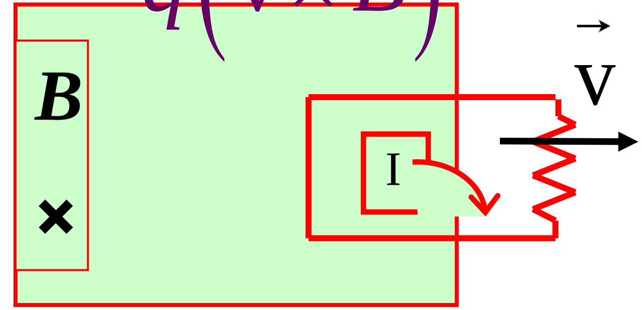
Note : $\gamma \rightarrow 1$ as $v \rightarrow 0$.

Lorents transformations transform the space-time coordinates of ONE EVENT.

Faraday's experiments

Reason here… $q\left(\vec{v}\times\vec{B}\right)$

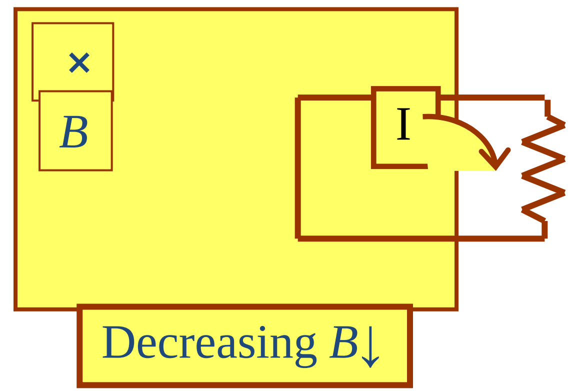




Current: identical!

Reason here…

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$



Strength of B *decreased*.
Nothing is moving,
but still, current seen!!!

$$I \propto \frac{dB}{dt}$$

Einstein:
Special Theory of Relativity

“So the "flux rule" that the emf in a circuit is equal to the rate of change of the magnetic flux through the circuit applies whether the flux changes because the field changes or because the circuit moves (or both).... Yet in our explanation of the rule we have used **two completely distinct laws** for the two cases : $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ for "field changes“,

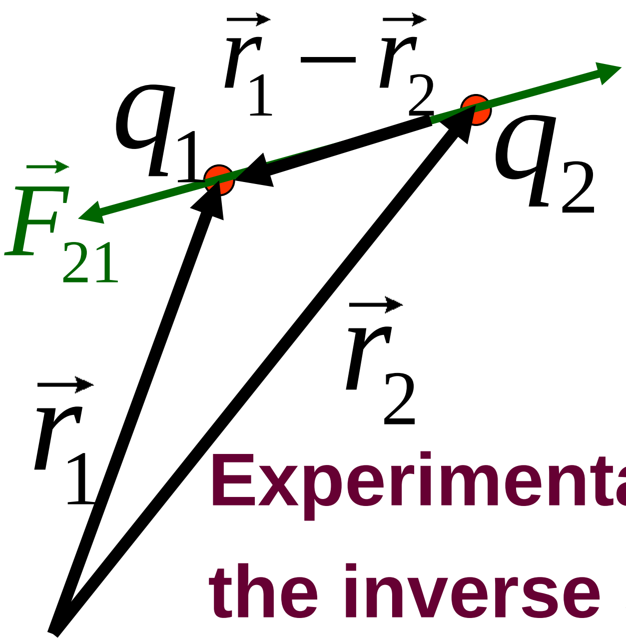
and $\vec{v} \times \vec{B}$ for "circuit moves" .

We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of *two different phenomena*.”

– ***Richard P. Feynman***,

The Feynman Lectures on Physics

We began with simple,
empirical foundations of classical electrodynamics



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}$$

Experimental recognition of the inverse square law:

Priestly (1767)

Robinson (1769)

Cavendish (1771)

Coulomb (1785)

Coulomb also advanced the view that negative charges exist, that they did not merely represent absence of a positive charge.

Rest mass of the photon

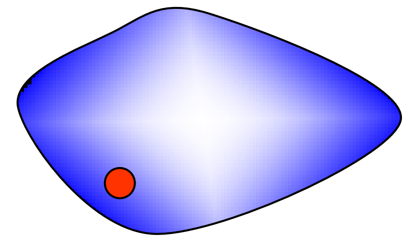
$$\frac{1}{(\text{distance})^2}$$

Range of the Coulomb potential

At what rate does the potential between two charges diminish with distance?

$$\oint\!\!\oint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oint\!\!\oint \left(\frac{q}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}'}{\left|\vec{r}-\vec{r}'\right|^3} \right) \bullet \overrightarrow{dS}$$

$$\oint\!\!\oint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oint\!\!\oint \left(\frac{q}{4\pi\varepsilon_0} \frac{\hat{u}}{\left|\vec{r}-\vec{r}'\right|^2} \right) \bullet \overrightarrow{dS}$$



$$dS\cos\xi = d\Omega\left|\vec{r}-\vec{r}'\right|^2$$

Independent of shape!

$$\oint\!\!\oint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oint\!\!\oint \left(\frac{q}{4\pi\varepsilon_0} \frac{dS\cos\xi}{\left|\vec{r}-\vec{r}'\right|^2} \right)$$

$$\oint\!\!\oint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \oint\!\!\oint \left(\frac{q}{4\pi\varepsilon_0} \frac{d\Omega\cancel{\left|\vec{r}-\vec{r}'\right|^2}}{\cancel{\left|\vec{r}-\vec{r}'\right|^2}} \right)$$

$$= \frac{q}{\varepsilon_0}$$

Also, the result is completely independent of just where inside the arbitrary region is the charge placed!

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}$$

Oersted,
Ampere
Biot-Savart

$$\oint\vec{B}\bullet\overrightarrow{dl}=\mu_0 I_{enclosed}$$

$$\oint\vec{E}\bullet\overrightarrow{dl}=-\frac{\partial\Phi_B}{\partial t}=-\frac{\partial}{\partial t}\iint\vec{B}\bullet\overrightarrow{dS}$$

← Faraday,
Lenz

Maxwell added a term corresponding to changing electric flux, similar to the term for changing magnetic flux of Faraday-Lenz law

$$\oint\vec{B}\bullet\overrightarrow{dl}=\mu_0 I_{enclosed}+\mu_0\varepsilon_0\frac{\partial}{\partial t}\iint\vec{E}\bullet\overrightarrow{dS}$$

Oersted,Ampere - Maxwell

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

The equations of James Clerk Maxwell

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\oiint \vec{E}(\vec{r}) \bullet \overrightarrow{dS} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E} \bullet \overrightarrow{dl} = -\frac{\partial \Phi_B}{\partial t}$$

$$\oiint \vec{B}(\vec{r}) \bullet \overrightarrow{dS} = 0$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

SYMMETRY!

SYMMETRY!

$$\oint \vec{B} \bullet \overrightarrow{dl} = \mu_0 I_{enclosed} +$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$+ \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint \vec{E} \bullet \overrightarrow{dS}$$

The equations of James Clerk Maxwell

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Changing magnetic field produces a rotational electric field.

$$\vec{\nabla} \bullet \vec{B} = 0$$

$$\mathrm{v} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c$$

c: constant.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Changing electric field produces a rotational magnetic field.

Maxwell's equations involve derivatives with respect to space and time, and they unify electro-magnetic phenomena and light/optics.

Space?

Time?

Feynman's observations!

Special Theory of Relativity (STR)

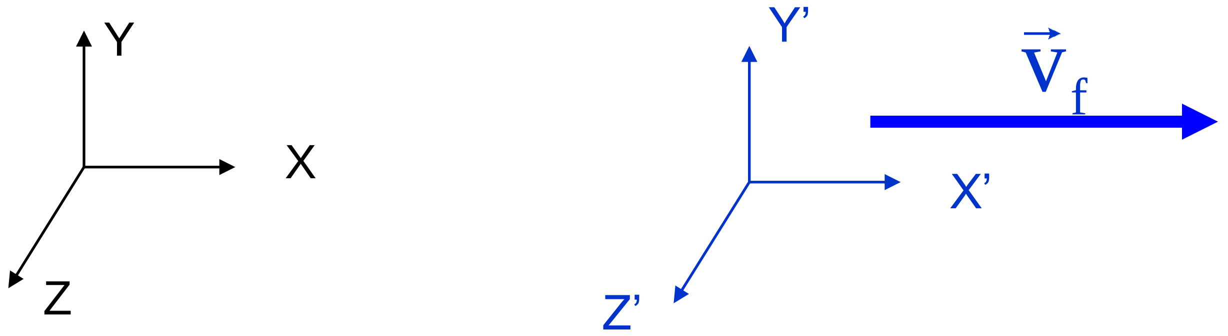
connects all this up.

$$\vec{F} = q\left[\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right]$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Charge particle dynamics observed in different INERTIAL frames of reference

Examine trajectories of charged particles in EM fields, as observed by two observers both in their respective inertial frames. S' moves with respect to S at a constant velocity $\vec{v}_f$ along the X-direction.



$$\frac{d\vec{p}}{dt} = \vec{F} \quad \text{where } \vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$\frac{d\vec{p}}{dt} = \vec{F} \quad \text{where } \vec{F} = q\left[\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right]$$

$$x, y, z, t \rightarrow x', y', z', t'$$

$$\vec{r} = \vec{r}(t); \qquad \boxed{\vec{r}\,' = \vec{r}\,'(t')}$$

$$\boxed{\left(\vec{E}, \vec{B}\right) \rightarrow \left(\vec{E}\,', \vec{B}\,'\right)}$$

$$\frac{d\vec{p}\,'}{dt'} = \vec{F}\,' \quad \text{where } \vec{F}\,' = q\left[\vec{E}\,' + \vec{\mathrm{v}}\,' \times \vec{B}\,'\right]$$

$$\boxed{\vec{r}\,' = \vec{r}\,'(t')}$$

We shall take a break here…….

Questions ?

Comments ?

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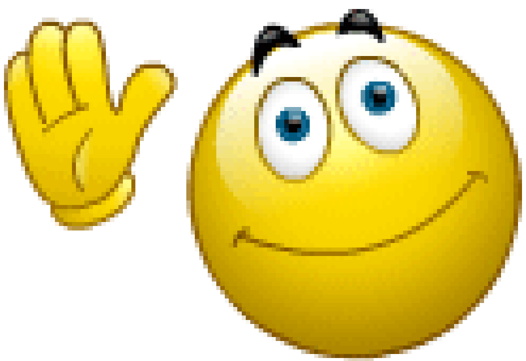

Next: L34

Unit 10 – Electrodynamics & STR

STiCM

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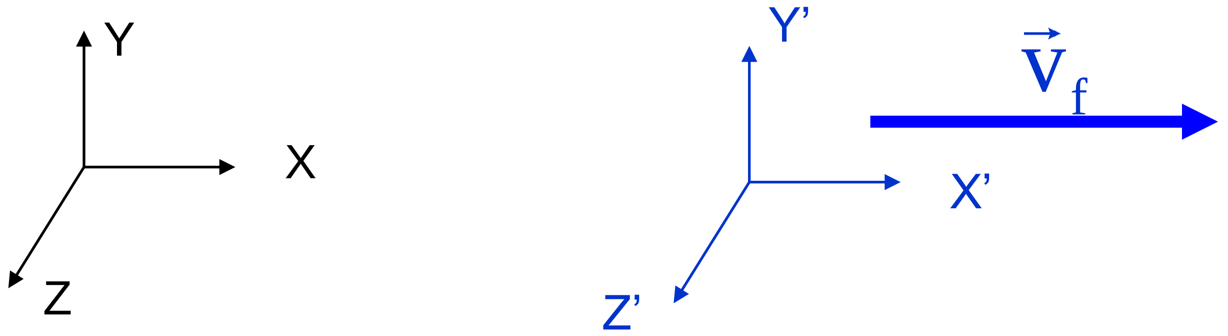
STiCM Lecture 34

Unit 10 : **Classical Electrodynamics**

Electrodynamics & Special Theory of Relativity

Examine trajectories of charged particles in EM fields, as observed by two observers both in their respective inertial frames. S‘ moves with respect to S at a constant velocity $\vec{v}_f$ along the X-direction.



$$\frac{d\vec{p}}{dt} = \vec{F} \quad \text{where } \vec{F} = q\left[\vec{E} + \vec{v}\times\vec{B}\right]$$

Speed of light: does not change…

…from one inertial frame of reference to another……

STR ⟷ ED

… it is 'time' that changes!

$$\frac{d\vec{p}}{dt} = \vec{F} \quad \text{where } \vec{F} = q\left[\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right]$$

$$x, y, z, t \rightarrow x', y', z', t'$$

$$\vec{r} = \vec{r}(t); \quad \boxed{\vec{r}\,' = \vec{r}\,'(t')}$$

$$\boxed{\left(\vec{E}, \vec{B}\right) \rightarrow \left(\vec{E}\,', \vec{B}\,'\right)}$$

$$\frac{d\vec{p}\,'}{dt'} = \vec{F}\,' \quad \text{where } \vec{F}\,' = q\left[\vec{E}\,' + \vec{\mathrm{v}}\,' \times \vec{B}\,'\right]$$

$$\boxed{\vec{r}\,' = \vec{r}\,'(t')}$$

$$x' = \gamma_f \left(x - \mathrm{v}_f t\right), \quad y' = y, \quad z' = z, \quad t' = \gamma_f \left(t - \frac{\mathrm{v}_f}{c^2} x\right)$$

where $\gamma_f = 1 \Big/ \sqrt{1 - \frac{\mathrm{v}_f^2}{c^2}}$

$$E'_x = E_x$$

$$E'_y = \gamma_f \left[E_y - \mathrm{v}_f B_z\right]$$

$$E'_z = \gamma_f \left[E_z - \mathrm{v}_f B_y\right]$$

$$B'_x = B_x$$

$$B'_y = \gamma_f \left[B_y + \frac{\mathrm{v}_f}{c^2} E_z\right]$$

$$B'_z = \gamma_f \left[B_z + \frac{\mathrm{v}_f}{c^2} E_y\right]$$

Unity of electric & magnetic phenomena – - note the constructs of linear superposition.

Demonstration of the 'STR $\leftrightarrow$ ED' educational software

'Motion of Charged Particles in Electromagnetic Fields and Special Theory of Relativity',

P. Chaitanya Das, G. Srinivasa Murty,
K. Satish Kumar, T A.Venkatesh
and P.C. Deshmukh

Resonance, Vol. 9, Number 7, 77-85 (2004)

You can download the software from this link:

http://www.physics.iitm.ac.in/~labs/amp/homepage/dasandmurthy.htm

Charge of the particle: -1.602e-19 C electron

Mass of the particle: 9.1e-31 Kg

Case 1

$E_x = 0.0$ $B_x = 0.1$ $v_x = 4.6e7$

$E_y = 0.0$ $B_y = 0.0$ $v_y = 2.65e8$

$E_z = 0.0$ $B_z = 0.0$ $v_z = 0.0$

$v_{rel} = 2\ e8$

Units: Electric field E in V/m, Magnetic field B in Wb/m^2 and velocity in m/s

Case 2

$E_x = 0.0$ $B_x = 0.05$ $v_x = 0.0$

$E_y = 0.0$ $B_y = 0.0$ $v_y = 0.0$

$E_z = 10e3$ $B_z = 0.0$ $v_z = 0.0$

$v_{rel} = 1.5e8$

Case 3

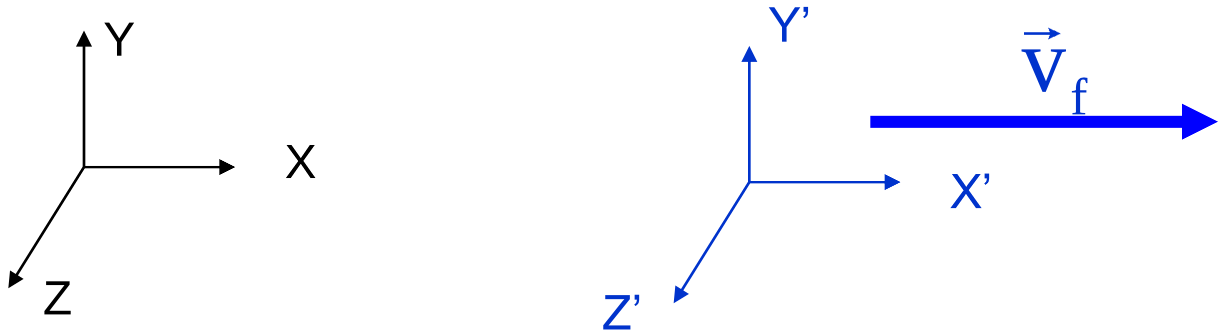
$E_x = 35e3$ $B_x=0.05$ $v_x = 0.0$

$E_y = 0.0$ $B_y=0.0$ $v_y = 2.65e7$

$E_z = 0.0$ $B_z=0.0$ $v_z = 0.0$

$v_{rel} = -2.5e8$

Examine trajectories of charged particles in EM fields, as observed by two observers both in their respective inertial frames. S' moves with respect to S at a constant velocity $\vec{V}_f$ along the X-direction.



Charge of the particle: -1.602e-19 C electron

Mass of the particle: 9.1e-31 Kg

Units: Electric field E in V/m, Magnetic field B in Wb/m^2 and velocity in m/s

Case 1

Ex = 0.0 Bx = 0.1 vx = 4.6e7

Ey = 0.0 By = 0.0 vy = 2.65e8

Ez = 0.0 Bz = 0.0 vz = 0.0

v_{rel} = 2 e8

Case 2

Ex = 0.0 Bx = 0.05 vx = 0.0

Ey = 0.0 By = 0.0 vy = 0.0

Ez = 10e3 Bz = 0.0 vz = 0.0

v_{rel} = 1.5e8

Case 3

Ex = 35e3 Bx=0.05 vx = 0.0

Ey = 0.0 By=0.0 vy = 2.65e7

Ez = 0.0 Bz=0.0 vz = 0.0

v_{rel} = -2.5e8

Electrodynamics in tensor notation

We provide a *very brief* introduction;

- once the structure of the equations is understood, ordinary matrix algebra is sufficient to interpret the relations.

Detailed work-out is left as rather straight-forward exercises.

EM field expressed as derivable from 'potential'

contravariant 4-vector

$$x^{\mu} = (x^0, \vec{x}) = (x^0, x^1, x^2, x^3)$$

$$= (ct, x, y, z)$$

$$g^{\mu\nu} = g_{\mu\nu}$$

$$x^{\mu} = g^{\mu\nu} x_{\nu}$$

$$x_{\mu} = g_{\mu\nu} x^{\nu}$$

covariant 4-vector

$$x_{\mu} = (x_0 = ct, -\vec{x})$$

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$x^{\mu} = g^{\mu\nu} x_{\nu}$$

$$\begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A^{\mu} = \left(\frac{\phi}{c}, \vec{A} \right)$$

EM Potentials

$$F^{\mu\nu} = \frac{\partial A^{\nu}}{\partial x_{\mu}} - \frac{\partial A^{\mu}}{\partial x_{\nu}}$$

EM Fields

$$Notation:$$

$$\partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}} \text{ and } \partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}$$

If a frame of reference $\bar{S}$ *moves w.r.t.* S *along* X*-axis at speed* $\left|\vec{v}_f\right|$, *the Lorentz transformation is:*

$$\bar{x} = \gamma_f \left(x - v_f t\right), \quad \bar{y} = y, \quad \bar{z} = z, \quad \bar{t} = \gamma_f \left(t - \frac{v_f}{c^2} x\right)$$

$$\begin{bmatrix} \bar{a}^0 \\ \bar{a}^1 \\ \bar{a}^2 \\ \bar{a}^3 \end{bmatrix} = \begin{bmatrix} \gamma_f & -\gamma_f \beta & 0 & 0 \\ -\gamma_f \beta & \gamma_f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a^0 \\ a^1 \\ a^2 \\ a^3 \end{bmatrix}$$

$$\bar{a}^{\mu} = \Lambda^{\mu}_{\nu} \; a^{\nu}$$

where $\gamma_f = 1 \Big/ \sqrt{1 - \frac{v_f^2}{c^2}}$

$$\beta = \frac{v_f}{c}$$

The EM field is conveniently expressed *as an antisymmetric tensor that has the following form:*

$$t^{\mu\nu} = \begin{bmatrix} t^{00}=0 & t^{01} & t^{02} & t^{03} \\ t^{10}=-t^{01} & t^{11}=0 & t^{12} & t^{13} \\ t^{20}=-t^{02} & t^{21}=-t^{12} & t^{22}=0 & t^{23} \\ t^{30}=-t^{03} & t^{31}=-t^{13} & t^{32}=-t^{23} & t^{33}=0 \end{bmatrix}$$

$$F^{\mu\nu} = \begin{bmatrix} F^{00}=0 & F^{01}=\dfrac{E_x}{c} & F^{02}=\dfrac{E_y}{c} & F^{03}=\dfrac{E_z}{c} \\ F^{10}=-F^{01} & F^{11}=0 & F^{12}=B_z & F^{13}=-B_y \\ F^{20}=-F^{02} & F^{21}=-F^{12} & F^{22}=0 & F^{23}=B_x \\ F^{30}=-F^{03} & F^{31}=-F^{13} & F^{32}=-F^{23} & F^{33}=0 \end{bmatrix}$$

$$\bar{a}_{\mu} = \Lambda^{\mu}_{\nu} a^{\nu} : \text{ Transformation rule for } 1^{\text{st}} \text{ rank tensor}$$

4-vector

$$\bar{t}^{\mu\nu} = \Lambda^{\mu}_{\lambda} \Lambda^{\nu}_{\sigma} t^{\lambda\sigma} :$$

Transformation rule for 2^{nd} rank tensor

$$\frac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \mu_0 J^{\mu} : \text{ Maxwell's equations}$$

where $J^{\mu} = (c\rho, J_x, J_y, J_z)$ is the Current Density 4-Vector.

$$\frac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \mu_0 J^{\mu} : \quad F^{00} = 0 \quad F^{01} = \frac{E_x}{c} \quad F^{02} = \frac{E_y}{c} \quad F^{03} = \frac{E_z}{c}$$

$J^{\mu} = (c\rho, J_x, J_y, J_z)$ is the Current Density 4-Vector.

For μ=0:

$$\frac{\partial F^{0\nu}}{\partial x^{\nu}} = \sum_{\nu=0}^{3} \frac{\partial F^{0\nu}}{\partial x^{\nu}} = \frac{\partial F^{00}}{\partial x^{0}} + \frac{\partial F^{01}}{\partial x^{1}} + \frac{\partial F^{02}}{\partial x^{2}} + \frac{\partial F^{03}}{\partial x^{3}} = \mu_0 J^{0}$$

$$i.e.\ \frac{1}{\mathrm{c}}\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right] = \mu_0 c \rho$$

$$\sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c$$

$$\rightarrow \vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$

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[6] Charles A. Brau 'Modern Problems in Classical Electrodynamics' Oxford Univ. Press, 2004.

[7] P. Chaitanya Das, G. Srinivasa Murty, K. Satish Kumar, T A. Venkatesh and P.C. Deshmukh
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We shall take a break here…….

Questions ?

Comments ?

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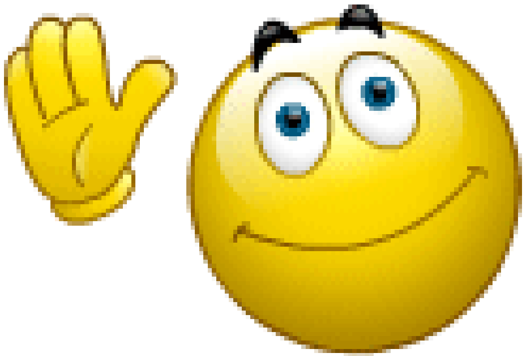

Next: L35

Unit 11 – CHAOTIC DYNAMICAL SYSTEMS