# STiCM

# Select / Special Topics in Classical Mechanics

P. C. Deshmukh

Department of Physics
Indian Institute of Technology Madras
Chennai 600036

pcd@physics.iitm.ac.in

School of Basic Sciences
Indian Institute of Technology Mandi
Mandi 175001

pcdeshmukh@iitmandi.ac.in

## STiCM Lecture 40:

## The Scope, and Limitations, of Classical Mechanics

# The Scope,
# and Limitations,
# of Classical Mechanics

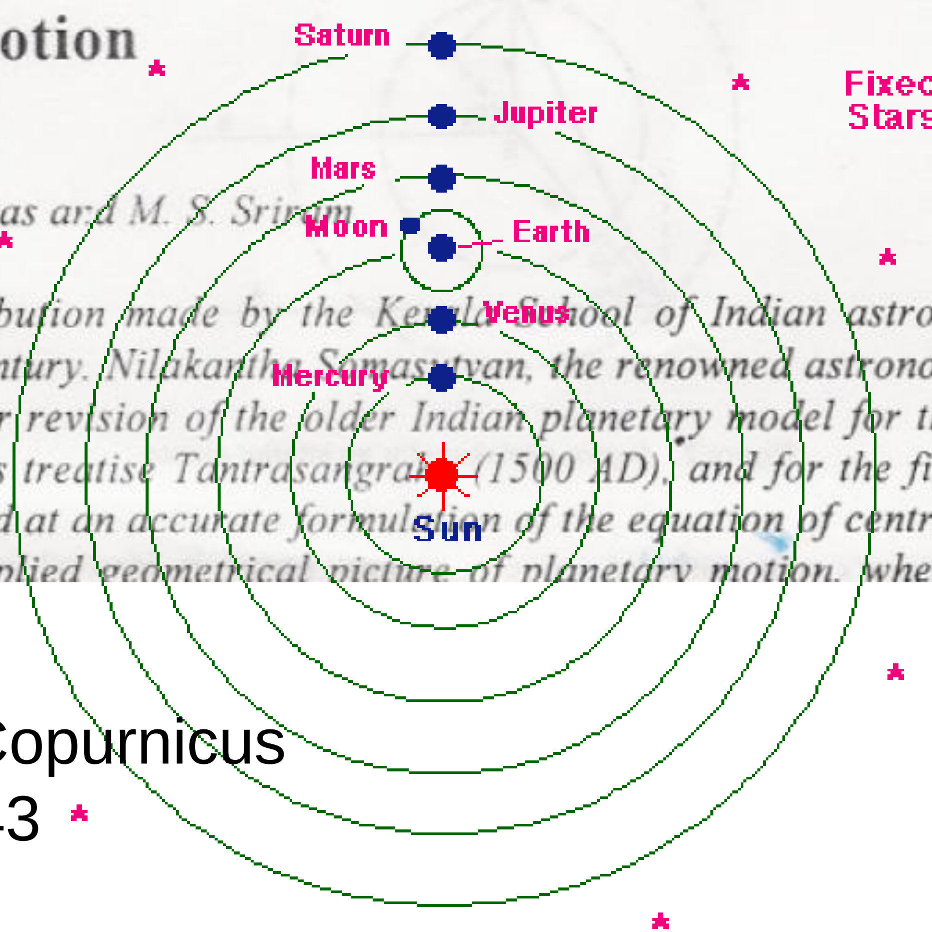
widely accepted, used for a long time;

traditional in style or idea

## Modification of the earlier Indian planetary theory by the Kerala astronomers (c. 1500 AD) and the implied heliocentric picture of planetary motion

*K. Ramasubramanian, M. D. Srinivas and M. S. Sriram*

*We report on a significant contribution made by the Kerala School of Indian astronomers to planetary theory in the fifteenth century. Nilakantha Somasutvan, the renowned astronomer of the Kerala School, carried out a major revision of the older Indian planetary model for the interior planets, Mercury and Venus, in his treatise Tantrasangraha (1500 AD), and for the first time in the history of astronomy, he arrived at an accurate formulation of the equation of centre for these planets. He also described the implied geometrical picture of planetary motion, where the five*





Nicolus Copurnicus
1473-1543

http://www.physics.iitm.ac.in/~labs/amp/kerala-astronomy.pdf

Albert Einstein: “We owe a lot to Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made.”

ARYABHATTA (in $5^{th}$ century) introduced new concepts: sphericity of the earth, rotation about its axis, revolution around the sun, explanation of eclipses….. estimated length of the year…..

BRAHMAGUPTA (in $7^{th}$ century) estimated the circumference of the earth to be around 5000 yoganas which in today’s units is close to the correct value as we know it now….

Central problem in ‘Mechanics’: How is the ‘mechanical state’ of a system described, and how does this ‘state’ evolve with time?

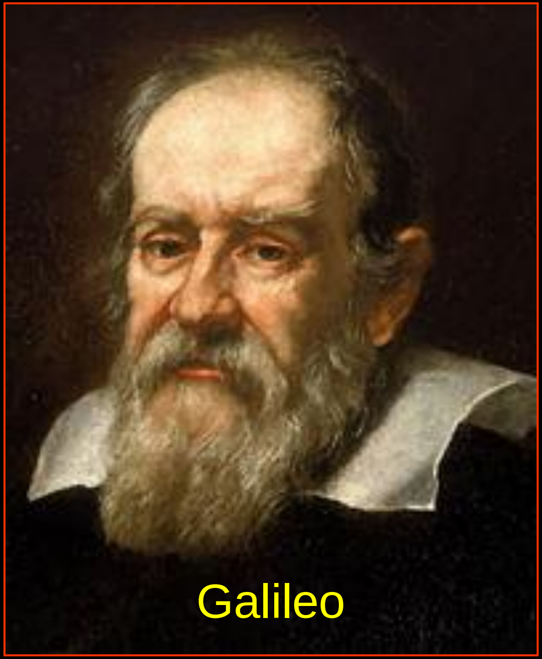
‘position’ and ‘velocity’: both needed to specify the mechanical state of a system?

The mechanical state of a system is characterized by its position and velocity, $(q,\dot{q})$

or, position and momentum, $(q,p)$

Or, equivalently by their well-defined functions:

$$L(q,\dot{q}) : Lagrangian$$
$$H(q,p) : \ Hamiltonian$$





$(q, \dot{q})$

$\vec{F} = m\vec{a}$

Linear Response.

Principle of causality.

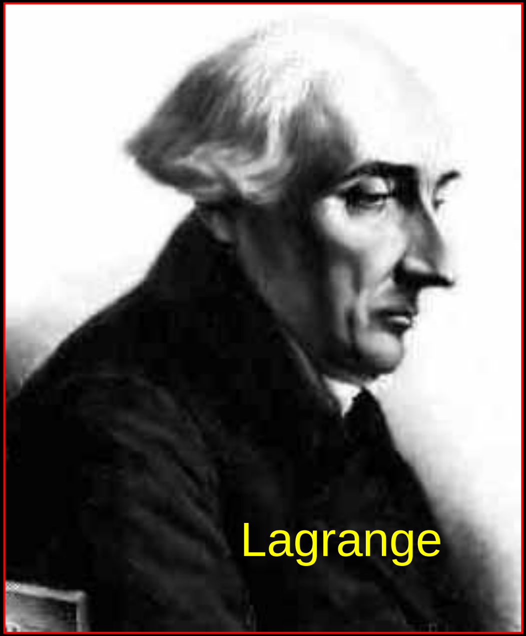
Principle of Variation

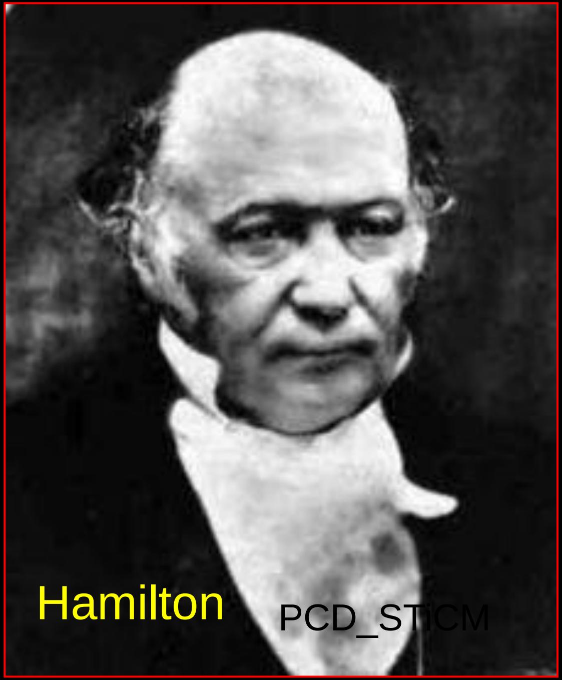
$L(q, \dot{q})$

$H(q, p)$

$$\int_{t_1}^{t_2} L(q, \dot{q}, t)\,dt$$

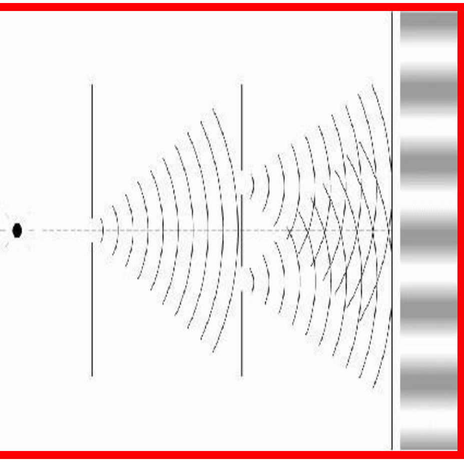
$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

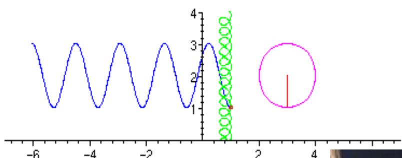





$$\dot{q} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial q_k}$$






Oscillations.

Small oscillations.

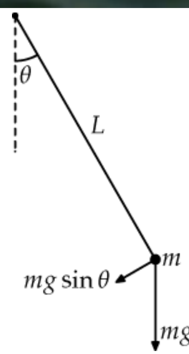
SHM.

Driven and damped oscillator.

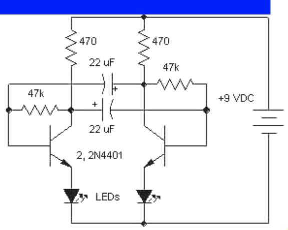
Resonance,

Quality factor.

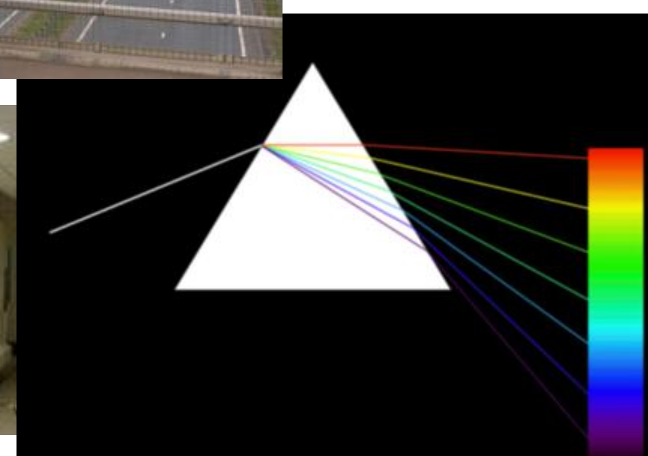
Waves.








**Flip-Flop Square Wave oscillator**



$$\frac{c}{v_\varphi} = n = \frac{\lambda_{vac}}{\lambda_\varphi}$$

$$n_r = n_r(\omega)$$

*My heart leaps up when I behold*
*A rainbow in the sky:*
*So was it when my life began;*
*So is it now I am a man;*
*So be it when I shall grow old,*
*Or let me die!…*
*- William Wordsworth*

R.I. of water for red is ~1.331

R.I. of water for blue is ~1.343

Questions:

1. Why is the red outside and blue inside?
2. Which part of this picture is the brightest, and why?

PCD_STiCM

Rainbow, seen from the 'Maid of the Mist' ride at the Niagara Falls, U.S.A., 18th July, 2009. - pcd

Are the conservation principles consequences of the laws of nature? Or, are the laws of nature the consequences of the symmetry principles that govern them?

Until Einstein's special theory of relativity,
it was believed that
conservation principles are the result of the laws of nature.

Since Einstein's work, however, physicists began to analyze the conservation principles as consequences of certain underlying symmetry considerations,
enabling the laws of nature to be revealed from this analysis.

Instead of introducing Newton's III law as a *fundamental principle,*

we *deduced* it (in Unit 1) from symmetry / invariance.

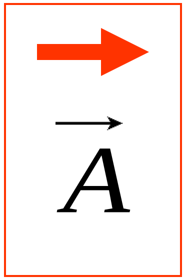
This approach places SYMMETRY *ahead of* LAWS OF NATURE.
It is this approach that is of greatest value to contemporary physics. This approach has its origins in the works of
Albert Einstein, Emmily Noether and Eugene Wigner.

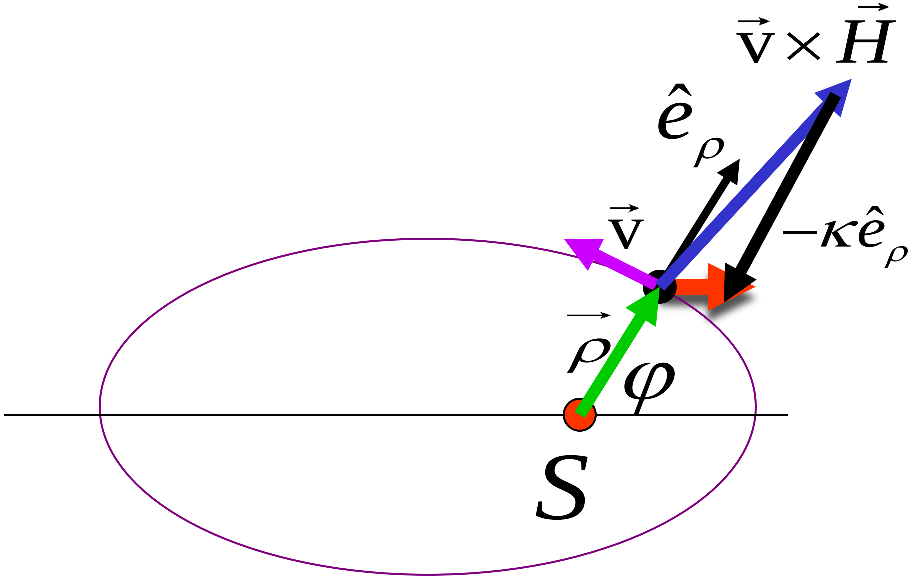

(1879 – 1955)

(1882 – 1935)

(1902 – 1995)

Laplace Runge Lenz Vector is constant for a strict $1/r$ potential.



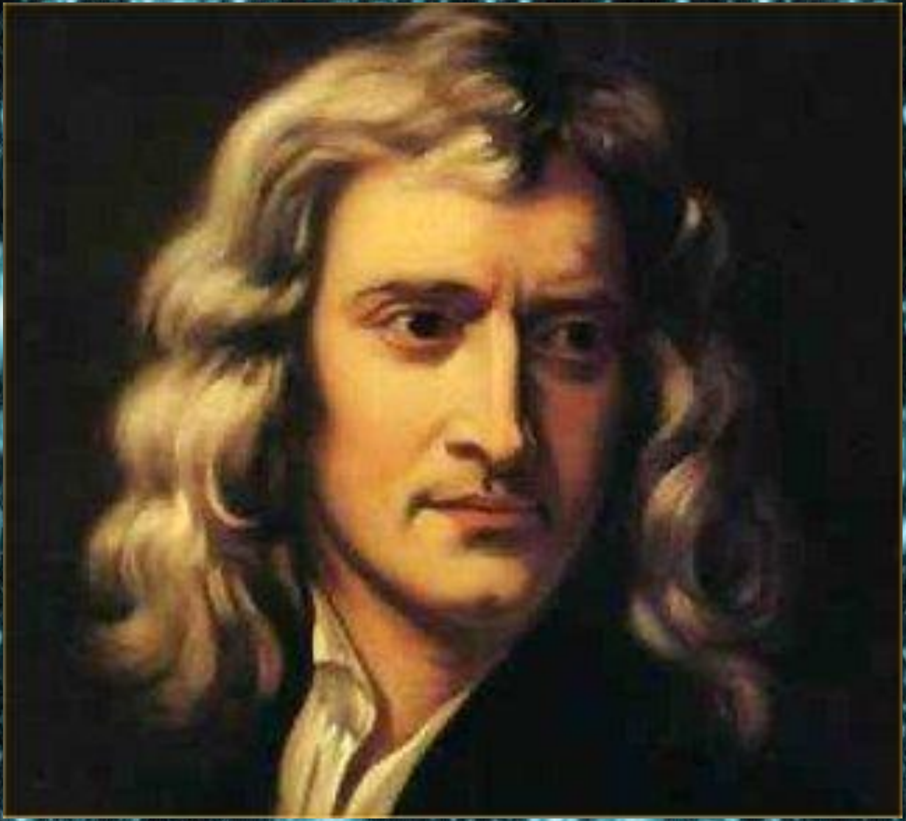



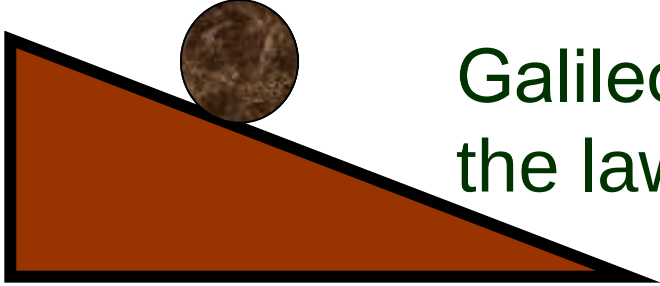

$$\vec{A} = \left(\vec{\mathrm{v}} \times \vec{H}\right) - \kappa \hat{e}_\rho$$

The $\odot\vec{H}$ (specific) angular momentum vector is *out* of the plane of this figure, toward us.

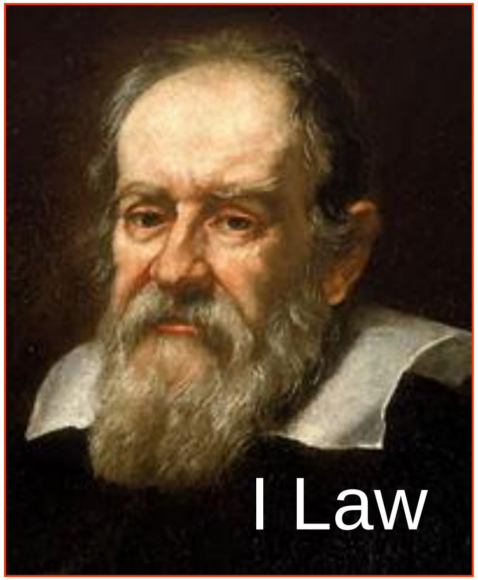
**Just where is the inertial frame?**

**Newton envisaged the inertial frame to be located in *deep space, amidst distant stars*.**

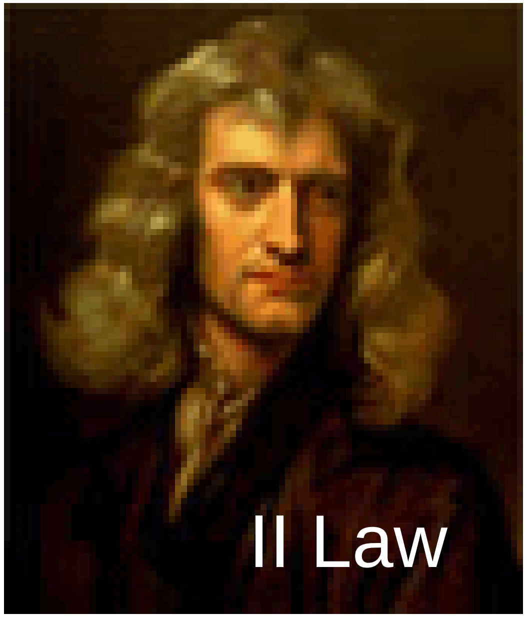




## Galileo's experiments that led him to the law of inertia.




Galileo Galilee
1564 - 1642

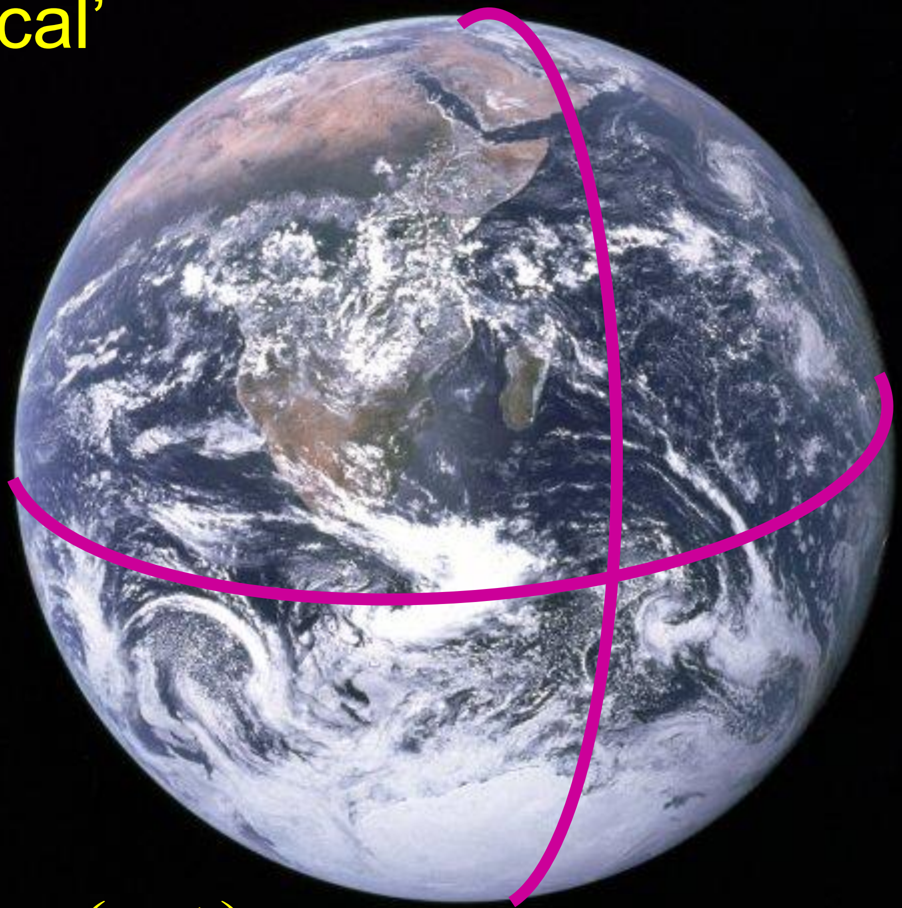



$\vec{F} = m\vec{a}$ Linear Response.

*Effect* is proportional to the *Cause*

Principle of causality.

# 3 ‘definitions’ of the ‘vertical’



$$m\left(\frac{d^2}{dt^2}\right)_R \vec{r} = m\left(\frac{d^2}{dt^2}\right)_I \vec{r} - m\left(\frac{d\vec{\omega}}{dt}\right)_R \times \vec{r} - 2m\vec{\omega} \times \left(\frac{d}{dt}\right)_R \vec{r}$$

$$- \mathrm{m}\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

$$\begin{pmatrix} -x \\ y \\ z \end{pmatrix} \leftarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

REFLECTION

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ y \\ z \end{pmatrix}$$

Left ⟷ Right

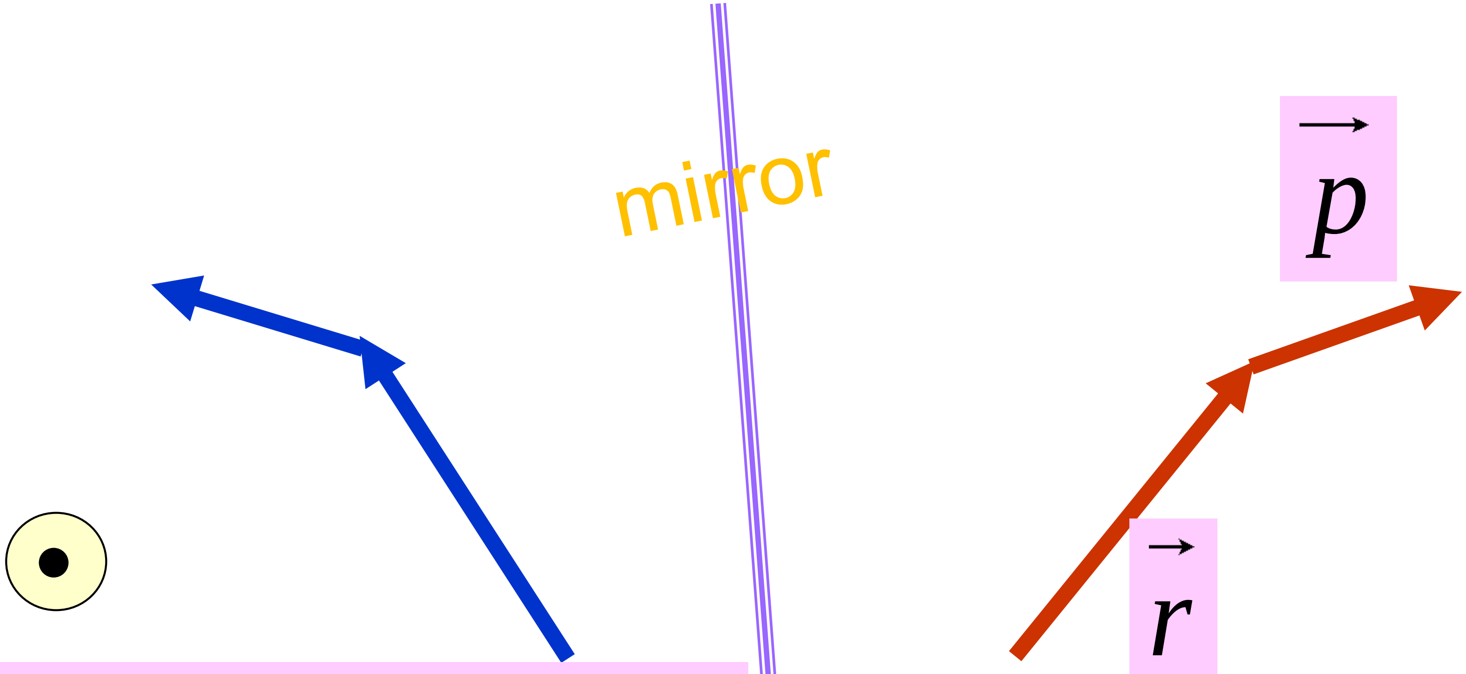
Top ⟷ Bottom ?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ -z \end{pmatrix}$$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{l} = \vec{r} \times \vec{p}$$

angular momentum



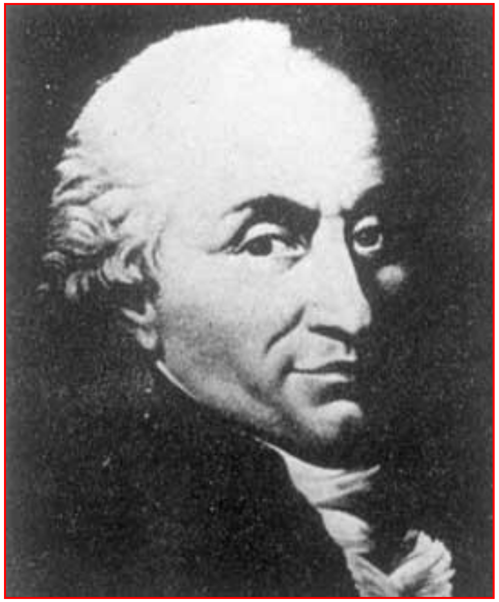

$$\vec{l}_{right-hand-cross-product} = \vec{r}_{image} \times \vec{p}_{image}$$

$$\otimes \quad \vec{l} = \vec{r} \times \vec{p}$$

# Galilean Relativity

$$\frac{d\vec{r}}{dt} - \vec{u}_c = \frac{d\vec{r}\,'}{dt}$$

# Speed of light ?



Charles Coulomb
1736-1806

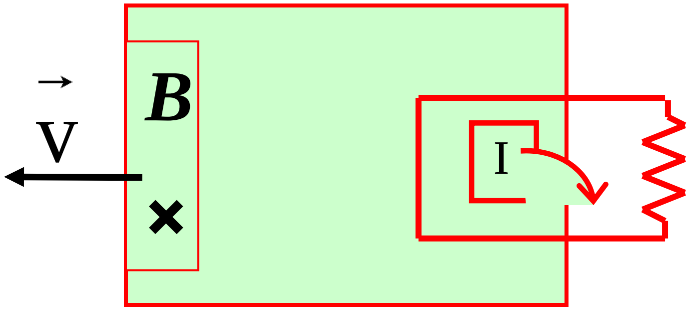
Andre Marie Ampere
1775-1836

Michael Faraday
1791-1867

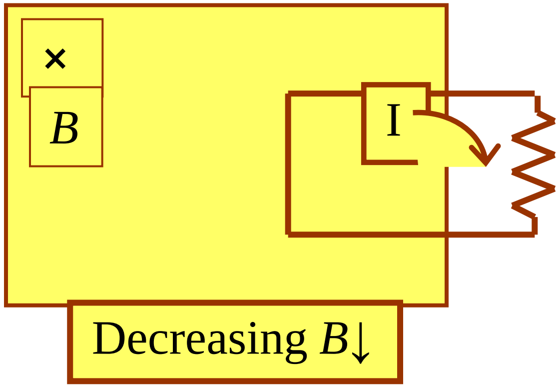
….. other developments in Physics ….

Carl Freidrich Gauss
1777-1855

# Faraday's experiments




**Current: identical!**



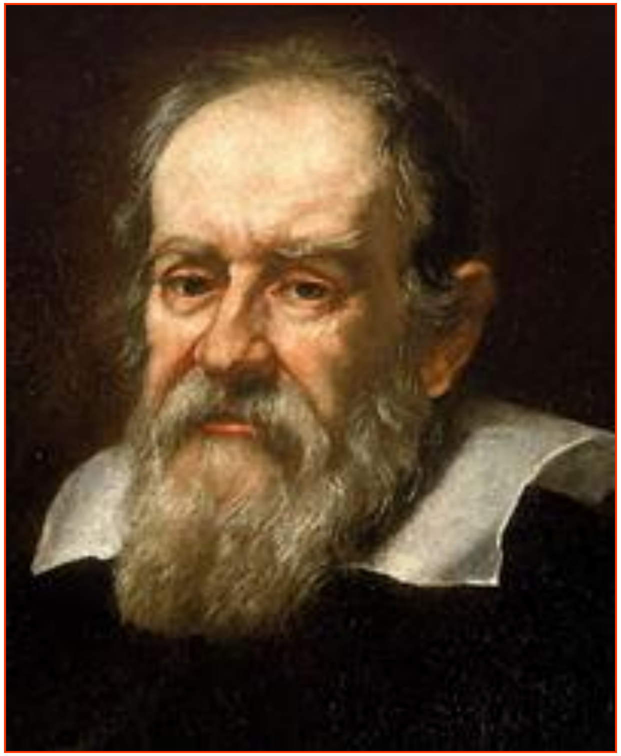

Strength of *B* *decreased*.
*Nothing* is moving,
but still, current seen!!!

$$I \propto \frac{dB}{dt}$$

Einstein:

Special Theory of Relativity

# Galilean & Lorentz Transformations.
# Special Theory of Relativity.



**Galileo Galilei**
**1564 - 1642**

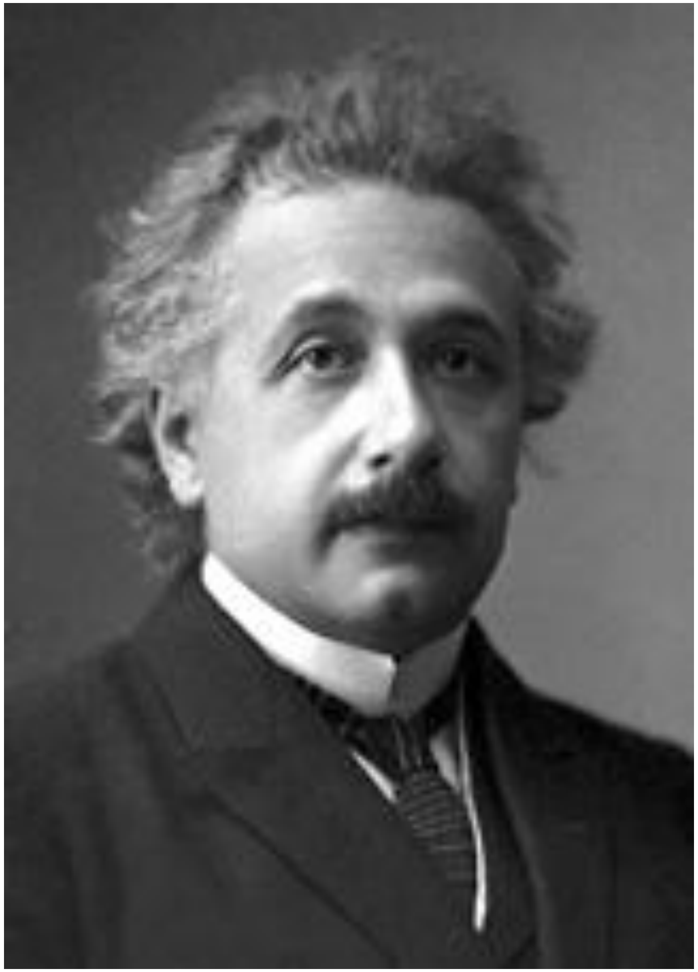
**Hendrik Antoon Lorentz**
1853-1928

**Smoking is injurious to health!**

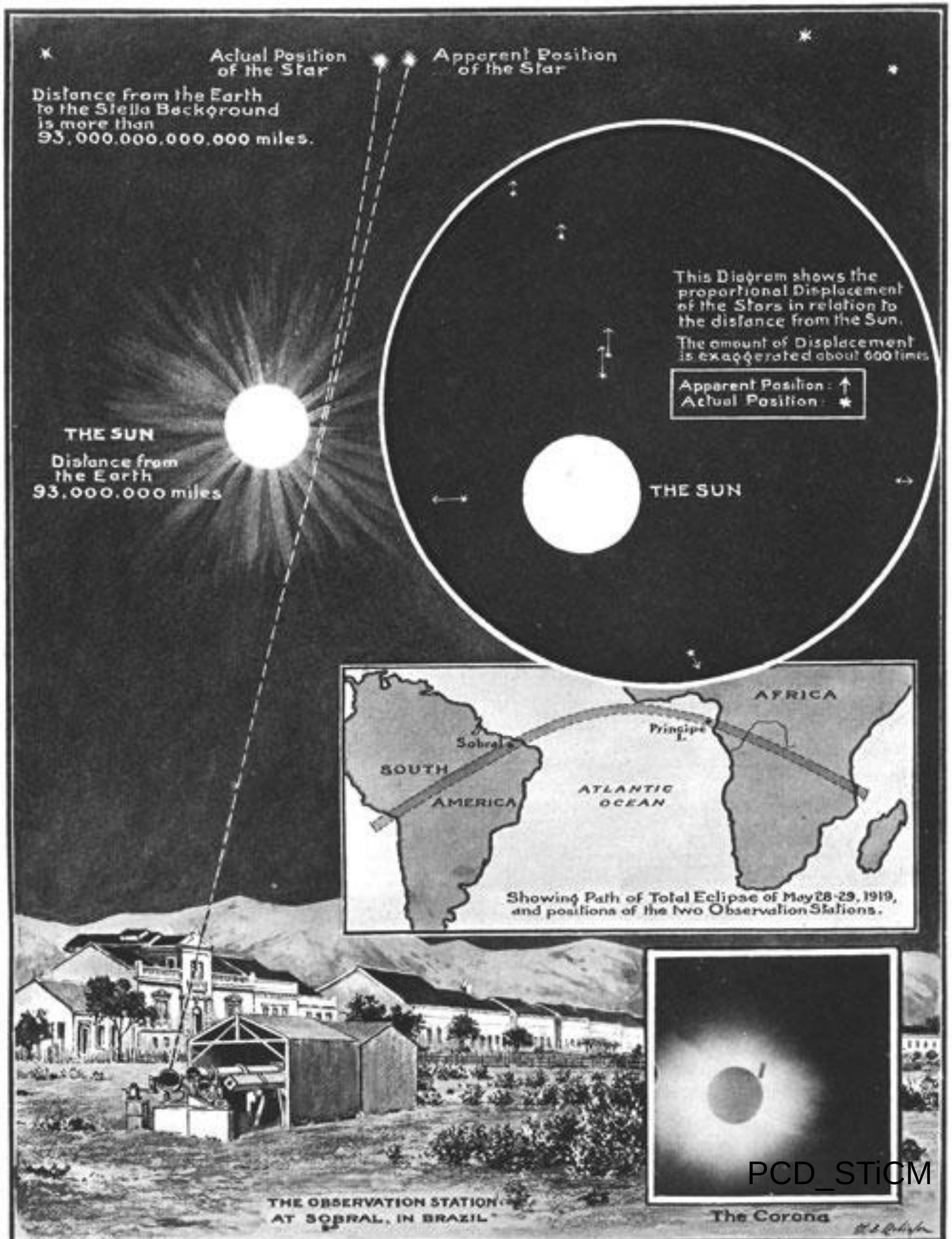
**Albert Einstein**
**1879-1955**

# Classical Electrodynamics
# and
# the Special Theory of Relativity.

Maxwell



Einstein



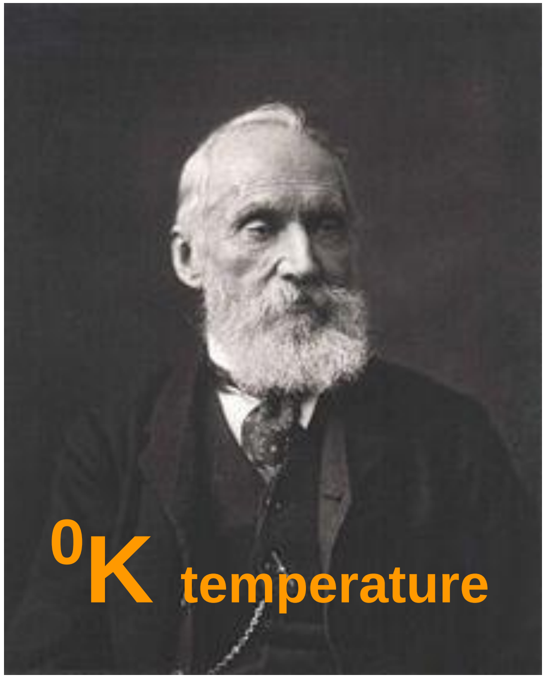

Arthur Eddington

http://denisdutton.com/einstein_eddington.htm

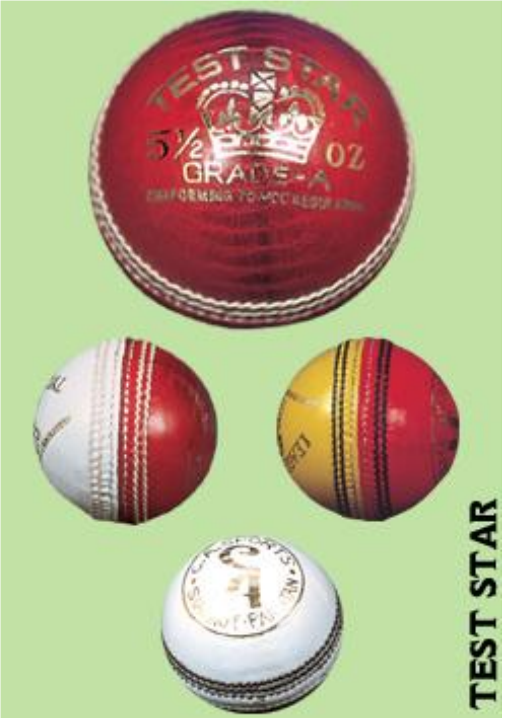
Observation station at
SOBRAL, BRAZIL

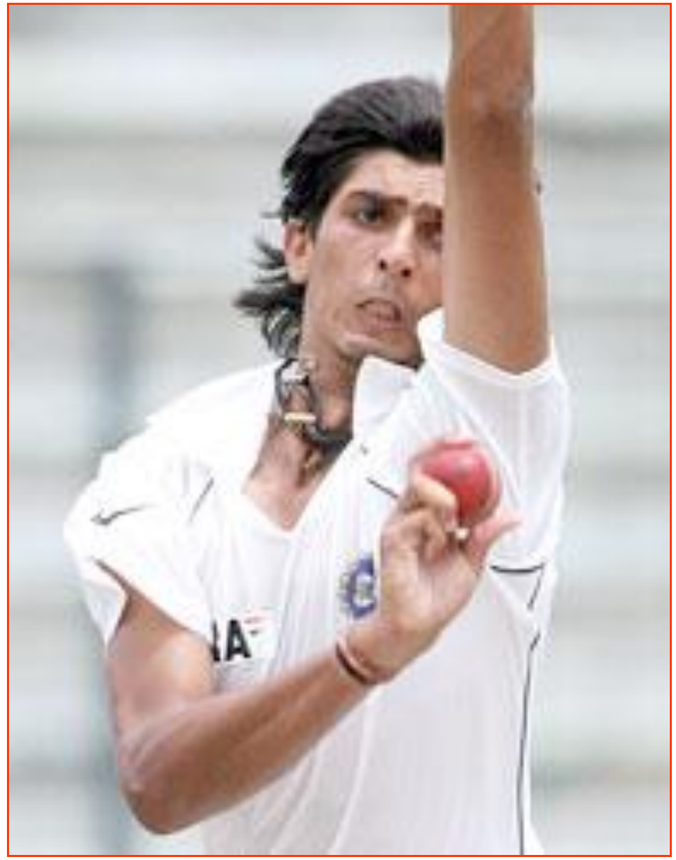
Johann
Carl
Friedrich
**Gauss**
**1777 - 1855**

**George Gabriel Stokes (1819–1903)**




**William Thomson, 1st Baron Kelvin (1824-1907)**

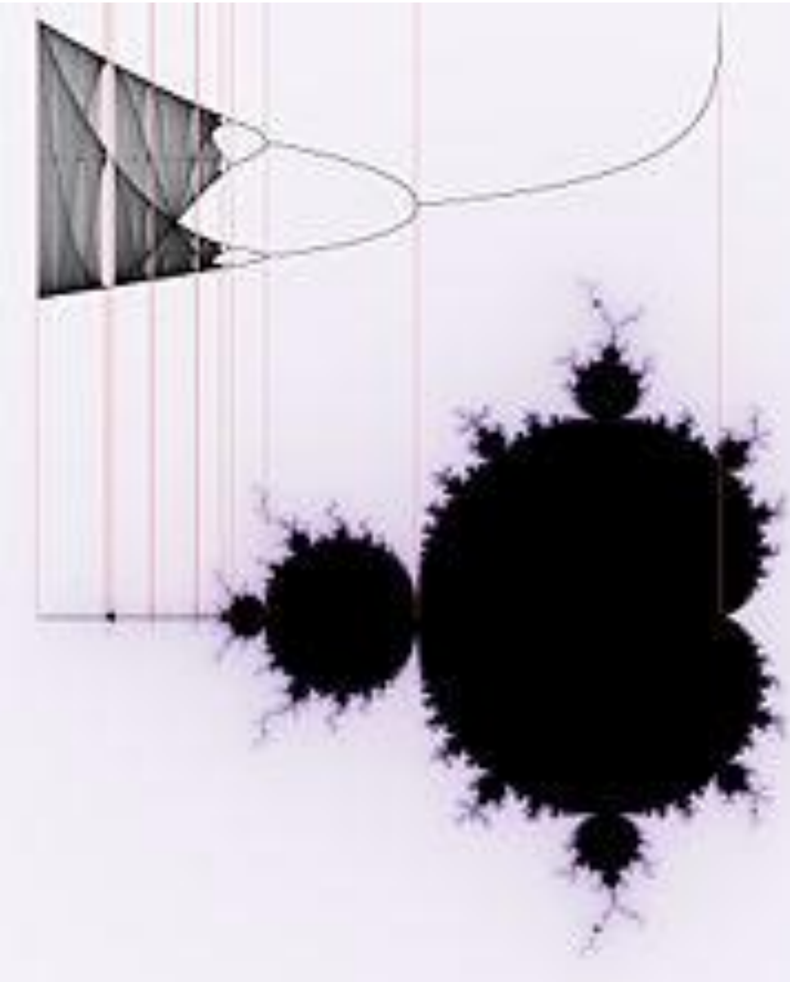


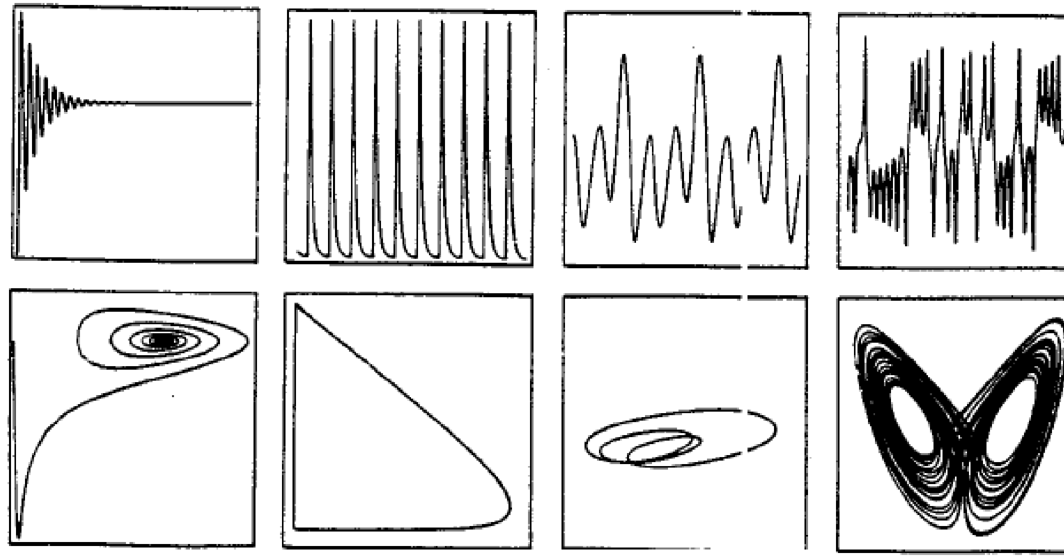



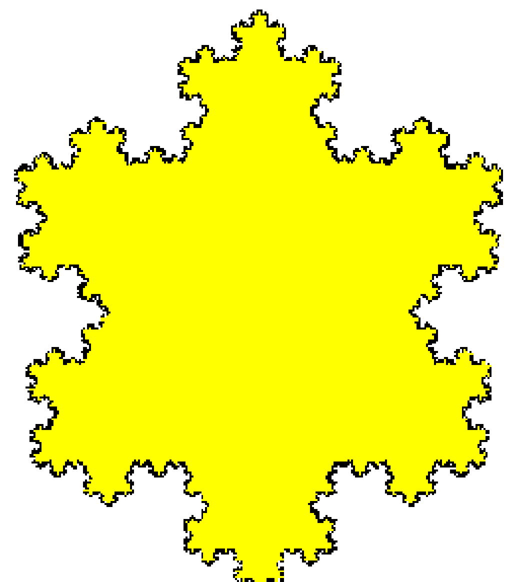
Ishant Sharma
Swing Bowling

Difference between the rough and shiny surface of a white ball is much more, and hence swings more!

# 'Chaos', bifurcation, strange attractors, fractals, self-similarity, Mandelbrot sets.







Principal premise of "classical mechanics"

mechanical system is characterized by position and velocity/momentum,

simultaneously and accurately.

Central problem in ‘Mechanics’:

How is the ‘mechanical state’ of a system described, and how does this ‘state’ evolve with time?

- Formulations due to Galileo/Newton,
- Lagrange and Hamilton.

(q,p) : How do we get these?

MEASUREMENT !

New approach required !

‘New approach’ is not required on account of the Heisenberg principle!

Rather,

the measurements of *q* and *p* are not compatible….

It is rather the HEISENBERG PRINCIPLE that RESULTS as an expression of this incompatibility!

….. so how could one describe the mechanical state of a system by (*q*,*p*) ?

$| \quad \rangle$ Mechanical State:
State vectors in Hilbert Space

$\Big|$ Characterize? Labels?
"Good" quantum numbers/labels $\Big\rangle$

$|$ Measurment: C.S.C.O. $\rangle$

Complete Set of Commuting Operators
Complete Set of Compatible Observables

$$i\hbar \frac{\partial}{\partial t} | \quad \rangle = H | \quad \rangle$$

Schrödinger Equation

Evolution of the Mechanical State of the system

Quantization! state vector : $| \quad \rangle$

dynamical variables: operators

$A| \qquad\qquad \rangle \rightarrow | \quad \text{label ?} \quad \rangle$

$| \text{ new vector } \rangle \;\; \alpha \;\; | \text{ old vector } \rangle$

eigenvalue equation

$A| \qquad\qquad \rangle = a\,| \text{ label ? } \rangle$

label

$A| \quad \rangle = a\,| \; a \; \rangle$

$A| \; a \; \rangle = a\,| \; a \; \rangle$

eigenvalue equation

$| \text{ label(s)?} \rangle$

$A| \quad \rangle = a| a \rangle$ (label)

$A| a \rangle = a| a \rangle$

eigenvalue equation

**Measurement: system 'collapses' into its eigenstate**

What *else* can be measured ?

C.S.C.O.

Complete Set of Compatible Observables

Complete Set of Commuting Operators

$B| a,b \rangle = b| a,b \rangle$

$[A,B]_- = AB - BA$

$CSCO: \{A,B,C,......\}$

Erwin Schrödinger

**1887 - 1961**

Werner Heisenberg

**1901 - 1976**

…… Any questions ?

pcd@physics.iitm.ac.in
pcdeshmukh@iitmandi.ac.in